The Mole and Avogadro's Number

The name mole (German *Mol*) is attributed to Wilhelm Ostwald who introduced the concept in the year 1902. It is an abbreviation for molecule (German *Molekül*), which is in turn derived from Latin *moles* "mass, massive structure". (From the Wikipedia article on the mole unit.)

A good site that introduces the mole concept and includes sample calculations and practice problems can be found here, from John Park's excellent ChemTeam site.

For some interesting background on Avogadro's number, see here. By T.A. Furtsch, Tennessee Technological University, Cookeville, TN.

Don't miss the interview with Count Amedeo Avogadro and his wife the Countess Felicita, located here (thanks to Kory Tonouchi, Roosevelt High, Honolulu, HI).

Avogadro's 1811 publication, "Essay on a Manner of Determining the Relative Masses of the Elementary Molecules of Bodies, and the Proportions in Which They Enter into These Compounds", may be found here (thanks to Carmen Giunta, Le Moyne College, Syracuse, NY).

A fun "mole" page to visit is here. It is a collection of student projects from Carondelet High School. Back in '98 they had a mole mystery described here.

Did you know we have a **mole day**? Find out about it here. And for some corny mole jokes, don't miss the Dictionary of Mole Day Terms & Jokes here!

Introduction

A **mole** of objects contains **Avogadro's number**, **6.022×10^{23}**, objects. Just as a dozen apples is 12 apples, a mole of apples is 6.022×10^{23} apples. A mole of iron atoms is 6.022×10^{23} iron atoms. A mole of water molecules is 6.022×10^{23} water molecules.

The NIST 2007 value of Avogadro's number is $6.022\ 141\ 79 \pm 0.000\ 000\ 30 \times 10^{23}$ mol^{-1}. For most calculations, a rounded value of 6.022×10^{23} (four significant figures) is satisfactory.

This is an incredibly large number. A mole of say, grapefruit, stacked together, would occupy the volume of the entire planet earth! And yet a mole of water molecules is in only about 18 milliliters of water. Since atoms and molecules are so small, there are gigantic numbers of them in ordinary gram quantities of substances such as what we weigh and use in the chemistry lab.

AMUs, Grams, and Moles

The value of Avogadro's number is actually chosen arbitrarily, based on the definition of the atomic mass unit, amu or u. **By definition, a single carbon-12 atom weights 12 amu exactly**. Therefore, one amu is one-twelfth the mass of a single carbon-12 atom.

Now, how many carbon-12 atoms would weigh exactly 12 grams?

From experiment, the actual mass of a single carbon-12 atom in grams has been determined. For example, using the method of mass spectrometry, the mass of a single carbon-12 atom has been measured to be about 1.993 X 10^{-23} g. From this we can calculate the number of carbon-12 atoms in 12 grams of carbon-12:

$$\frac{12\text{ g}}{1} \quad \text{X} \quad \frac{1\text{ carbon-12 atom}}{1.993 \text{ X } 10^{-23}\text{ g}} \quad = \quad 6.021 \text{ X } 10^{23}\text{ carbon-12 atoms}$$

(from Clugston and Flemming, *Advanced Chemistry*)

This is the basis of Avogadro's number. Better experimental methods have yielded the more accurate value of Avogadro's number we have today.

By definition, 12 grams of carbon-12 contain one mole, or Avogadro's number of, carbon-12 atoms.

We can also relate the two mass scales, grams and amu, as follows:

$$\frac{6.022 \text{ X } 10^{23}\text{ atoms of carbon-12}}{12\text{ g}} \quad \text{X} \quad \frac{12\text{ amu}}{1\text{ atom of carbon-12}} \quad = \quad \mathbf{6.022 \text{ X } 10^{23}\text{ amu / g}}.$$

That is, **1 g = 6.022 X 10^{23} amu**.

The average weight of a carbon atom found in nature is a little more than 12 amu, actually 12.0107 amu, because there is a small amount of heavier carbon-13 atoms present.

We can calculate the average weight of one mole of carbon atoms as follows:

$$\frac{1\text{ mole C}}{1} \quad \text{X} \quad \frac{6.022 \text{ X } 10^{23}\text{ C atoms}}{\text{mole C}} \quad \text{X} \quad \frac{12.0107\text{ amu}}{\text{C atom}} \quad \text{X} \quad \frac{1\text{ g}}{6.022 \text{ X } 10^{23}\text{ amu}} \quad = \quad 12.0107\text{ grams}$$

As we have with carbon-12, the weight of a single carbon atom, on average, is 12.0107 amu, and one mole of carbon atoms weighs 12.0107 grams, the same number.

What about other elements, does the same relationship hold? Indeed yes, the proportions of the weight of a single atom of carbon compared to a single atom of, say, iron is the same, whether we are comparing the weights of single atoms, one dozen atoms, or one mole of atoms. For example, it is known from experiment that, on average, an iron atom is 4.6496 times more massive than a carbon atom, which is 55.845 amu per iron atom. By proportion, one dozen iron atoms will be 4.6496 times more massive than one dozen carbon atoms. Likewise, one mole of iron atoms will be 4.6496 times more massive than one mole of carbon atoms, which is 55.845 grams per mole of iron atoms.

We therefore have two types of units for atomic weights, molecular weights, and formula weights:

1) **amu per single atom, molecule, or formula unit**
A single iron atom weighs 55.845 amu, or 55.845 amu / atom
A single water molecule weighs 18.0153 amu, or 18.0153 amu / molecule
A single "unit" or "formula unit" of NaCl weighs 58.443 amu, or 58.443 amu / formula unit

2) **grams per mole** of atoms, molecules, or formula units
One mole of iron atoms weighs 55.845 g, or 55.845 g / mol
One mole of H_2O molecules weighs 18.0153 g, or 18.0153 g / mol
One mole of NaCl formula units weighs 58.443 g, or 58.443 g / mol

In general, we can refer to the weight of one mole of a pure substance as its **molar mass**. If the substance is an element such as iron, the molar mass is the atomic weight of the substance. If the substance is molecular, like H_2O, we can call the molar mass the molecular weight of the substance. If the substance is ionic, rather than molecular, we can refer to the molar mass as the "formula weight" of the substance.

Since most chemical calculations involve converting between grams and moles, you should get into the habit of using **g/mol** units (and showing them in your work!). You will only occasionally need to use atomic mass units in calculations.

Important Formulas

The important calculation formulas to memorize are

moles = grams / molar mass

and rearranging,

grams = moles X molar mass

We use these two formulas more than any others in chemistry, because so often we are required to convert from grams to moles and moles to grams in chemical calculations.

Sample Calculations

Example 1. How many moles of iron are in 50.0 g of iron?

a) Plug in to the mole formula, moles $= \dfrac{50.0\text{ g}}{55.845\text{ g/mol}} =$ **0.895 mol**

b) Or, you can do the calculation like a conversion problem:

$$\frac{50.0\text{ g}}{1} \times \frac{1\text{ mol}}{55.845\text{ g}} = \mathbf{0.895\ mol}$$

Example 2. How many moles of carbon atoms and oxygen atoms are in 0.250 mol of CO_2 ?

Here we just have to look at the formula. Since one molecule of CO_2 contains one carbon atom, one mole of CO_2 molecules will contain one mole of carbon atoms. If we have 0.250 mol of CO_2, there will be **0.250 mol of carbon atoms** present.

$$\frac{0.250\text{ mol of }CO_2}{1} \times \frac{1\text{ mol of C}}{1\text{ mol of }CO_2} = \mathbf{0.250\ mol\ of\ C\ atoms}$$

Since there are two oxygen atoms in one CO_2 molecule, there are 2 X 0.250 mol = **0.500 mol of oxygen atoms** present in this amount:

$$\frac{0.250\text{ mol of }CO_2}{1} \times \frac{2\text{ mol of O}}{1\text{ mol of }CO_2} = \mathbf{0.500\ mol\ of\ O\ atoms}$$

Example 3. How many moles of copper atoms are in a copper penny weighing 3.10 g ? How many copper atoms are in the penny?

moles = 3.10 g / 63.546 g/mol = **0.0488 mol**

number of Cu atoms $= \dfrac{0.0488\text{ mol}}{1} \times \dfrac{6.022 \times 10^{23}\text{ atoms}}{1\text{ mol}} =$ **2.94 X 10^{22} atoms**

Example 4. A chemistry class has 15 men and 17 women in it. How many moles of students are in the class?

OK, that's not something we need to know every day, but can you do the calculation? Of course!

$$\frac{32\text{ students}}{1} \times \frac{1\text{ mol}}{6.022 \times 10^{23}\text{ students}} = \mathbf{5.314 \times 10^{-23}\ mol}$$

Example 5. How much do 1.00×10^{12} (a trillion) gold atoms weigh in grams?

$$\frac{1.00 \times 10^{12} \text{ atoms}}{1} \quad \times \quad \frac{1 \text{ mol}}{6.022 \times 10^{23} \text{ atoms}} \quad \times \quad \frac{196.967 \text{ g}}{\text{mol}} \quad = \quad \mathbf{3.27 \times 10^{-10} \text{ g}}$$

Example 6. The mass of the earth is 5.98×10^{24} kg. The mass of a baseball is 145 g. How many baseballs would have a mass equal to the mass of the earth? What is this number in moles?

$$\frac{5.98 \times 10^{24} \text{ kg}}{1} \quad \times \quad \frac{1 \text{ baseball}}{0.145 \text{ kg}} \quad = \quad \mathbf{4.12 \times 10^{25} \text{ baseballs}}$$

$$\frac{4.12 \times 10^{25} \text{ baseballs}}{1} \quad \times \quad \frac{1 \text{ mol}}{6.022 \times 10^{23} \text{ baseballs}} \quad = \quad \mathbf{68.5 \text{ moles}}$$

Example 7. A 6.00 M solution of HCl contains 6.00 mol of HCl per liter of solution. How many moles of HCl are in 125 mL of this solution?

$$\frac{0.125 \text{ L}}{1} \quad \times \quad \frac{6.00 \text{ mol}}{1 \text{ L}} \quad = \quad \mathbf{0.750 \text{ mol}}$$

Example 8. What is the mass in grams of 0.100 mol of glucose, $C_6H_{12}O_6$?

grams = moles X molar mass = 0.100 mol X 180.156 g/mol = **18.0 g**

Example 9. A sample of a certain hydrocarbon contains 0.090 mol of carbon and 0.36 mol of hydrogen. What is the empirical formula of the compound?

The subscripts in formulas are the relative numbers of each element in the compound. For example, one mole of $C_6H_{12}O_6$ contains 6 moles of carbon atoms, 12 moles of hydrogen atoms, and 6 moles of oxygen atoms.

The empirical formula is the simplest formula with whole-number subscripts. The empirical formula of the hydrocarbon is "$C_{0.090}H_{0.36}$". To convert to whole-number subscripts, divide each number of moles by the smallest value, which is 0.090. That gives C_1H_4 or $\mathbf{CH_4}$ as the empirical formula.

Example 10. Analysis of a compound showed it to be 40.0% C, 6.7% H, and 53.3% O by mass. What is the empirical formula of the compound?

First, for simplicity, assume you have 100 g of compound. Then, it contains 40.0 g of carbon, 6.7 g of hydrogen, and 53.3 g of oxygen. That's convenient! Now convert each amount to moles in order to get an empirical formula as in Example 9:

moles of C = 40.0 g / 12.0107 g/mol = 3.33 mol
moles of H = 6.7 g / 1.00794 g/mol = 6.7 mol
moles of O = 53.3 g / 15.9994 g/mol = 3.33 mol

This gives "$C_{3.33}H_{6.7}O_{3.33}$" which we can divide through by 3.33 to obtain the empirical formula $\mathbf{CH_2O}$.

Exercises

Some more practice problems can be found here (choose "Chapter 24 The Mole" when you get there.) You may have to cover part of your screen because the answers are right by the questions! (No peeking until you've first worked on the problem yourself!) These problems are from Don Voyce's chemistry site at the University of Hawai'i Kapi`olani Community College.

1. How many moles of the substance are in the following amounts?

a) 2.50 g of lead
b) 5.00 g of KBr
c) 3.75 g of $Ca_3(PO_4)_2$
d) 10.00 g of $CH_3CH_2CH_2CH_2CH_2CH_3$
e) 15.00 g of $CuSO_4 \bullet 5\ H_2O$
f) a cube of copper metal with an edge length of 1.00 in. The density of copper is 8.92 g/cm^3.

2. How many atoms, molecules, or formula units of the substance are in the following amounts?

a) 3.50 mol of O_2
b) 2.75 g of S_8
c) 5.50 g of $C_{12}H_{22}O_{11}$
d) 5.00 g of $Al(NO_3)_3$
e) 5.00 mL of ethanol, CH_3CH_2OH, density = 0.790 g/mL
f) a sphere of chromium metal, 0.343 mm in diameter. The density of chromium is 7.20 g/cm^3.

3. How many moles of phosphate ions are in 1.00 lb of $Ca_3(PO_4)_2$?
1 lb = 453.59237 g exactly

4. A student performed an experiment to determine the molecular weight of a gaseous compound. Using the ideal gas law PV = nRT, and knowing the pressure, temperature, and volume of the vapor, the student calculated the number of moles of gas, n, to be 0.0443 mol. The weight of the gas was 1.42 g. What is the molecular weight of this compound?

5. The stratosphere contains approximately 3 billion kilograms of ozone, O_3. How many moles of ozone is this?

6. At the end of the 18th century, a kilogram was defined as the mass of exactly one cubic decimeter of water at the temperature where the density of water is at a maximum (now known to be 0.999972 g/cm^3 at 3.98 °C and 1 atm pressure; see http://www.sizes.com/units/kilogram.htm). How many moles of water is this?

7. How many moles of water are in a snowflake weighing 5.0×10^{-5} g ? How many of these snowflakes would it take to have 1.0 kg of ice?

8. What is the percent by mass of carbon in sucrose, $C_{12}H_{22}O_{11}$?

9. What is the empirical formula of a compound that is 21.2% nitrogen, 6.1% hydrogen, 24.3% sulfur, and 48.4% oxygen by mass?

10. The mass of the earth is 5.98×10^{24} kg. How many moles of gold would have this mass? How many moles of aluminum would have this mass?

Answers

1. a) 0.0121 mol of Pb
b) 0.0420 mol of KBr
c) 0.0121 mol of $Ca_3(PO_4)_2$
d) 0.116 mol of C_6H_{14}
e) 0.0601 mol of $CuSO_4 \cdot 5\ H_2O$
f) 2.30 mol of Cu

2. a) 2.11×10^{24} O_2 molecules
b) 6.46×10^{21} S_8 molecules
c) 9.68×10^{21} $C_{12}H_{22}O_{11}$ molecules
d) 1.41×10^{22} $Al(NO_3)_3$ formula units
e) 5.16×10^{22} CH_3CH_2OH molecules
f) 4.76×10^{21} Cr atoms

3. 2.92 mol of PO_4^{3-} ions

4. 32.1 g/mol

5. 6×10^{10} mol of O_3

6. 55.5068 mol of H_2O using a molecular weight of 18.0153 g/mol

7. 2.8×10^{-6} mol of H_2O, 2×10^{7} or 20 million snowflakes

8. 42.1% carbon by mass

9. $N_2H_8SO_4$ (ammonium sulfate)

10. 3.04×10^{25} moles of Au, 2.22×10^{26} mol of Al