Name\_\_\_\_\_

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Divide using synthetic division.

1) 
$$\frac{x^5 + x^2 + 2}{x - 2}$$

Use synthetic division and the Remainder Theorem to find the indicated function value.

2)  $f(x) = 4x^4 + 3x^3 + 5x^2 - 6x + 52$ ; f(2)

Use the Rational Zero Theorem to list all possible rational zeros for the given function.

3)  $f(x) = -4x^4 + 4x^2 - 3x + 6$ 

Solve the polynomial equation. In order to obtain the first root, use synthetic division to test the possible rational roots. 4)  $x^3 + 2x^2 - 6x + 8 = 0$ 

Graph the rational function.

5) 
$$f(x) = \frac{x-2}{x^2 - x - 20}$$

Verify the identity

6) 
$$1 - \frac{\cos^2 x}{1 + \sin x} = \sin x$$

Find the exact value by using a sum or difference identity. 7)  $\sin 255^{\circ}$ 

#### Find the exact value of the expression.

8)  $\sin 265^\circ \cos 25^\circ - \cos 265^\circ \sin 25^\circ$ 

#### Use the given information to find the exact value of the expression.

9) Find  $\cos (\alpha - \beta)$ .  $\sin \alpha = \frac{7}{25}$ ,  $\alpha$  lies in quadrant II, and  $\cos \beta = \frac{2}{5}$ ,  $\beta$  lies in quadrant I.

#### Find the exact value under the given conditions.

10)  $\sin \alpha = \frac{3}{5}$ ,  $0 < \alpha < \frac{\pi}{2}$ ;  $\cos \beta = \frac{20}{29}$ ,  $0 < \beta < \frac{\pi}{2}$  Find  $\tan (\alpha + \beta)$ .

#### Use the given information to find the exact value of the expression.

11) Find  $\cos 2\theta$ .  $\sin \theta = \frac{7}{25}$ ,  $\theta$  lies in quadrant I.

#### Use a half-angle formula to find the exact value of the expression.

12) 
$$\sin \frac{3\pi}{8}$$

Express the product as a sum or difference. 13) sin 8x cos 4x

#### Express the sum or difference as a product.

14)  $\sin 6x + \sin 2x$ 

#### Find all solutions of the equation.

15)  $\cos x \csc x = 2 \cos x$ 

## Solve the equation on the interval $[0, 2\pi)$ .

16)  $2\sin^2 x = \sin x$ 

17)  $\sin 2x + \sin x = 0$ 

#### Solve the problem.

- 18) A guy wire to a tower makes a 70° angle with level ground. At a point 38 ft farther from the tower than the wire but on the same side as the base of the wire, the angle of elevation to the top of the tower is 37°. Find the length of the wire (to the nearest foot).
- 19) The distance from home plate to dead center field in Sun Devil Stadium is 400 feet. A baseball diamond is a square with a distance from home plate to first base of 90 feet. How far is it from first base to dead center field?

## The rectangular coordinates of a point are given. Find polar coordinates of the point.

20)  $(-4\sqrt{2}, -4\sqrt{2})$ 

#### Polar coordinates of a point are given. Find the rectangular coordinates of the point.

21) (-3, 120°)

## Use DeMoivre's Theorem to find the indicated power of the complex number. Write answer in rectangular form. 22) $(-\sqrt{3} + i)^6$

#### Find all the complex roots. Write the answer in the indicated form.

23) The complex cube roots of  $27(\cos 234^\circ + i \sin 234^\circ)$  (polar form)

#### Find the specified vector or scalar.

24)  $\mathbf{u} = -2\mathbf{i} - 6\mathbf{j}$ ,  $\mathbf{v} = -6\mathbf{i} + 8\mathbf{j}$ ; Find  $\mathbf{u} + \mathbf{v}$ .

Write a vector v in terms of i and j whose magnitude  $\|v\|$  and direction angle  $\theta$  are given.

25)  $\|\mathbf{v}\| = 10, \ \theta = 120^{\circ}$ 

#### Solve the problem.

26) The magnitude and direction of two forces acting on an object are 35 pounds, N45°E, and 55 pounds, S30°E, respectively. Find the magnitude, to the nearest hundredth of a pound, and the direction angle, to the nearest tenth of a degree, of the resultant force.

#### Use the given vectors to find the specified scalar.

27)  $\mathbf{u} = 6\mathbf{i} + 4\mathbf{j}$  and  $\mathbf{v} = -8\mathbf{i} - 7\mathbf{j}$ ; Find  $\mathbf{u} \cdot \mathbf{v}$ .

#### Solve the problem.

28) A person is pulling a freight cart with a force of 50 pounds. How much work is done in moving the cart 30 feet if the cart's handle makes an angle of 22° with the ground?

#### Find the foci of the ellipse whose equation is given.

29)  $16(x-2)^2 + 36(y+2)^2 = 576$ 

#### Find the standard form of the equation of the hyperbola satisfying the given conditions.

30) Endpoints of transverse axis: (-3, 0), (3, 0); foci: (-7, 0), (-7, 0)

Find the location of the center, vertices, and foci for the hyperbola described by the equation.

$$31)\frac{(x+4)^2}{36} - \frac{(y-1)^2}{25} = 1$$

Find the standard form of the equation of the parabola using the information given.

32) Vertex: (5, -7); Focus: (4, -7)

#### Convert the equation to the standard form for a parabola by completing the square on x or y as appropriate.

33)  $x^2 + 4x - 2y + 8 = 0$ 

#### Find the vertex, focus, and directrix of the parabola with the given equation.

34) 
$$(x + 2)^2 = -20(y + 3)$$

#### Solve the problem.

35) A bridge is built in the shape of a parabolic arch. The bridge arch has a span of 150 feet and a maximum height of 35 feet. Find the height of the arch at 15 feet from its center.

### Write the appropriate rotation formulas so that in a rotated system the equation has no x'y'-term.

36)  $2x^2 + 3xy + 2y^2 - 8x + 8y = 0$ 

#### Eliminate the parameter t. Find a rectangular equation for the plane curve defined by the parametric equations.

37) x = 2t - 1,  $y = t^2 + 7$ ;  $-4 \le t \le 4$ 

#### Write the first four terms of the sequence defined by the recursion formula.

38)  $a_1 = 4$  and  $a_n = 3a_{n-1}$  for  $n \ge 2$ 

#### Solve the problem.

39) A new exhibit is scheduled to open at the local museum. Museum officials expect that 9000 people will visit the exhibit in its first week, and that the number of visitors will drop by 10 people per week after the first week during the first 6 months. Find the total number of visitors expected in the exhibit's first 7 weeks.

# Use the formula for the general term (the nth term) of a geometric sequence to find the indicated term of the sequence with the given first term, a<sub>1</sub>, and common ratio, r.

40) Find  $a_5$  when  $a_1 = 4$ , r = 3.

#### Solve the problem.

- 41) Looking ahead to retirement, you sign up for automatic savings in a fixed-income 401K plan that pays 6.5% per year compounded annually. You plan to invest \$2000 at the end of each year for the next 20 years. How much will your account have in it at the end of 20 years?
- 42) A pendulum bob swings through an arc 40 inches long on its first swing. Each swing thereafter, it swings only 60% as far as on the previous swing. How far will it swing altogether before coming to a complete stop? Round to the nearest inch when necessary.

#### Use mathematical induction to prove that the statement is true for every positive integer n.

43) 1 • 3 + 2 • 3 + 3 • 3 + ... + 3n = 
$$\frac{3n(n+1)}{2}$$

Use the Binomial Theorem to expand the binomial and express the result in simplified form. 44)  $(3x + 4)^5$ 

## Find the term indicated in the expansion.

45)  $(x^2 + y^4)^9$ ; 6th term

#### Complete the table for the function and find the indicated limit.

6) 
$$\lim_{x \to 2} x^2 + 8x - 2$$

4

The graph of a function, f, is given. Use the graph to find the indicated limit, or state that a limit does not exist. 47)  $\lim_{x \to 1} f(x)$ 

x



Use properties of limits to find the indicated limit. It may be necessary to rewrite an expression before limit properties can be applied.

48)  $\lim_{x \to 2} (x^2 + 8x - 2)$ 

49)  $\lim_{x \to 4} \frac{\sqrt{x-2}}{x-4}$ 

Use the definition of continuity to determine whether f is continuous at a.

$$50) f(x) = \frac{5}{x+3}$$
$$a = 0$$

Find the slope of the tangent line to the graph of f at the given point.

51) 
$$f(x) = -4x^2 + 7x$$
 at (5, -65)

Find the derivative of f at x. That is, find f '(x).

52)  $f(x) = x^2 - 6x - 6; x = 6$ 

#### Solve the problem.

53) A foul tip of a baseball is hit straight upward from a height of 4 feet with an initial velocity of 48 feet per second. The function  $s(t) = -16t^2 + 48t$  describes the ball's height above the ground, s(t), in feet, t seconds after it was hit. What is the instantaneous velocity of the ball 2.3 seconds after it was hit? Answer Key Testname: MATH 2412 RFF12



# Answer Key Testname: MATH 2412 RFF12

 $21)\left(\frac{3}{2}, \frac{-3\sqrt{3}}{2}\right)$ 22) -64 23) 3(cos 78° + i sin 78°), 3(cos198° + i sin 198°), 3(cos 318° + i sin 318°) 24) -8**i** + 2**j** 25) **v** =  $-5i + 5\sqrt{3}j$ 26)  $\|\mathbf{F}\| = 57.04; \ \theta = -23.6^{\circ}$ 27) -76 28) 1390.8 foot-pounds 29) foci at  $(2 + 2\sqrt{5}, -2)$  and  $(2 - 2\sqrt{5}, -2)$  $30)\frac{x^2}{9} - \frac{y^2}{40} = 1$ 31) Center: (-4, 1); Vertices: (-10, 1) and (2, 1); Foci:  $(-4 - \sqrt{61}, 1)$  and  $(-4 + \sqrt{61}, 1)$ 32)  $(y + 7)^2 = -4(x - 5)$ 33)  $(x + 2)^2 = 2(y - 2)$ 34) vertex: (-2, -3) focus: (-2, -8) directrix: y = 235) 33.6 ft 36)  $x = \frac{\sqrt{2}}{2}(x' - y'); y = \frac{\sqrt{2}}{2}(x' + y')$ 37)  $y = \frac{1}{4}x^2 + \frac{1}{2}x + \frac{29}{4}; -9 \le x \le 7$ 38) 4, 12, 36, 108 39) 62,790 visitors 40) 324 41) \$77,651 42) 100 inches

43) S<sub>1</sub>: 
$$1 \cdot 3 \stackrel{?}{=} \frac{(3 \cdot 1)(1 + 1)}{2}$$
  
 $3 \stackrel{?}{=} \frac{3 \cdot 2}{2}$   
 $3 = 3 \checkmark$   
S<sub>k</sub>:  $1 \cdot 3 + 2 \cdot 3 + 3 \cdot 3 + \ldots + 3k = \frac{3k(k + 1)}{2}$   
S<sub>k+1</sub>:  $1 \cdot 3 + 2 \cdot 3 + 3 \cdot 3 + \ldots + 3(k + 1) = \frac{3(k + 1)(k + 2)}{2}$ 

We work with  $S_k$ . Because we assume that  $S_k$  is true, we add the next consecutive term, namely 3(k+1), to both sides."

$$1 \cdot 3 + 2 \cdot 3 + 3 \cdot 3 + \dots + 3k + 3(k+1) = \frac{3k(k+1)}{2} + 3(k+1)$$
  

$$1 \cdot 3 + 2 \cdot 3 + 3 \cdot 3 + \dots + 3(k+1) = \frac{3k(k+1)}{2} + \frac{6(k+1)}{2}$$
  

$$1 \cdot 3 + 2 \cdot 3 + 3 \cdot 3 + \dots + 3(k+1) = \frac{(k+1)(3k+6)}{2}$$
  

$$1 \cdot 3 + 2 \cdot 3 + 3 \cdot 3 + \dots + 3(k+1) = \frac{3(k+1)(k+2)}{2}$$

We have shown that if we assume that  $S_k$  is true, and we add (3(k+1) to both sides of  $S_k$ , then  $S_{k+1}$  is also true. By the principle of mathematical induction, the statement  $S_n$  is true for every positive integer n.

- 44)  $243x^5 + 1620x^4 + 4320x^3 + 5760x^2 + 3840x + 1024$
- 45) 126x<sup>8</sup>y<sup>20</sup>
- 46) 16.810; 17.880; 17.988; 18.012; 18.120; 19.210
- limit = 18.0
- 47) Does not exist
- 48) 18
- 49) 1/4
- 50) Continuous
- 51) -33
- 52) 6
- 53) -25.6 feet per second