the sphere, will the person be harmed upon touching the inside of the sphere?
4. A student who grew up in a tropical country and is studying in the United States may have no experience with static electricity sparks and shocks until his or her first American winter. Explain.
5. If a suspended object $A$ is attracted to a charged object $B$, can we conclude that A is charged? Explain.
6. Consider point $A$ in Figure CQ23.6 located an arbitrary distance from two positive point charges in otherwise empty space. (a) Is it possible for an electric field to exist at point $A$ in empty space? Explain. (b) Does charge exist at this point? Explain. (c) Does a force exist at this point? Explain.


Figure CQ23.6
7. In fair weather, there is an electric field at the surface of the Earth, pointing down into the ground. What is the sign of the electric charge on the ground in this situation?
8. A charged comb often attracts small bits of dry paper that then fly away when they touch the comb. Explain why that occurs.
9. A balloon clings to a wall after it is negatively charged by rubbing. (a) Does that occur because the wall is positively charged? (b) Why does the balloon eventually fall?
10. Consider two electric dipoles in empty space. Each dipole has zero net charge. (a) Does an electric force exist between the dipoles; that is, can two objects with zero net charge exert electric forces on each other? (b) If so, is the force one of attraction or of repulsion?
11. (a) Would life be different if the electron were positively charged and the proton were negatively charged? (b) Does the choice of signs have any bearing on physical and chemical interactions? Explain your answers.

## Problems

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        WebAssign The problems found in this chapter may be assigned online in Enhanced WebAssign
1. denotes straightforward problem; 2. denotes intermediate problem;
3. denotes challenging problem
1. full solution available in the Student Solutions Manual/Study Guide
1. denotes problems most often assigned in Enhanced WebAssign; these provide students with targeted feedback and either a Master It tutorial or a Watch It solution video.
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## Section 23.1 Properties of Electric Charges

1. Find to three significant digits the charge and the mass of the following particles. Suggestion: Begin by looking up the mass of a neutral atom on the periodic table of the elements in Appendix C. (a) an ionized hydrogen atom, represented as $\mathrm{H}^{+}$(b) a singly ionized sodium atom, $\mathrm{Na}^{+}$(c) a chloride ion $\mathrm{Cl}^{-}$(d) a doubly ionized calcium atom, $\mathrm{Ca}^{++}=\mathrm{Ca}^{2+}$ (e) the center of an ammonia molecule, modeled as an $\mathrm{N}^{3-}$ ion (f) quadruply ionized nitrogen atoms, $\mathrm{N}^{4+}$, found in plasma in a hot star (g) the nucleus of a nitrogen atom (h) the molecular ion $\mathrm{H}_{2} \mathrm{O}^{-}$
2. (a) Calculate the number of electrons in a small, electrically neutral silver pin that has a mass of 10.0 g . Silver has 47 electrons per atom, and its molar mass is $107.87 \mathrm{~g} / \mathrm{mol}$. (b) Imagine adding electrons to the pin until the negative charge has the very large value 1.00 mC . How many electrons are added for every $10^{9}$ electrons already present?

## Section 23.2 Charging Objects by Induction

## Section 23.3 Coulomb's Law

3. Review. A molecule of DNA (deoxyribonucleic acid) is $2.17 \mu \mathrm{~m}$ long. The ends of the molecule become singly ionized: negative on one end, positive on the other. The
Q|C denotes asking for quantitative and conceptual reasoning
$\mathbf{S}$ denotes symbolic reasoning problem
$\mathbf{M}$ denotes Master It tutorial available in Enhanced WebAssign
$\mathbf{G P}$ denotes guided problem
shaded denotes "paired problems" that develop reasoning with
symbols and numerical values
helical molecule acts like a spring and compresses $1.00 \%$ upon becoming charged. Determine the effective spring constant of the molecule.
4. Nobel laureate Richard Feynman (1918-1988) once said that if two persons stood at arm's length from each other and each person had $1 \%$ more electrons than protons, the force of repulsion between them would be enough to lift a "weight" equal to that of the entire Earth. Carry out an order-of-magnitude calculation to substantiate this assertion.
5. Q|C A $7.50-\mathrm{nC}$ point charge is located 1.80 m from a $4.20-\mathrm{nC}$ point charge. (a) Find the magnitude of the electric force that one particle exerts on the other. (b) Is the force attractive or repulsive?
6. $\mathbf{Q} \mid \mathbf{C}$ (a) Find the magnitude of the electric force between a $\mathrm{Na}^{+}$ion and a $\mathrm{Cl}^{-}$ion separated by 0.50 nm . (b) Would the answer change if the sodium ion were replaced by $\mathrm{Li}^{+}$and the chloride ion by $\mathrm{Br}^{-}$? Explain.
7. Q|C (a) Two protons in a molecule are $3.80 \times 10^{-10} \mathrm{~m}$ apart. Find the magnitude of the electric force exerted by one proton on the other. (b) State how the magnitude of this force compares with the magnitude of the gravitational force exerted by one proton on the other. (c) What If? What
must be a particle's charge-to-mass ratio if the magnitude of the gravitational force between two of these particles is equal to the magnitude of electric force between them?
8. Three point charges lie along a straight line as shown in Figure P23.8, where $q_{1}=6.00 \mu \mathrm{C}, q_{2}=1.50 \mu \mathrm{C}$, and $q_{3}=$ $-2.00 \mu \mathrm{C}$. The separation distances are $d_{1}=3.00 \mathrm{~cm}$ and $d_{2}=2.00 \mathrm{~cm}$. Calculate the magnitude and direction of the net electric force on (a) $q_{1}$, (b) $q_{2}$, and (c) $q_{3}$.


Figure P23.8
9. Three point charges are arranged as shown in Figure P23.9. Find (a) the magnitude and (b) the direction of the electric force on the particle at the origin.


Figure P23.9
10. Two small metallic spheres, each of mass $m=0.200 \mathrm{~g}$, are suspended as pendulums by light strings of length $L$ as shown in Figure P23.10. The spheres are given the same electric charge of 7.2 nC , and they come to equilibrium when each string is at an angle of $\theta=$ $5.00^{\circ}$ with the vertical. How long are the strings?


Figure P23.10
11. Q|C Two small beads having positive charges $q_{1}=3 q$ and $q_{2}=q$ are fixed at the opposite ends of a horizontal insulating rod of length $d=1.50 \mathrm{~m}$. The bead with charge $q_{1}$ is at the origin. As shown in Figure P23.11, a third small, charged bead is free to slide on the rod. (a) At what position $x$ is the third bead in equilibrium? (b) Can the equilibrium be stable?


Figure P23.11 Problems 11 and 12.
12. $\mathbf{Q} \mid \mathbf{C} \mathbf{S}$ Two small beads having charges $q_{1}$ and $q_{2}$ of the same sign are fixed at the opposite ends of a horizontal insulating rod of length $d$. The bead with charge $q_{1}$ is at the origin. As shown in Figure P23.11, a third small, charged bead is free to slide on the rod. (a) At what position $x$ is the third bead in equilibrium? (b) Can the equilibrium be stable?

M Three charged particles are located at the corners of an equilateral triangle as shown in Figure P23.13. Calculate the total electric force on the $7.00-\mu \mathrm{C}$ charge.


Figure P23.13 Problems 13 and 22.
14. S Review. Two identical particles, each having charge $+q$, are fixed in space and separated by a distance d. A third particle with charge $-Q$ is free to move and lies initially at rest on the perpendicular bisector of the two fixed charges a distance $x$ from the midpoint between those charges (Fig. P23.14). (a) Show that if $x$ is small compared with $d$, the motion of $-Q$ is simple harmonic along the perpendicular bisector. (b) Determine the period of


Figure P23.14 that motion. (c) How fast will the charge $-Q$ be moving when it is at the midpoint between the two fixed charges if initially it is released at a distance $a \ll d$ from the midpoint?
15. Review. In the Bohr theory of the hydrogen atom, an electron moves in a circular orbit about a proton, where the radius of the orbit is $5.29 \times 10^{-11} \mathrm{~m}$. (a) Find the magnitude of the electric force exerted on each particle. (b) If this force causes the centripetal acceleration of the electron, what is the speed of the electron?
16. GP Particle A of charge $3.00 \times 10^{-4} \mathrm{C}$ is at the origin, particle $B$ of charge $-6.00 \times 10^{-4} \mathrm{C}$ is at $(4.00 \mathrm{~m}, 0)$, and particle C of charge $1.00 \times 10^{-4} \mathrm{C}$ is at $(0,3.00 \mathrm{~m})$. We wish to find the net electric force on C . (a) What is the $x$ component of the electric force exerted by A on C? (b) What is the $y$ component of the force exerted by $A$ on $C$ ? (c) Find the magnitude of the force exerted by $B$ on $C$. (d) Calculate the $x$ component of the force exerted by $B$ on $C$. (e) Calculate the $y$ component of the force exerted by $B$ on C. (f) Sum the two $x$ components from parts (a) and (d) to obtain the resultant $x$ component of the electric force acting on C. (g) Similarly, find the $y$ component of the resultant force vector acting on C. (h) Find the magnitude and direction of the resultant electric force acting on C .
17. $\mathbf{S}$ A point charge $+2 Q$ is at the origin and a point charge $-Q$ is located along the $x$ axis at $x=d$ as in Figure P23.17. Find a symbolic expression for the net force on a third point charge $+Q$ located along the $y$ axis at $y=d$.
18. Why is the following situation


Figure P23.17 impossible? Two identical dust particles of mass $1.00 \mu \mathrm{~g}$ are floating in empty space, far from any external sources of large gravitational or electric
fields, and at rest with respect to each other. Both particles carry electric charges that are identical in magnitude and sign. The gravitational and electric forces between the particles happen to have the same magnitude, so each particle experiences zero net force and the distance between the particles remains constant.
19. Two identical conducting small spheres are placed with their centers 0.300 m apart. One is given a charge of 12.0 nC and the other a charge of -18.0 nC . (a) Find the electric force exerted by one sphere on the other. (b) What If? The spheres are connected by a conducting wire. Find the electric force each exerts on the other after they have come to equilibrium.

## Section 23.4 The Electric Field

20. A small object of mass 3.80 g and charge $-18.0 \mu \mathrm{C}$ is suspended motionless above the ground when immersed in a uniform electric field perpendicular to the ground. What are the magnitude and direction of the electric field?
21. M In Figure P23.21, determine the point (other than infinity) at which the electric field is zero.

22. Three charged particles are at the corners of an equilateral triangle as shown in Figure P23.13. (a) Calculate the electric field at the position of the $2.00-\mu \mathrm{C}$ charge due to the $7.00-\mu \mathrm{C}$ and $-4.00-\mu \mathrm{C}$ charges. (b) Use your answer to part (a) to determine the force on the $2.00-\mu \mathrm{C}$ charge.
23. Three point charges are located on a circular arc as shown in Figure P23.23. (a) What is the total electric field at $P$, the center of the arc? (b) Find the electric force that would be exerted on a $-5.00-\mathrm{nC}$ point charge placed at $P$.


Figure P23.23
24. Q|C S Two charged particles are located on the $x$ axis. The first is a charge $+Q$ at $x=-a$. The second is an unknown charge located at $x=+3 a$. The net electric field these charges produce at the origin has a magnitude of $2 k_{e} Q / a^{2}$. Explain how many values are possible for the unknown charge and find the possible values.
25. S Four charged particles are at the corners of a square of side $a$ as shown in Figure P23.25. Determine (a) the electric field at the location of charge $q$ and (b) the total electric force exerted on $q$.


Figure P23.25
26. S Three point charges lie along a circle of radius $r$ at angles of $30^{\circ}, 150^{\circ}$, and $270^{\circ}$ as shown in Figure P23.26. Find a symbolic expression for the resultant electric field at the center of the circle.
27. $\mathbf{S}$ Two equal positively charged particles are at opposite corners of a trapezoid as shown in Figure P23.27. Find symbolic expressions for the total electric field at (a) the point $P$ and (b) the point $P^{\prime}$.
28. Q|C Consider $n$ equal positively charged particles each of magnitude $Q / n$ placed


Figure P23.26


Figure P23.27 symmetrically around a circle of radius $a$. (a) Calculate the magnitude of the electric field at a point a distance $x$ from the center of the circle and on the line passing through the center and perpendicular to the plane of the circle. (b) Explain why this result is identical to the result of the calculation done in Example 23.7.

## Section 23.5 Electric Field of a Continuous Charge Distribution

29. A rod 14.0 cm long is uniformly charged and has a total charge of $-22.0 \mu \mathrm{C}$. Determine (a) the magnitude and (b) the direction of the electric field along the axis of the rod at a point 36.0 cm from its center.
30. A uniformly charged disk of radius 35.0 cm carries charge with a density of $7.90 \times 10^{-3} \mathrm{C} / \mathrm{m}^{2}$. Calculate the electric field on the axis of the disk at (a) 5.00 cm , (b) 10.0 cm , (c) 50.0 cm , and (d) 200 cm from the center of the disk.
31. M A uniformly charged ring of radius 10.0 cm has a total charge of $75.0 \mu \mathrm{C}$. Find the electric field on the axis of the ring at (a) 1.00 cm , (b) 5.00 cm , (c) 30.0 cm , and (d) 100 cm from the center of the ring.
32. Q|C Example 23.8 derives the exact expression for the electric field at a point on the axis of a uniformly charged disk. Consider a disk of radius $R=3.00 \mathrm{~cm}$ having a uniformly distributed charge of $+5.20 \mu \mathrm{C}$. (a) Using the result of Example 23.8, compute the electric field at a point on the axis and 3.00 mm from the center. (b) What If? Explain how the answer to part (a) compares with the field computed from the near-field approximation $E=\sigma / 2 \epsilon_{0}$. (We will derive this expression in Chapter 24.) (c) Using the result of Example 23.8, compute the electric field at a point on the axis and 30.0 cm from the center of the disk. (d) What If? Explain how the answer to part (c) compares with the electric field obtained by treating the disk as a $+5.20-\mu \mathrm{C}$ charged particle at a distance of 30.0 cm .
33. S A continuous line of charge lies along the $x$ axis, extending from $x=+x_{0}$ to positive infinity. The line carries positive charge with a uniform linear charge density $\lambda_{0}$. What are (a) the magnitude and (b) the direction of the electric field at the origin?
34. S The electric field along the axis of a uniformly charged disk of radius $R$ and total charge $Q$ was calculated
in Example 23.8. Show that the electric field at distances $x$ that are large compared with $R$ approaches that of a particle with charge $Q=\sigma \pi R^{2}$. Suggestion: First show that $x /\left(x^{2}+R^{2}\right)^{1 / 2}=\left(1+R^{2} / x^{2}\right)^{-1 / 2}$ and use the binomial expansion $(1+\delta)^{n} \approx 1+n \delta$, when $\delta \ll 1$.
35. M A uniformly charged insulating rod of length 14.0 cm is bent into the shape of a semicircle as shown in Figure P23.35. The rod has a total charge of $-7.50 \mu \mathrm{C}$. Find (a) the magnitude and (b) the direction of the electric field at $O$, the center of the semicircle.
36. Q|C S A uniformly charged rod of


Figure P23.35 length $L$ and total charge $Q$ lies along the $x$ axis as shown in Figure P23.36. (a) Find the components of the electric field at the point $P$ on the $y$ axis a distance $d$ from the origin. (b) What are the approximate values of the field components when $d \gg$ $L$ ? Explain why you would expect these results.


Figure P23.36


Figure P23.37
38. S (a) Consider a uniformly charged, thin-walled, right circular cylindrical shell having total charge $Q$, radius $R$, and length $\ell$. Determine the electric field at a point a distance $d$ from the right side of the cylinder as shown in Figure P23.38. Suggestion: Use the result of Example 23.7 and treat the cylinder as a collection of ring charges. (b) What If? Consider now a solid cylinder with the same dimensions and carrying the same charge, uniformly distributed through its volume. Use the result of Example 23.8 to find the field it creates at the same point.


Figure P23.38

## Section 23.6 Electric Field Lines

39. A negatively charged rod of finite length carries charge with a uniform charge per unit length. Sketch the electric field lines in a plane containing the rod.
40. A positively charged disk has a uniform charge per unit area $\sigma$ as described in Example 23.8. Sketch the electric field lines in a plane perpendicular to the plane of the disk passing through its center.
41. Figure P23.41 shows the electric field lines for two charged particles separated by a small distance. (a) Determine the ratio $q_{1} / q_{2}$.
(b) What are the signs of $q_{1}$ and $q_{2}$ ?
S Three equal positive charges $q$ are at the corners of an equilateral triangle of side $a$ as shown in Figure P23.42. Assume the three charges together create an electric field. (a) Sketch the field lines in the plane of the charges. (b) Find the location of one point (other than $\infty$ ) where the electric field is zero. What are (c) the magnitude and (d) the direction of the electric field at $P$ due to the two charges at the base?


Figure P23.41


Figure P23.42

## Section 23.7 Motion of a Charged Particle in a Uniform Electric Field

43. An electron and a proton are each placed at rest in a uniform electric field of magnitude 520 N/C. Calculate the speed of each particle 48.0 ns after being released.
44. A proton is projected in the positive $x$ direction into a region of a uniform electric field $\overrightarrow{\mathbf{E}}=\left(-6.00 \times 10^{5}\right) \hat{\mathbf{i}} \mathrm{N} / \mathrm{C}$ at $t=0$. The proton travels 7.00 cm as it comes to rest. Determine (a) the acceleration of the proton, (b) its initial speed, and (c) the time interval over which the proton comes to rest.
45. $M$ A proton accelerates from rest in a uniform electric field of $640 \mathrm{~N} / \mathrm{C}$. At one later moment, its speed is $1.20 \mathrm{Mm} / \mathrm{s}$ (nonrelativistic because $v$ is much less than the speed of light). (a) Find the acceleration of the proton. (b) Over what time interval does the proton reach this speed? (c) How far does it move in this time interval? (d) What is its kinetic energy at the end of this interval?
46. Q|C Two horizontal metal plates, each 10.0 cm square, are aligned 1.00 cm apart with one above the other. They are given equal-magnitude charges of opposite sign so that a uniform downward electric field of $2.00 \times 10^{3} \mathrm{~N} / \mathrm{C}$ exists in the region between them. A particle of mass $2.00 \times$ $10^{-16} \mathrm{~kg}$ and with a positive charge of $1.00 \times 10^{-6} \mathrm{C}$ leaves the center of the bottom negative plate with an initial speed of $1.00 \times 10^{5} \mathrm{~m} / \mathrm{s}$ at an angle of $37.0^{\circ}$ above the horizontal. (a) Describe the trajectory of the particle. (b) Which plate does it strike? (c) Where does it strike, relative to its starting point?
47. S The electrons in a particle beam each have a kinetic energy $K$. What are (a) the magnitude and (b) the direction of the electric field that will stop these electrons in a distance $d$ ?
48. GP Protons are projected with an initial speed $v_{i}=$ $9.55 \mathrm{~km} / \mathrm{s}$ from a field-free region through a plane and into a region where a uniform electric field $\overrightarrow{\mathbf{E}}=-720 \hat{\mathbf{j}} \mathrm{~N} / \mathrm{C}$ is
present above the plane as shown in Figure P23.48. The initial velocity vector of the protons makes an angle $\theta$ with the plane. The protons are to hit a target that lies at a horizontal distance of $R=1.27 \mathrm{~mm}$ from the point where the protons cross the plane and enter the electric field. We wish to find the angle $\theta$ at which the protons must pass through the plane to strike the target. (a) What analysis model describes the horizontal motion of the protons above the plane? (b) What analysis model describes the vertical motion of the protons above the plane? (c) Argue that Equation 4.13 would be applicable to the protons in this situation. (d) Use Equation 4.13 to write an expression for $R$ in terms of $v_{i}, E$, the charge and mass of the proton, and the angle $\theta$. (e) Find the two possible values of the angle $\theta$. (f) Find the time interval during which the proton is above the plane in Figure P23.48 for each of the two possible values of $\theta$.


Figure P23.48
49. M A proton moves at $4.50 \times 10^{5} \mathrm{~m} / \mathrm{s}$ in the horizontal direction. It enters a uniform vertical electric field with a magnitude of $9.60 \times 10^{3} \mathrm{~N} / \mathrm{C}$. Ignoring any gravitational effects, find (a) the time interval required for the proton to travel 5.00 cm horizontally, (b) its vertical displacement during the time interval in which it travels 5.00 cm horizontally, and (c) the horizontal and vertical components of its velocity after it has traveled 5.00 cm horizontally.

## Additional Problems

50. A small sphere of charge $q_{1}=0.800 \mu \mathrm{C}$ hangs from the end of a spring as in Figure P23.50a. When another small sphere of charge $q_{2}=-0.600 \mu \mathrm{C}$ is held beneath the first sphere as in Figure P23.50b, the spring stretches by $d=$ 3.50 cm from its original length and reaches a new equilibrium position with a separation between the charges of $r=5.00 \mathrm{~cm}$. What is the force constant of the spring?
51. A small block of mass $m$ and charge $Q$ is placed on an insulated, frictionless, inclined plane of angle $\theta$ as in Figure P23.51. An electric field is applied parallel to the incline. (a) Find an expression for the magnitude of the electric
field that enables the block to remain at rest. (b) If $m=$ $5.40 \mathrm{~g}, Q=-7.00 \mu \mathrm{C}$, and $\theta=$ $25.0^{\circ}$, determine the magnitude and the direction of the electric field that enables the block to remain at rest on the incline.


Figure P23.51
52. Three solid plastic cylinders all have radius 2.50 cm and length 6.00 cm . Find the charge of each cylinder given the following additional information about each one. Cylinder (a) carries charge with uniform density $15.0 \mathrm{nC} / \mathrm{m}^{2}$ everywhere on its surface. Cylinder (b) carries charge with uniform density $15.0 \mathrm{nC} / \mathrm{m}^{2}$ on its curved lateral surface only. Cylinder (c) carries charge with uniform density $500 \mathrm{nC} / \mathrm{m}^{3}$ throughout the plastic.
53. S Consider an infinite number of identical particles, each with charge $q$, placed along the $x$ axis at distances $a$, $2 a, 3 a, 4 a, \ldots$ from the origin. What is the electric field at the origin due to this distribution? Suggestion: Use

$$
1+\frac{1}{2^{2}}+\frac{1}{3^{2}}+\frac{1}{4^{2}}+\ldots=\frac{\pi^{2}}{6}
$$

54. A particle with charge -3.00 nC is at the origin, and a particle with negative charge of magnitude $Q$ is at $x=50.0 \mathrm{~cm}$. A third particle with a positive charge is in equilibrium at $x=20.9 \mathrm{~cm}$. What is $Q$ ?
55. S A line of charge starts at $x=+x_{0}$ and extends to positive infinity. The linear charge density is $\lambda=\lambda_{0} x_{0} / x$, where $\lambda_{0}$ is a constant. Determine the electric field at the origin.
56. Two small silver spheres, each with a mass of 10.0 g , are separated by 1.00 m . Calculate the fraction of the electrons in one sphere that must be transferred to the other to produce an attractive force of $1.00 \times 10^{4} \mathrm{~N}$ (about 1 ton) between the spheres. The number of electrons per atom of silver is 47.
57. A uniform electric field of magnitude $640 \mathrm{~N} / \mathrm{C}$ exists between two parallel plates that are 4.00 cm apart. A proton is released from rest at the positive plate at the same instant an electron is released from rest at the negative plate. (a) Determine the distance from the positive plate at which the two pass each other. Ignore the electrical attraction between the proton and electron. (b) What If? Repeat part (a) for a sodium ion $\left(\mathrm{Na}^{+}\right)$and a chloride ion $\left(\mathrm{Cl}^{-}\right)$.
58. Q|C Two point charges $q_{\mathrm{A}}=-12.0 \mu \mathrm{C}$ and $q_{\mathrm{B}}=45.0 \mu \mathrm{C}$ and a third particle with unknown charge $q_{\mathrm{C}}$ are located on the $x$ axis. The particle $q_{\mathrm{A}}$ is at the origin, and $q_{\mathrm{B}}$ is at $x=$ 15.0 cm . The third particle is to be placed so that each particle is in equilibrium under the action of the electric forces exerted by the other two particles. (a) Is this situation possible? If so, is it possible in more than one way? Explain. Find (b) the required location and (c) the magnitude and the sign of the charge of the third particle.
59. M A charged cork ball of mass 1.00 g is suspended on a light string in the presence of a uniform electric field as shown


Figure P23.59
Problems 59 and 60.
in Figure P23.59. When $\overrightarrow{\mathbf{E}}=(3.00 \hat{\mathbf{i}}+5.00 \hat{\mathbf{j}}) \times 10^{5} \mathrm{~N} / \mathrm{C}$, the ball is in equilibrium at $\theta=37.0^{\circ}$. Find (a) the charge on the ball and (b) the tension in the string.
60. S A charged cork ball of mass $m$ is suspended on a light string in the presence of a uniform electric field as shown in Figure P23.59. When $\overrightarrow{\mathbf{E}}=A \hat{\mathbf{i}}+B \hat{\mathbf{j}}$, where $A$ and $B$ are positive numbers, the ball is in equilibrium at the angle $\theta$. Find (a) the charge on the ball and (b) the tension in the string.
61. Three charged particles are aligned along the $x$ axis as shown in Figure P23.61. Find the electric field at (a) the position $(2.00 \mathrm{~m}, 0)$ and (b) the position ( $0,2.00 \mathrm{~m}$ ).


Figure P23.61
62. Four identical charged particles $(q=+10.0 \mu \mathrm{C})$ are located on the corners of a rectangle as shown in Figure P23.62. The dimensions of the rectangle are $L=60.0 \mathrm{~cm}$ and $W=15.0 \mathrm{~cm}$. Calculate (a) the magnitude and (b) the direction of the total electric force exerted on


Figure P23.62 the charge at the lower left corner by the other three charges.
63. A line of positive charge is formed into a semicircle of radius $R=60.0 \mathrm{~cm}$ as shown in Figure P23.63. The charge per unit length along the semicircle is described by the expres$\operatorname{sion} \lambda=\lambda_{0} \cos \theta$. The total charge on the semicircle is $12.0 \mu \mathrm{C}$. Calculate the total force on a charge of


Figure P23.63 $3.00 \mu \mathrm{C}$ placed at the center of curvature $P$.
64. Why is the following situation impossible? An electron enters a region of uniform electric field between two parallel plates. The plates are used in a cathode-ray tube to adjust the position of an electron beam on a distant fluorescent screen. The magnitude of the electric field between the plates is $200 \mathrm{~N} / \mathrm{C}$. The plates are 0.200 m in length and are separated by 1.50 cm . The electron enters the region at a speed of $3.00 \times 10^{6} \mathrm{~m} / \mathrm{s}$, traveling parallel to the plane of the plates in the direction of their length. It leaves the plates heading toward its correct location on the fluorescent screen.
65. Two small spheres hang in equilibrium at the bottom ends of threads, 40.0 cm long, that have their top ends tied to the same fixed point. One sphere has mass 2.40 g and charge +300 nC . The other sphere has the same mass and
charge +200 nC . Find the distance between the centers of the spheres.
66. Three identical point charges, each of mass $m=0.100 \mathrm{~kg}$, hang from three strings as shown in Figure P23.66. If the lengths of the left and right strings are each $L=30.0 \mathrm{~cm}$ and the angle $\theta$ is $45.0^{\circ}$, determine the value of $q$.


Figure P23.66
67. Review. Two identical blocks resting on a frictionless, horizontal surface are connected by a light spring having a spring constant $k=100 \mathrm{~N} / \mathrm{m}$ and an unstretched length $L_{i}=0.400 \mathrm{~m}$ as shown in Figure P23.67a. A charge $Q$ is slowly placed on each block, causing the spring to stretch to an equilibrium length $L=0.500 \mathrm{~m}$ as shown in Figure P23.67b. Determine the value of $Q$, modeling the blocks as charged particles.


Figure P23.67 Problems 67 and 68.
68. S Review. Two identical blocks resting on a frictionless, horizontal surface are connected by a light spring having a spring constant $k$ and an unstretched length $L_{i}$ as shown in Figure P23.67a. A charge $Q$ is slowly placed on each block, causing the spring to stretch to an equilibrium length $L$ as shown in Figure P23.67b. Determine the value of $Q$, modeling the blocks as charged particles.
69. Two hard rubber spheres, each of mass $m=15.0 \mathrm{~g}$, are rubbed with fur on a dry day and are then suspended with two insulating strings of length $L=5.00 \mathrm{~cm}$ whose support points are a distance $d=3.00 \mathrm{~cm}$ from each other as shown in Figure


Figure P23.69 P23.69. During the rubbing process, one sphere receives exactly twice the charge of the other. They are observed to hang at equilibrium, each at an angle of $\theta=10.0^{\circ}$ with the vertical. Find the amount of charge on each sphere.
70. S Show that the maximum magnitude $E_{\text {max }}$ of the electric field along the axis of a uniformly charged ring occurs at $x=$ $a / \sqrt{2}$ (see Fig. 23.16) and has the value $Q /\left(6 \sqrt{3} \pi \epsilon_{0} a^{2}\right)$.
71. Q|C S Two small spheres of mass $m$ are suspended from strings of length $\ell$ that are connected at a common point. One sphere has charge $Q$ and the other charge 2Q. The strings make angles $\theta_{1}$ and $\theta_{2}$ with the vertical. (a) Explain how $\theta_{1}$ and $\theta_{2}$ are related. (b) Assume $\theta_{1}$ and $\theta_{2}$ are small. Show that the distance $r$ between the spheres is approximately

$$
r \approx\left(\frac{4 k_{e} Q^{2} \ell}{m g}\right)^{1 / 3}
$$

72. S Two identical beads each have a mass $m$ and charge $q$. When placed in a hemispherical bowl of radius $R$ with frictionless, nonconducting walls, the beads move, and at equilibrium, they are a distance dapart (Fig. P23.72).


Figure P23.72
(a) Determine the charge $q$ on each bead. (b) Determine the charge required for $d$ to become equal to $2 R$.
73. Q|C Review. A $1.00-\mathrm{g}$ cork ball with charge $2.00 \mu \mathrm{C}$ is suspended vertically on a $0.500-\mathrm{m}$-long light string in the presence of a uniform, downward-directed electric field of magnitude $E=1.00 \times 10^{5} \mathrm{~N} / \mathrm{C}$. If the ball is displaced slightly from the vertical, it oscillates like a simple pendulum. (a) Determine the period of this oscillation. (b) Should the effect of gravitation be included in the calculation for part (a)? Explain.
74. S Review. A negatively charged particle $-q$ is placed at the center of a uniformly charged ring, where the ring has a total positive charge $Q$ as shown in Figure P23.74. The particle, confined to move along the $x$ axis, is moved a small distance $x$ along the axis (where $x \ll a$ )


Figure P23.74 and released. Show that the particle oscillates in simple harmonic motion with a frequency given by

$$
f=\frac{1}{2 \pi}\left(\frac{k_{e} q Q}{m a^{3}}\right)^{1 / 2}
$$

## Challenge Problems

75. S Identical thin rods of length $2 a$ carry equal charges $+Q$ uniformly distributed along their lengths. The rods lie along the $x$ axis with their centers separated by a distance $b>2 a$ (Fig. P23.75). Show that the magnitude of the force exerted by the left rod on the right one is

$$
F=\left(\frac{k_{e} Q^{2}}{4 a^{2}}\right) \ln \left(\frac{b^{2}}{b^{2}-4 a^{2}}\right)
$$



Figure P23.75
76. Inez is putting up decorations for her sister's quinceañera (fifteenth birthday party). She ties three light silk ribbons together to the top of a gateway and hangs a rubber balloon from each ribbon (Fig. P23.76). To include the effects of the gravitational and buoyant forces on it, each balloon can be modeled as a particle of mass 2.00 g ,


Figure P23.76 with its center 50.0 cm from the point of support. Inez rubs the whole surface of each balloon with her woolen scarf, making the balloons hang separately with gaps between them. Looking directly upward from below the balloons, Inez notices that the centers of the hanging balloons form a horizontal equilateral triangle with sides 30.0 cm long. What is the common charge each balloon carries?
77. S Eight charged particles, each of magnitude $q$, are located on the corners of a cube of edge $s$ as shown in Figure P23.77. (a) Determine the $x, y$, and $z$ components of the total force exerted by the other charges on the charge located at point $A$. What are (b) the magnitude and (c) the direction of this total force?


Figure P23.77 Problems 77 and 78.
78. $\mathbf{S}$ Consider the charge distribution shown in Figure P23.77. (a) Show that the magnitude of the electric field at the center of any face of the cube has a value of $2.18 k_{e} q / s^{2}$. (b) What is the direction of the electric field at the center of the top face of the cube?
79. S Review. An electric dipole in a uniform horizontal electric field is displaced slightly from its equilibrium position as shown in Figure P23.79, where $\theta$ is small. The separation of the charges is $2 a$, and each of the two particles
has mass $m$. (a) Assuming the dipole is released from this position, show that its angular orientation exhibits simple harmonic motion with a frequency

$$
f=\frac{1}{2 \pi} \sqrt{\frac{q E}{m a}}
$$

What If? (b) Suppose the masses of the two charged particles in the dipole are not the same even though each particle continues to have charge $q$. Let the masses of the particles be $m_{1}$ and $m_{2}$. Show that the frequency of the oscillation in this case is

$$
f=\frac{1}{2 \pi} \sqrt{\frac{q E\left(m_{1}+m_{2}\right)}{2 a m_{1} m_{2}}}
$$



Figure P23.79
80. Q|C Two particles, each with charge 52.0 nC , are located on the $y$ axis at $y=25.0 \mathrm{~cm}$ and $y=-25.0 \mathrm{~cm}$. (a) Find the vector electric field at a point on the $x$ axis as a function of $x$. (b) Find the field at $x=36.0 \mathrm{~cm}$. (c) At what location is the field $1.00 \hat{\mathbf{i}} \mathrm{kN} / \mathrm{C}$ ? You may need a computer to solve this equation. (d) At what location is the field $16.0 \hat{\mathrm{i}} \mathrm{kN} / \mathrm{C}$ ?
81. A line of charge with uniform density $35.0 \mathrm{nC} / \mathrm{m}$ lies along the line $y=-15.0 \mathrm{~cm}$ between the points with coordinates $x=0$ and $x=40.0 \mathrm{~cm}$. Find the electric field it creates at the origin.
82. S A particle of mass $m$ and charge $q$ moves at high speed along the $x$ axis. It is initially near $x=-\infty$, and it ends up near $x=+\infty$. A second charge $Q$ is fixed at the point $x=0$, $y=-d$. As the moving charge passes the stationary charge, its $x$ component of velocity does not change appreciably, but it acquires a small velocity in the $y$ direction. Determine the angle through which the moving charge is deflected from the direction of its initial velocity.

