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Math2415-TestReview2-Fall2016

Multiple Choice
Identify the choice that best completes the statement or answers the question.

____ 1. Find and simplify the function $f(x,y) = \int_x^y \frac{9}{t}\,dt$ at the given value $(10,5)$.

a. $-9\ln(10)$
b. $-18\ln(2)$
c. $9\ln(2)$
d. $-9\ln(2)$
e. $18\ln(10)$

____ 2. Find and simplify $\dfrac{f(x+\Delta x,y)-f(x,y)}{\Delta x}$ for the given function $f(x,y) = 7x+2y^2$.

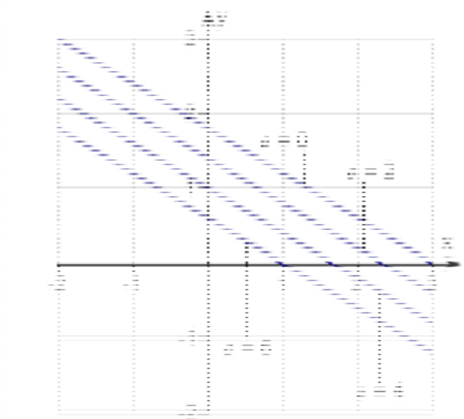
a. $7\Delta x$
b. $2y^2,\ \Delta x \neq 0$
c. $7,\ \Delta x \neq 0$
d. $7x$
e. $2,\ \Delta x \neq 0$

____ 3. Describe the domain of the function $f(x,y) = \ln(6-x-y)$.

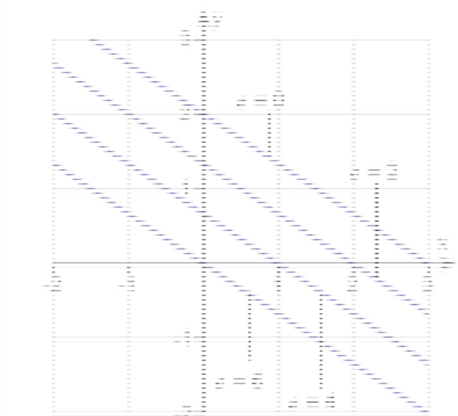
a. $\{(x,y): x \geq 0,\ y \geq 0\}$
b. $\{(x,y): y \geq 0\}$
c. $\{(x,y): y < -x+6\}$
d. $\{(x,y): y \leq x-6\}$
e. $\{(x,y): y \leq -x+6\}$

____ 4. Sketch the level curves of the function $z = 8 - 2x - 5y$ for the given c-values $c = 0, 2, 4, 6$.

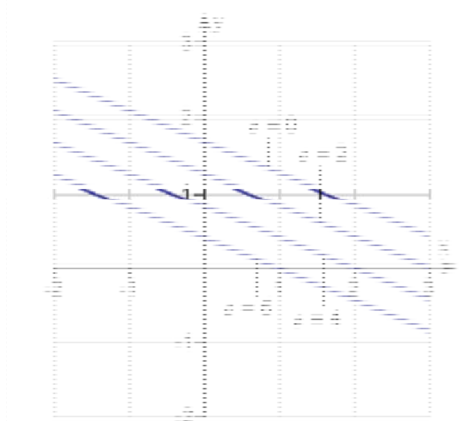
a.



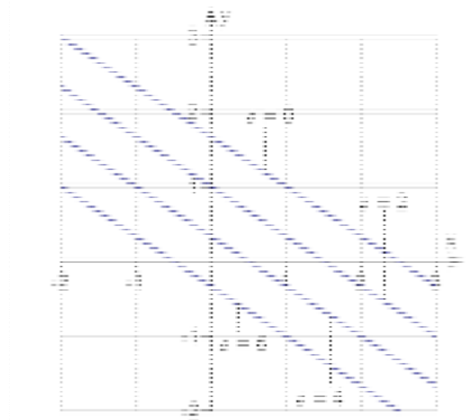
b.



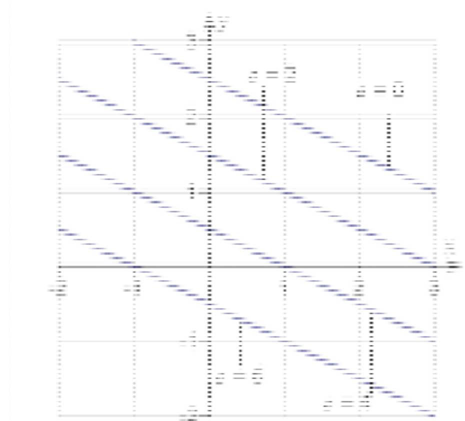
c.



d.



e.



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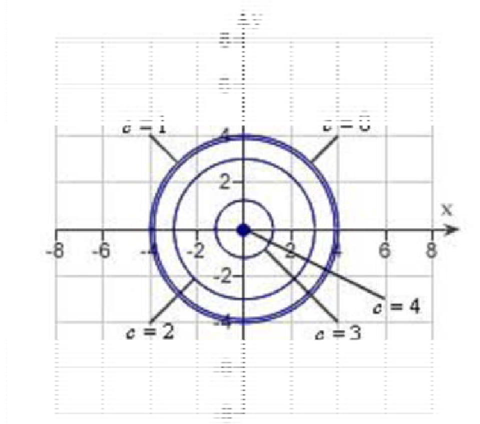
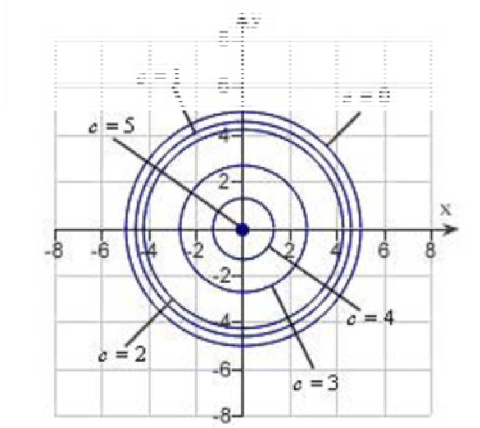
____ 5. Sketch the level curves for the function $z=\sqrt{16-x^2-y^2}$ for the given c-values $c=0,1,2,3,4,5$.

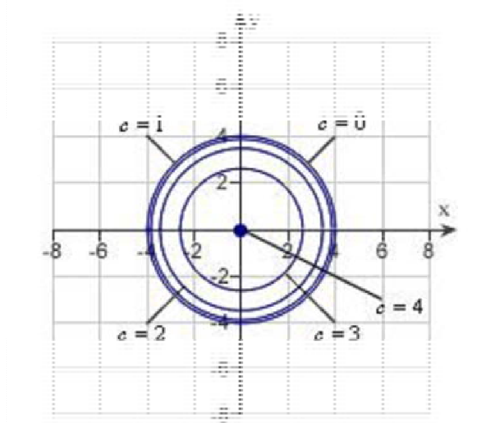
a.



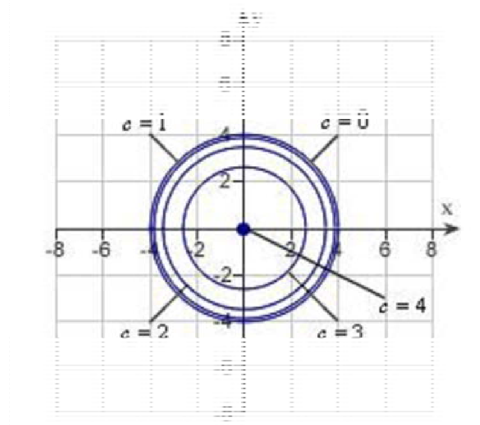
b.



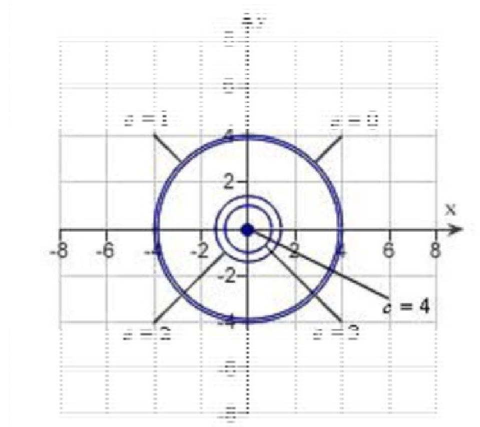
c.



d.



e.



____ 6.

Find the partial derivative f_x for the function $f(x,y) = \sqrt{28x + y^3}$.

a. $f_x(x,y) = \dfrac{14}{\sqrt{28x + y^3}}$

b. $f_x(x,y) = \dfrac{393x}{\sqrt{28x + y^3}}$

c. $f_x(x,y) = \dfrac{197}{\sqrt{28x + y^3}}$

d. $f_x(x,y) = \dfrac{1}{2\sqrt{28x + y^3}}$

e. $f_x(x,y) = \dfrac{28x}{\sqrt{28x + y^3}}$

____ 7. For $f(x,y) = \sin(7xy)$, evaluate f_x at the point $\left(2, \frac{\pi}{4}\right)$.

a. $f_x\left(2, \frac{\pi}{4}\right) = \frac{7\pi}{4}$

b. $f_x\left(2, \frac{\pi}{4}\right) = 0$

c. $f_x\left(2, \frac{\pi}{4}\right) = 7$

d. $f_x\left(2, \frac{\pi}{4}\right) = \frac{7\pi}{2}$

e. $f_x\left(2, \frac{\pi}{4}\right) = 14$

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____ 8. For $f(x,y)$, find all values of x and y such that $f_x(x,y) = 0$ and $f_y(x,y) = 0$ simultaneously.

$f(x,y) = 10x^3 - 2xy + 10y^3$

a. $(0,0),\ \left(\frac{1}{15},\frac{1}{15}\right)$

b. $\left(\frac{1}{15},\frac{1}{15}\right)$

c. $\left(-\frac{1}{15},-\frac{1}{15}\right),\ \left(\frac{1}{15},\frac{1}{15}\right)$

d. $\left(-\frac{1}{15},-\frac{1}{15}\right)$

e. $(0,0)$

____ 9. Find $\frac{dw}{dt}$ using the appropriate Chain Rule for $w = x^2 + y^2$ where $x = 7t$ and $y = 3t$.

a. $116t$
b. $58t$
c. $104t$
d. $32t$
e. $174t$

____ 10. Let $w = xy\cos z$, where $x = t^3$, $y = t^5$, and $z = \arccos t$. Find $\dfrac{dw}{dt}$.

a. $\dfrac{dw}{dt} = 8t^8 - \sqrt{1-t^2}$

b. $\dfrac{dw}{dt} = 8t^8 + \sqrt{1-t^2}$

c. $\dfrac{dw}{dt} = t^8\left(8 + \sqrt{1-t^2}\right)$

d. $\dfrac{dw}{dt} = 9t^8$

e. $\dfrac{dw}{dt} = t^8\left(8 - \sqrt{1-t^2}\right)$

____ 11. Find $\dfrac{\partial w}{\partial s}$ using the appropriate Chain Rule for $w = y^3 - 8x^2y$ where $x = e^s$ and $y = e^t$, and evaluate the partial derivative at $s = -2$ and $t = 3$. Round your answer to two decimal places.

a. –8.83
b. –5.89
c. –6.62
d. –2.72
e. –8.09

____ 12. Find $\dfrac{\partial w}{\partial s}$ using the appropriate Chain Rule for $w = x^2 + y^2 + z^2$ where $x = 8t\sin s$, $y = 8t\cos s$, and $z = 8st^2$.

a. $64st^4$
b. $128s^4t$
c. $16s^4t$
d. $128st^4$
e. $16st^4$

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____ 13. Differentiate implicitly to find $\frac{dy}{dx}$.

$x^2 - 4xy + y^2 - 10x + y - 9 = 0$

a. $\frac{dy}{dx} = -\frac{2x - 4y - 10}{2y - 4x + 1}$

b. $\frac{dy}{dx} = \frac{2x + 4y + 10}{2y + 4x + 1}$

c. $\frac{dy}{dx} = \frac{2x - 4y - 10}{2y - 4x + 1}$

d. $\frac{dy}{dx} = \frac{2x + 4y - 10}{2y + 4x - 1}$

e. $\frac{dy}{dx} = -\frac{2x + 4y + 10}{2y + 4x + 1}$

____ 14. Differentiate implicitly to find $\frac{\partial z}{\partial y}$, given $3x + \sin(10y + z) = 0$.

a. 3

b. 10

c. $-\frac{3}{\cos(y+z)}$

d. $-\frac{10}{\cos(y+z)}$

e. –10

____ 15. \The radius of a right circular cylinder is increasing at a rate of 9 inches per minute, and the height is decreasing at a rate of 7 inches per minute. What is the rate of change of the surface area when the radius is 12 inches and the height is 32 inches?

a. 936π in.^{2} / min
b. 984π in.^{2} / min
c. 876π in.^{2} / min
d. 840π in.^{2} / min
e. 968π in.^{2} / min

____ 16. Find the directional derivative of the function at P in the direction of $\vec{\mathbf{v}}$.

$$f(x,y,z) = xy + yz + xz,\ \ P(1,1,1),\ \ \vec{\mathbf{v}} = 4\hat{\mathbf{i}} + 5\hat{\mathbf{j}} - 3\hat{\mathbf{k}}$$

a. $4/5\sqrt{2}$
b. $-8/5\sqrt{2}$
c. $12/5\sqrt{2}$
d. $24/5\sqrt{2}$
e. $-24/5\sqrt{2}$

____ 17. Find the directional derivative of the function $g(x,y,z) = xye^z$ at $P(3,8,0)$ in the direction of $Q(0,0,0)$. Round your answer to two decimal places.

a. –11.64
b. –1.87
c. –5.86
d. –0.70
e. –5.62

____ 18. Use the gradient to find the directional derivative of the function at P in the direction of Q.

$$g(x,y) = x^2 + y^2 + 1, \quad P(4,8), \quad Q(12,24)$$

a. $-4\sqrt{5}$
b. $8\sqrt{5}$
c. $16\sqrt{5}$
d. $4\sqrt{5}$
e. $-8\sqrt{5}$

____ 19. Use the gradient to find a normal vector to the graph of the equation at the given point.

$2x^2 - y = 8, \quad (6, 64)$

a. $24\hat{\mathbf{i}} + \hat{\mathbf{j}}$

b. $24\hat{\mathbf{i}} - \hat{\mathbf{j}}$

c. $-24\hat{\mathbf{i}} - 64\hat{\mathbf{j}}$

d. $-24\hat{\mathbf{i}} - \hat{\mathbf{j}}$

e. $24\hat{\mathbf{i}} + 64\hat{\mathbf{j}}$

____ 20. The temperature at the point (x, y) on a metal plate is $T = \dfrac{9x}{x^2 + y^2}$. Find the direction of greatest increase in heat from the point $(7, 9)$. Round all numerical values in your answer to three decimal places.

a. $0.101\hat{\mathbf{i}} - 0.023\hat{\mathbf{j}}$

b. $0.034\hat{\mathbf{i}} - 0.017\hat{\mathbf{j}}$

c. $0.023\hat{\mathbf{i}} - 0.034\hat{\mathbf{j}}$

d. $0.038\hat{\mathbf{i}} - 0.101\hat{\mathbf{j}}$

e. $0.017\hat{\mathbf{i}} - 0.067\hat{\mathbf{j}}$

____ 21. Find the path of a heat-seeking particle placed at point $P(16, 16)$ on a metal plate with a temperature field $T(x, y) = 401 - 6x^2 - 3y^2$.

a. $y^2 = 32x$
b. $y^2 = 32x^2$
c. $y = 256x^2$
d. $y^2 = 16x$
e. $y = 16x^2$

____ 22. Find the unit normal vector to the surface $10x + 10y + 5z = 0$ at the point $(0,0,0)$.

a. $10\mathbf{i} + 10\mathbf{j} + 5\mathbf{k}$

b. $\frac{2}{3}\mathbf{i} + \frac{2}{3}\mathbf{j} + \frac{1}{4}\mathbf{k}$

c. $\frac{2}{3}\mathbf{i} + \frac{2}{3}\mathbf{j} + \frac{1}{3}\mathbf{k}$

d. $\frac{2}{45}\mathbf{i} + \frac{2}{45}\mathbf{j} + \frac{1}{80}\mathbf{k}$

e. $\frac{2}{45}\mathbf{i} + \frac{2}{45}\mathbf{j} + \frac{1}{45}\mathbf{k}$

____ 23. Find a unit normal vector to the surface $x^2 + y^2 + z^2 = 51$ at the point $(7,1,1)$.

a. $\frac{1}{\sqrt{50}}\langle 7,0,1\rangle$

b. $\frac{1}{\sqrt{50}}\langle 7,1,0\rangle$

c. $\frac{1}{\sqrt{3}}\langle 1,1,1\rangle$

d. $\frac{1}{\sqrt{51}}\langle 7,1,1\rangle$

e. $-\frac{1}{\sqrt{50}}\langle 7,0,1\rangle$

____ 24. Find an equation of the tangent plane to the surface $g(x,y) = x^2 - y^2$ at the point $(2,3,-5)$.

a. $4(x-2) - 6(y-3) + (z+5) = 0$
b. $4(x-2) - 6(y-3) - (z+5) = 0$
c. $2(x-2) - 3(y-3) - (z+5) = 0$
d. $4(x-2) + 3(y-3) - (z+5) = 0$
e. $4(x-2) + 6(y-3) + (z+5) = 0$

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____ 25. Find symmetric equations of the normal line to the surface $3x+2y+3z=26$ at the point $(2,4,4)$.

a. $\dfrac{x-2}{6}=\dfrac{y-4}{9}=\dfrac{z-4}{6}$

b. $\dfrac{3x-2}{1}=\dfrac{2y-4}{1}=\dfrac{3z-4}{1}$

c. $\dfrac{x-2}{4}=\dfrac{y-4}{4}=\dfrac{z-4}{8}$

d. $\dfrac{x-2}{3}=\dfrac{y-4}{2}=\dfrac{z-4}{3}$

e. $\dfrac{x-3}{2}=\dfrac{y-2}{4}=\dfrac{z-3}{4}$

____ 26. Find symmetric equations of the normal line to the surface $3xy-z=0$ at the point $(-2,-4,24)$.

a. $\dfrac{x+2}{12}=\dfrac{y+4}{6}=\dfrac{z-24}{1}$

b. $-\dfrac{x+2}{12}=-\dfrac{y+4}{6}=z-24$

c. $-\dfrac{x+2}{6}=-\dfrac{y+4}{12}=z-24$

d. $3(x-2)=3(y-4)=-(z-24)$

e. $3(x-2)=3(y-4)=(z-24)$

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____ 27. Find symmetric equations of the tangent line to the curve of intersection of the surfaces $z = x^2 + y^2$, $z = 13 - y$ at the point $(1, -3, 10)$.

a. $\dfrac{x-1}{-7} = \dfrac{y+3}{2} = \dfrac{z-10}{-2}$

b. $\dfrac{x-1}{5} = \dfrac{y+3}{2} = \dfrac{z-10}{-2}$

c. $\dfrac{x+5}{1} = \dfrac{y+2}{3} = \dfrac{z-2}{10}$

d. $\dfrac{x+5}{1} = \dfrac{y-2}{-3} = \dfrac{z-2}{10}$

e. $\dfrac{x-1}{5} = \dfrac{y-3}{2} = \dfrac{z-10}{-2}$

____ 28. Find the angle of inclination θ of the tangent plane to the surface $x^2 - 2y^2 + z = 0$ at the point $(3, 3, 9)$. Round your answer to two decimal places.

a. 57.14°
b. 4.26°
c. 4.25°
d. 85.74°
e. 89.68°

____ 29. Examine the function $f(x, y) = \left(-2x^2 - 2y^2 + 2x - 4y + 1\right)$ for relative extrema.

a. relative minimum: $\left(\dfrac{1}{2}, -1, \dfrac{5}{2}\right)$

b. relative maximum: $\left(\dfrac{1}{2}, -1, \dfrac{3}{2}\right)$

c. relative maximum: $(0, 0, 1)$

d. relative maximum: $\left(\dfrac{1}{2}, -1, \dfrac{7}{2}\right)$

e. relative minimum: $(0, 0, 1)$

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____ 30. Examine the function $f(x,y) = 5x^2 - 6y^2 - 30x - 36y + 9$ for relative extrema and saddle points.

a. saddle point: $(3, 3, -198)$

b. relative minimum: $(3, -3, 18)$

c. relative minimum: $(-3, -3, 198)$

d. saddle point: $(3, -3, 18)$

e. saddle point: $(-3, -3, 198)$

____ 31. Examine the function $f(x,y) = x^3 - 18xy + y^3 + 7$ for relative extrema and saddle points.

a. saddle point: $(0, 0, 7)$; relative minimum: $(6, 6, -209)$
b. saddle point: $(6, 6, -209)$; relative minimum: $(0, 0, 7)$
c. relative minimum: $(0, 0, 7)$; relative maximum: $(6, 6, -209)$
d. relative minimum: $(6, 6, -209)$; relative maximum: $(0, 0, 7)$
e. saddle points: $(0, 0, 7)$, $(6, 6, -209)$

____ 32. Determine whether there is a relative maximum, a relative minimum, a saddle point, or insufficient information to determine the nature of the function $f(x,y)$ at the critical point (x_0, y_0), if
$f_{xx}(x_0, y_0) = 16$, $f_{yy}(x_0, y_0) = -2$, $f_{xy}(x_0, y_0) = -1$.

a. relative maximum
b. relative minimum
c. saddle point
d. insufficient information

____ 33. List the critical points of the function $f(x,y) = (x-9)^5 (y+5)^3$ for which the Second Partial Test fails.

a. Test fails at $(-9, a)$ and $(b, 5)$.
b. Test fails only at $(9, -5)$.
c. Test fails at $(9, a)$ and $(b, -5)$.
d. Test fails at $(9, a)$ and $(b, 5)$.
e. Test fails only at $(9, 5)$.

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____ 34. Find the critical points of the function $f(x,y,z) = (x+7)^2 + (7-y)^2 + (z+6)^2$, and from the form of the function, determine whether a relative maximum or a relative minimum occurs at each point.

a. relative minimum at $(7,-7,6)$
b. relative maximum at $(-7,7,-6)$
c. relative maximum at $(7,-7,6)$
d. relative minimum at $(-7,7,-6)$
e. no relative extrema

____ 35. Use Lagrange multipliers to minimize the function $f(x,y) = x^2 - y^2$ subject to the following constraint:

$x - 6y + 45 = 0$

Assume that x and y are positive.

a. $-\dfrac{405}{7}$

b. 0

c. $-\dfrac{7}{405}$

d. $\dfrac{405}{7}$

e. no absolute minimum

____ 36. Use Lagrange multipliers to maximize the function $f(x,y) = \sqrt{77 - x^2 - y^2}$ subject to the following constraint.

$x + y - 12 = 0$

Assume that x and y are positive.

a. $\sqrt{5}$

b. 5

c. 149

d. $\sqrt{149}$

e. no absolute maximum

____ 37. Use Lagrange multipliers to find the maximum value of $f\left(x,y,z\right) = xyz$ where $x > 0, y > 0$, and $z > 0$, subject to the constraint $x + y + z - 51 = 0$.

a. 4096
b. 4896
c. 5832
d. 5202
e. 4913

____ 38. Use Lagrange multipliers to find the minimum distance from the line $6x + 7y = -1$ to the point $\left(0,0\right)$.

a. $\frac{1}{85}$

b. $\frac{\sqrt{13}}{13}$

c. $\frac{\sqrt{13}}{85}$

d. $\frac{1}{13}$

e. $\frac{\sqrt{85}}{85}$

____ 39. Use Lagrange multipliers to find the minimum distance from the circle $(x-2)^2 + y^2 = 1$ to the point $\left(0,11\right)$. Round your answer to two decimal places.

a. 2.07
b. 4.28
c. 103.64
d. 10.18
e. 2.09

____ 40. Use Lagrange multipliers to find the minimum distance from the plane $x + y + z = 5$ to the point $\left(8,7,7\right)$. Round your answer to two decimal places.

a. 3.00
b. 15.67
c. 9.00
d. 9.81
e. 96.33

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Answer Section

MULTIPLE CHOICE

1. ANS: D PTS: 1 DIF: Medium REF: Section 13.1
OBJ: Evaluate and simplify a multivariable function MSC: Skill
2. ANS: C PTS: 1 DIF: Medium REF: Section 13.1
OBJ: Simplify a difference quotient for a multivariable function MSC: Skill
3. ANS: C PTS: 1 REF: Section 13.1
OBJ: Describe the domain of a multivariable function MSC: Concept
4. ANS: C PTS: 1 DIF: Medium REF: Section 13.1
OBJ: Graph level curves of a multivariable function MSC: Skill
5. ANS: C PTS: 1 DIF: Medium REF: Section 13.1
OBJ: Graph level curves of a multivariable function MSC: Skill
6. ANS: A PTS: 1 DIF: Medium REF: Section 13.3
OBJ: Calculate a partial derivative of a function MSC: Skill
7. ANS: B PTS: 1 DIF: Medium REF: Section 13.3
OBJ: Evaluate the partial derivative of a function at a point MSC: Skill
8. ANS: A PTS: 1 DIF: Medium REF: Section 13.3
OBJ: Identify all points where the first partial derivatives are simultaneously zero
MSC: Skill
9. ANS: A PTS: 1 DIF: Easy REF: Section 13.5
OBJ: Differentiate a function using the appropriate Chain Rule MSC: Skill
10. ANS: D PTS: 1 DIF: Medium REF: Section 13.5
OBJ: Differentiate a function using the appropriate Chain Rule MSC: Skill
11. ANS: B PTS: 1 DIF: Medium REF: Section 13.5
OBJ: Differentiate a multivariable function using the appropriate Chain Rule and evaluate for given values
MSC: Skill
12. ANS: D PTS: 1 DIF: Medium REF: Section 13.5
OBJ: Differentiate a multivariable function using the appropriate Chain Rule
MSC: Skill
13. ANS: A PTS: 1 DIF: Medium REF: Section 13.5
OBJ: Differentiate an equation in two variables implicitly MSC: Skill
14. ANS: E PTS: 1 DIF: Medium REF: Section 13.5
OBJ: Differentiate an equation in three variables implicitly MSC: Skill
15. ANS: D PTS: 1 DIF: Medium REF: Section 13.5
OBJ: Interpret derivatives as rates of change MSC: Application
16. ANS: C PTS: 1 DIF: Medium REF: Section 13.6
OBJ: Calculate the directional derivative of a function at a point
MSC: Skill
17. ANS: E PTS: 1 DIF: Medium REF: Section 13.6
OBJ: Calculate the directional derivative of a function at a point in the direction of another point
MSC: Skill
18. ANS: B PTS: 1 DIF: Medium REF: Section 13.6
OBJ: Calculate the directional derivative of a function at a point in the direction of another point
MSC: Skill

19. ANS: B PTS: 1 DIF: Medium REF: Section 13.6
OBJ: Identify a normal vector to a level curve of a function MSC: Skill
NOT: Section 13.6
20. ANS: E PTS: 1 DIF: Medium REF: Section 13.6
OBJ: Identify the direction of greatest increase from a given point in applications
MSC: Application
21. ANS: D PTS: 1 DIF: Medium REF: Section 13.6
OBJ: Construct a function in applications using the properties of the gradient of a function
MSC: Application
22. ANS: C PTS: 1 DIF: Easy REF: Section 13.7
OBJ: Construct a unit normal vector to a surface at a given point
MSC: Skill
23. ANS: D PTS: 1 DIF: Medium REF: Section 13.7
OBJ: Construct a unit normal vector to a surface at a given point
MSC: Skill
24. ANS: B PTS: 1 DIF: Easy REF: Section 13.7
OBJ: Write an equation of the plane tangent to a surface at a specified point
MSC: Skill
25. ANS: D PTS: 1 DIF: Easy REF: Section 13.7
OBJ: Write a set of symmetric equations for a line normal to a surface at a specified point
MSC: Skill
26. ANS: A PTS: 1 DIF: Medium REF: Section 13.7
OBJ: Write a set of symmetric equations for a line normal to a surface at a specified point
MSC: Skill
27. ANS: B PTS: 1 DIF: Medium REF: Section 13.7
OBJ: Write a set of symmetric equations for a line tangent to the curve of intersection of two surfaces
MSC: Skill
28. ANS: D PTS: 1 DIF: Medium REF: Section 13.7
OBJ: Calculate the angle of inclination of a tangent plane to a surface at a given point
MSC: Skill
29. ANS: D PTS: 1 DIF: Easy REF: Section 13.8
OBJ: Identify the relative extrema of a function MSC: Skill
30. ANS: D PTS: 1 DIF: Medium REF: Section 13.8
OBJ: Identify the relative extrema and saddle points of a function
MSC: Skill
31. ANS: A PTS: 1 DIF: Medium REF: Section 13.8
OBJ: Identify the relative extrema and saddle points of a function
MSC: Skill
32. ANS: C PTS: 1 DIF: Easy REF: Section 13.8
OBJ: Identify the nature of a function at a critical point using the Second Partials Test
MSC: Skill
33. ANS: C PTS: 1 DIF: Easy REF: Section 13.8
OBJ: List the critical points of a function for which the Second Partial Test fails
MSC: Skill
34. ANS: D PTS: 1 DIF: Easy REF: Section 13.8
OBJ: Identify all critical points of a function and use the graph to test the nature of the function at the critical points MSC: Skill

35. ANS: A PTS: 1 DIF: Medium REF: Section 13.10
OBJ: Calculate the minimum of a two-variable function with constraints using Lagrange multipliers
MSC: Skill
36. ANS: A PTS: 1 DIF: Medium REF: Section 13.10
OBJ: Calculate the maximum of a two-variable function with constraints using Lagrange multipliers
MSC: Skill
37. ANS: E PTS: 1 DIF: Medium REF: Section 13.10
OBJ: Calculate the minimum of a three-variable function with constraints using Lagrange multipliers
MSC: Skill
38. ANS: E PTS: 1 DIF: Medium REF: Section 13.10
OBJ: Calculate the minimum distance from a line to a point using Lagrange multipliers
MSC: Application
39. ANS: D PTS: 1 DIF: Medium REF: Section 13.10
OBJ: Calculate the minimum distance from a circle to a point using Lagrange multipliers
MSC: Application
40. ANS: D PTS: 1 DIF: Medium REF: Section 13.10
OBJ: Calculate the minimum distance from a circle to a point using Lagrange multipliers
MSC: Application