Chapter 3: Gas Reservoir Deliverability (Well Inflow Performance)

Gas production rate
Gas Well Inflow Performance Relationship (IPR)
Analytical method (based on theory)
Empirical Method (based on experience/experiment)
Production System

Understanding the principles of fluid flow through the production system is important in estimating the performance of wells and optimizing well and reservoir productivity. In the most general sense, the production system is the system that transports reservoir fluids from the subsurface reservoir to the surface, processes and treats the fluids, and prepares the fluids for storage and transfer to a purchaser. The basic elements of the production system include the reservoir; wellbore; tubular goods and associated equipment; surface wellhead, flowlines, and processing equipment; and artificial lift equipment.

- The reservoir is the source of fluids for the production system. It furnishes the primary energy for the production system.
- The wellbore serves as the conduit for access to the reservoir from the surface. The cased wellbore houses the tubing and associated subsurface production equipment, such as packers.
- The tubing serves as the primary conduit for fluid flow from the reservoir to the surface; fluids may also be transported through the tubing-casing annulus.
- The wellhead, flowlines, and processing equipment represent the surface mechanical equipment required to control and process reservoir fluids at the surface and prepare them for transfer to a purchaser. Surface equipment includes the wellhead equipment and associated valving, chokes, manifolds, flowlines, separators, treatment equipment, metering devices, and storage vessels.
\[ \Delta p_1 = \bar{p}_R - p_{wfs} \quad \text{Loss in reservoir} \]
\[ \Delta p_2 = p_{wfs} - p_{wf} \quad \text{Loss in completion} \]
\[ \Delta p_3 = p_{wf} - p_{wh} \quad \text{Loss in tubing} \]
\[ \Delta p_4 = p_{wh} - p_s \quad \text{Loss in flowline} \]
\[ \Delta p_T = \bar{p}_R - p_s \quad \text{Total pressure loss} \]
Deliverability Testing

Reducing the size of the well bore or increasing the pressure of the system into which the well must produce, increases the resistance to flow and therefore reduces the Deliverability of the well. The Deliverability Test allows prediction of flow rates for different line and reservoir pressures.
Deliverability testing goes under several names such as "Back-Pressure Testing", "4-Point Testing", "Open Flow Potential Testing", and "AOF Testing". The terms "Open Flow Potential" and "Absolute Open Flow" refer to the theoretical maximum flow rate from the reservoir. A "Deliverability Test" usually requires the well to be produced at several rates.
A general solution to pseudosteady state flow in a radial-flow gas reservoir is expressed as (Economides 1994):

\[ q = \frac{kh[m(\bar{p}) - m(p_{wf})]}{1424T \left[ \ln \left( \frac{0.472r_e}{r_w} \right) + s + Dq \right]} \]

where \( q \) is the gas production rate in Mscf/d, \( k \) is the effective permeability to gas in md, \( h \) is the thickness of pay zone in ft, \( m(\bar{p}) \) is the real gas pseudopressure in psi^2/cp at the reservoir pressure \( \bar{p} \) in psi, \( m(p_{wf}) \) is the real gas pseudopressure in psi^2/cp at the flowing bottom hole pressure \( p_{wf} \), \( T \) is the reservoir temperature in R, \( \gamma_w \) is the radius of drainage area in ft, \( \gamma_w \) is wellbore radius in ft, \( s \) is skin factor, and \( D \) is the non-Darcy coefficient in d/Mscf. The skin factor and non-Darcy coefficient can be estimated on the basis of pressure transient analyses.
At pressures $< 2000$ psia

$$m(p) = \int_{p_b}^{p} \frac{2p}{\mu z} dp \approx \frac{p^2 - p_b^2}{\mu z}$$

$$q = \frac{kh(\bar{p}^2 - p_{wf}^2)}{1424 \bar{\mu} \bar{z} T \left[ \ln \left( \frac{0.472r_e}{r_w} \right) + s + Dq \right]}$$

At pressures $> 3000$ psia

Compressed gases behave like liquids

$$q = \frac{kh(\bar{p} - p_{wf})}{141.2 \times 10^3 \bar{B}_g \bar{\mu} \left[ \ln \left( \frac{0.472r_e}{r_w} \right) + s + Dq \right]}$$

where $\bar{B}_g$ is the average formation volume factor of gas in rb/scf.
Example Problem 3.1

A gas well produces 0.65 specific gravity natural gas with N₂, CO₂, and H₂S of mole fractions 0.1, 0.08, and 0.02, respectively. The well diameter is 7-7/8 inches. It drains gas from a 78-ft thick pay zone in an area of 160 acres. The average reservoir pressure is 4,613 psia. Reservoir temperature is 180 °F. Assuming a Darcy skin factor of 5 and a non-Darcy coefficient of 0.001 day/Mscf, estimate the deliverability of the gas reservoir under pseudosteady state flow condition at a flowing bottom hole pressure of 3,000 psia.

1 acre = 43560 ft²

Solution

Theoretical Deliverability.xls

\[ q = \frac{kh (\bar{p} - p_{wf})}{141.2 \times 10^3 B_g \mu \left[ \ln \left( \frac{0.472 r_e}{r_w} \right) + s + Dq \right]} \]
Gas Deliverability - Empirical

\[ q = \frac{kh[m(\bar{p}) - m(p_{wf})]}{1424T \left[ \ln \left( \frac{0.472r_e}{r_w} \right) + s + Dq \right]} \]

**Forchheimer model**

\[ m(\bar{p}) - m(p_{wf}) = Aq + Bq^2 \]

**Backpressure model**

Rawlins and Schellhardt

\[ q = C[m(\bar{p}) - m(p_{wf})]^n \]

\[
A = \frac{[m(\bar{p}) - m(p_{wf1})] - Bq_1^2}{q_1} \\
B = \frac{[m(\bar{p}) - m(p_{wf1})]q_2 - [m(\bar{p}) - m(p_{wf2})]q_1}{q_1q_2 - q_2^2q_1} \\
C = \frac{q_1}{\left[ m(\bar{p}) - m(p_{wf1}) \right]^n} \\
n = \frac{\log \left( \frac{q_1}{q_2} \right)}{\log \left( \frac{m(\bar{p}) - m(p_{wf1})}{m(\bar{p}) - m(p_{wf2})} \right)}
\]

A, B, C, and \( n \) are empirical constants that can be determined based on test points. The value of \( n \) is usually between 0.5 and 1.
Forchheimer model

\[ m(\bar{p}) - m(p_{wf}) = Aq + Bq^2 \]

\[ \bar{p}^2 - p_{wf}^2 = Aq + Bq^2 \]

\[ A = \frac{(\bar{p}^2 - p_{wf1}^2) - Bq_1^2}{q_1} \]

\[ B = \frac{(\bar{p}^2 - p_{wf1}^2)q_2 - (\bar{p}^2 - p_{wf2}^2)q_1}{q_1^2q_2 - q_2^2q_1} \]

Backpressure model

\[ q = C[m(\bar{p}) - m(p_{wf})]^n \]

\[ q = C(\bar{p}^2 - p_{wf}^2)^n \]

\[ n = \frac{\log\left(\frac{q_1}{q_2}\right)}{\log\left(\frac{\bar{p}^2 - p_{wf1}^2}{\bar{p}^2 - p_{wf2}^2}\right)} \]

\[ C = \frac{q_1}{\left[\bar{p}^2 - p_{wf1}^2\right]^n} \]
Example Problem 3.2

A gas well produces 0.65 specific gravity natural gas with \( \text{N}_2 \), \( \text{CO}_2 \), and \( \text{H}_2\text{S} \) of mole fractions 0.1, 0.08, and 0.02, respectively. The average reservoir pressure is 4,505 psia. Reservoir temperature is 180 °F. The well was tested at two flow rates:

<table>
<thead>
<tr>
<th>Test point 1</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Flow rate:</td>
<td>1,152 Mscf/d</td>
</tr>
<tr>
<td>Bottom hole pressure:</td>
<td>3,025 psia</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Test point 2</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Flow rate:</td>
<td>1,548 Mscf/d</td>
</tr>
<tr>
<td>Bottom hole pressure:</td>
<td>1,685 psia</td>
</tr>
</tbody>
</table>

Estimate the deliverability of the gas reservoir under a pseudosteady state flow condition at a flowing bottom hole pressure of 1,050 psia.

Empirical Deliverability.xls
Inflow Performance Relationship (IPR) Curve

Theoretical

The graph shows the relationship between the flowing bottom hole pressure (psia) and the gas production rate (Mscf/d) for two different approaches: $p$ (black line) and $p^2$ (red line). The pressure decreases as the production rate increases, illustrating the inverse relationship between these two variables.
Empirical

<table>
<thead>
<tr>
<th>$p_{wf}$ (psia)</th>
<th>Forchheimer</th>
<th>Backpressure</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>1704</td>
<td>1709</td>
</tr>
<tr>
<td>239</td>
<td>1701</td>
<td>1706</td>
</tr>
<tr>
<td>464</td>
<td>1693</td>
<td>1698</td>
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<td>688</td>
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<td>1660</td>
<td>1663</td>
</tr>
<tr>
<td>1137</td>
<td>1634</td>
<td>1637</td>
</tr>
<tr>
<td>1362</td>
<td>1603</td>
<td>1605</td>
</tr>
</tbody>
</table>
**Absolute Open Flow**

Early estimates of gas well performance were conducted by opening the well to the atmosphere and then measuring the flow rate. Such “open flow” practices were wasteful of gas, sometimes dangerous to personnel and equipment, and possibly damaging to the reservoir. They also provided limited information to estimate productive capacity under varying flow conditions. The idea, however, did leave the industry with the concept of absolute open flow (AOF). AOF is a common indicator of well productivity and refers to the maximum rate at which a well could flow against a theoretical atmospheric backpressure at the reservoir.
I.P.R.

- Back Pressure (psi)
- Gas Rate (MMcfd)

AOF Deliverability

AOF
Chapter 4:
Wellbore Performance
(Well Outflow Performance)

Single-Phase Liquid Flow
Multiphase Flow
The pressure drop experienced in lifting reservoir fluids to the surface is one of the main factors affecting well deliverability. As much as 80% of the total pressure loss in a flowing well may occur in lifting the reservoir fluid to the surface. Wellbore flow performance relates to estimating the pressure-rate relationship in the wellbore as the reservoir fluids move to the surface through the tubulars.

The flow path through the wellbore may include flow through perforations, a screen and liner, and packers before entering the tubing for flow to the surface. The tubing may contain completion equipment that acts as flow restrictions, such as:

- Profile nipples
- Sliding sleeves
- Subsurface flow-control devices

In addition, the tubing string may be composed of multiple tubing diameters or allow for tubing/annulus flow to the surface. At the surface, the fluid must pass through wellhead valves, surface chokes, and through the flowline consisting of surface piping, valves, and fittings to the surface-processing equipment. The pressure drop experienced as the fluid moves from the reservoir sandface to the surface is a function of the mechanical configuration of the wellbore, the properties of the fluids, and the producing rate.
Well Outflow Performance

- The achievable oil production rate from a well is determined by wellhead pressure and the flow performance of production string, that is, tubing, casing.
- The flow performance of production string depends on geometries of the production string and properties of fluids being produced. The fluids in oil wells include oil, water, gas, and sand.
- Wellbore performance analysis involves establishing a relationship between tubular size, wellhead and bottom-hole pressure, fluid properties, and fluid production rate.
- Understanding wellbore flow performance is vitally important to production engineers for designing oil well equipment and optimizing well production conditions.
- Petroleum can be produced through tubing, casing, or both in a well depending on which flow path has better performance. Producing through tubing is a better option in most cases. The traditional term outflow performance relationship (OPR) or tubing performance relationship (TPR) is used (other terms such as vertical lift performance (VLP) or vertical flow performance (VFP) have been used). However, the mathematical models are also valid for casing flow and casing-tubing annular flow as long as hydraulic diameter is used.
Pressure loss through the wellbore

The flow path through the wellbore may include flow through perforations, a screen and liner, and packers before entering the tubing for flow to the surface. The tubing may contain completion equipment that acts as flow restrictions. In addition, the tubing string may be composed of multiple tubing diameters or allow for tubing/annulus flow to the surface. At the surface, the fluid must pass through wellhead valves, surface chokes, and through the flowline consisting of surface piping, valves, and fittings to the surface-processing equipment. The pressure drop experienced as the fluid moves from the reservoir sandface to the surface is a function of the mechanical configuration of the wellbore, the properties of the fluids, and the producing rate.

Relationships to estimate this pressure drop in the wellbore are based on the mechanical energy equation for flow between two points in a system as

$$\frac{p_1}{\rho} + \frac{g}{g_c}Z_1 + a\frac{v_1^2}{2g_c} = \frac{p_2}{\rho} + \frac{g}{g_c}Z_2 + a\frac{v_2^2}{2g_c} + W + E_l.$$  

For most practical applications, there is no work done by or on the fluid and the kinetic energy correction factor is assumed to be one.
We are interested in the determination of TPR and pressure traverse along the well string. **Tubing Performance Relationship (TPR)** is defined as a relation between tubing size, fluid properties, fluid flow rate, wellhead pressure, and bottom hole pressure. In most engineering analyses, it is desired to know the bottom hole pressure at a given wellhead pressure and flow rate in a gas well. There are two theoretical approaches:

1. **Single-Phase Flow**: assumes only a single phase is flowing (gas only or oil only)
2. **Mist Flow**: assumes multiple phases (up to four: water, oil, gas, sand) are flowing through the tube due to high velocity. When natural gas flows to the surface in a producing gas well, the gas carries liquids to the surface if the velocity of the gas is high enough. A high gas velocity results in a *mist flow* pattern in which liquids are finely dispersed in the gas. Consequently, a low volume of liquid is present in the tubing or production conduit, resulting in a pressure drop caused by gravity acting on the flowing fluids.

Methods to estimate the pressure drop in tubulars for single-phase liquid, single-phase vapor, and multiphase flow are based on this fundamental relationship:

$$ \frac{dp}{dL} = \frac{g}{g_c} \rho \sin \theta + \frac{\rho v}{g_c} \frac{dv}{dL} + \frac{f \rho v^2}{2g_c d} \, . $$
The first law of thermodynamics (conservation of energy) governs gas flow in tubing.

\[
\frac{p_1}{\rho} + \frac{g}{g_c} Z_1 + \alpha \frac{v_1^2}{2g_c} = \frac{p_2}{\rho} + \frac{g}{g_c} Z_2 + \alpha \frac{v_2^2}{2g_c} + W + E_1.
\]

The effect of kinetic energy change is negligible due to the fact that the variation in tubing diameter is insignificant in most gas wells. With no shaft work device installed along the tubing string, the first law of thermodynamics yields the following mechanical balance equation:

\[
\frac{dP}{\rho} + \frac{g}{g_c} dZ + \frac{f v^2 dL}{g_c D_i} = 0
\]

\[
\frac{zRT}{29\gamma_g} \frac{dP}{P} + \left\{ \frac{g}{g_c} \cos \theta + \frac{8 f Q_{sc}^2 P_{sc}^2}{\pi^2 g_c D_i^5 T_{sc}^2} \left[ \frac{zT}{P} \right]^2 \right\} dL = 0
\]
Average Temperature and Compressibility Factor Method

\[
P_{wf}^2 = \text{Exp}(s)p_{hf}^2 + \frac{8f [\text{Exp}(s) - 1] Q_{sc}^2 P_{sc}^2 \bar{z}^2 \bar{T}^2}{\pi^2 g_c D_i^5 T_{sc}^2 \cos \theta}
\]

In US unit notations  \( \text{(q in Mscf/d)} \)

\[
p_{wf}^2 = \text{Exp}(s)p_{hf}^2 + \frac{6.67 \times 10^{-4}[\text{Exp}(s) - 1] f q_{sc}^2 \bar{z}^2 \bar{T}^2}{d_i^5 \cos \theta}
\]

Moody factor (fully turbulent flow)

\[
f = \begin{cases} 
0.01750 & \text{for } d_i \leq 4.277 \text{ in} \\
0.01603 & \text{for } d_i > 4.277 \text{ in}
\end{cases}
\]

Guo (fully turbulent flow)

\[
f = \left[ \frac{1}{1.74 - 2 \log \left( \frac{2e}{d_i} \right)} \right]^2
\]
Problem

Suppose that a vertical well produces 2 MMscf/d of 0.71 gas-specific gravity gas through a 2 7/8-in tubing set to the top of a gas reservoir at a depth of 10,000 ft. At tubing head, the pressure is 800 psia and the temperature is 150 °F; the bottom hole temperature is 200 °F. The relative roughness of tubing is about 0.0006. Calculate the pressure profile along the tubing length and plot the results.

Cullender and Smith Method

\[ p_{mf} = p_{hf} + \frac{18.75 \gamma_g L}{I_{mf} + I_{hf}} \]

\[ p_{wf} = p_{mf} + \frac{18.75 \gamma_g L}{I_{wf} + I_{mf}} \]

The integrant denoted with symbol \( I \), is

\[ I = \frac{p}{zT} \left[ 0.001 \cos \theta \left( \frac{p}{zT} \right)^2 + 0.666 \frac{f q_{sc}^2}{d_i^5} \right] \]

Example Problem 4.2

Solve the problem in Example Problem 4.1 with the Cullender and Smith Method.

Cullender-Smith.xls
Gas well deliqufication, also referred to as "gas well dewatering", is the general term for technologies used to remove water or condensates build-up from producing gas wells. When natural gas flows to the surface in a producing gas well, the gas carries liquids to the surface if the velocity of the gas is high enough. A high gas velocity results in a mist flow pattern in which liquids are finely dispersed in the gas. Consequently, a low volume of liquid is present in the tubing or production conduit, resulting in a pressure drop caused by gravity acting on the flowing fluids. As the gas velocity in the production tubing drops with time, the velocity of the liquids carried by the gas declines even faster. Flow patterns of liquids on the walls of the conduit cause liquid to accumulate in the bottom of the well, which can either slow or stop gas production altogether.
The TPR equations presented in the previous section are not valid for multiphase gas wells. To analyze TPR of multiphase gas wells, a gas-oil-water-solid four-phase flow model is presented in this section. It is warned that the four-phase flow model is valid only for gas wells producing multiphase fluid with gas being the main component. Specifically, the model can be used with good accuracy when mist flow exists in the wellbore. When the flow velocity drops to below a critical velocity at which the liquid droplets cannot be carried up to surface by gas, annular flow or even slug flow may develop in the well. TPR equations for annular flow and slug flow are available from literature of oil well performance.
The gas-oil-water-solid four-phase flow model was first presented by Guo (2001) for coal-bed methane production wells. Guo formulated the governing equation assuming homogeneous mixture of the four phases, which may exist in misting flow. According to Guo, Sun, and Ghalambor (2004) the following equation can be used for calculating pressure $P$ (in lbf/ft$^2$) at depth $L$:

$$b(P - P_{hf}) + \frac{1 - 2bM}{2} \ln \left| \frac{(P + M)^2 + N}{(P_{hf} + M)^2 + N} \right|$$

$$\frac{M + \frac{b}{c}N - bM^2}{\sqrt{N}} \left[ \tan^{-1}\left( \frac{P + M}{\sqrt{N}} \right) - \tan^{-1}\left( \frac{P_{hf} + M}{\sqrt{N}} \right) \right]$$

$$= a \left( \cos \theta + d^2 e \right) L$$
\[ a = \frac{0.0765\gamma_g Q_{sc} + 350\gamma_o q_o + 350\gamma_w q_w + 62.4\gamma_s q_s}{4.07T_{av} Q_{sc}} \]

\[ b = \frac{5.615q_o + 5.615q_w + q_s}{4.07T_{av} Q_{sc}} \]

\[ c = 0.00678 \frac{T_{av} Q_{sc}}{A} \]

\[ d = \frac{0.00166}{A} (5.615q_o + 5.615q_w + q_s) \]

\[ e = \frac{f}{2gD_i} \]

\[ M = \frac{cde}{\cos \theta + d^2e} \]

\[ N = \frac{c^2e \cos \theta}{(\cos \theta + d^2e)^2} \]
Example Problem 4.3

Solve the problem in Example Problem 4.1 for bottom hole pressure with the following additional data:

- Condensate Gas Ratio (CGR): 0.02 bbl/Mscf
- Water Cut (WC): 50%
- Oil gravity: 60 0API
- Water-specific gravity: 1.03
- Sand production: 0.1 ft3/d
- Sand-specific gravity: 2.65

Solution

This example problem is solved with the spreadsheet program MistFlow.xls. Table 4-3 shows the appearance of the spreadsheet for the data input and results sections. It indicates a flowing bottom hole pressure of 1,103 psia.
Class Assignment

4-1 Suppose 3 MMscf/d of 0.75 specific gravity gas are produced through a 3 1/2-in (di = 3 in) tubing string set to the top of a gas reservoir at a depth of 8,000 ft. At tubing head, the pressure is 1,000 psia and the temperature is 120 °F; the bottom hole temperature is 180 °F. The relative roughness of the tubing is about 0.0006. Calculate the flowing bottom hole pressure with three methods:

a) the average temperature and compressibility factor method;
b) the Cullender and Smith method; and
c) the fourphase flow method. Make comments on your results.

- Condensate Gas Ratio (CGR): 0.02 bbl/Mscf
- Water Cut (WC): 40%
- Oil gravity: 45 API
- Water-specific gravity: 1.03
- Sand production: 0.1 ft³/d
- Sand-specific gravity: 2.35

4-2 Solve Problem 4-1 for gas production through a K-55, 17 lb/ft, 5 1/2-in casing.
4-3 Suppose 2 MMscf/d of 0.65 specific gravity gas are produced through a 2 7/8-in (2.259-in ID) tubing string set to the top of a gas reservoir at a depth of 5,000 ft. Tubing head pressure is 300 psia and the temperature is 100 0F; the bottom hole temperature is 150 0F. The relative roughness of the tubing is about 0.0006. Calculate the flowing bottom pressure with the average temperature and compressibility factor method.
Chapter 5: Choke Performance

Subsonic and Sonic Flow
Temperature at Choke
Intro

Multiphase flow occurs in almost all producing oil and gas/condensate wells. Every flowing well has some devices to control the flow rate for maintaining sufficient back pressure to prevent formation damage (sand production), to protect surface equipments, to prevent water/gas coning, to stabilize the flow rate and to produce the reservoir at the most efficient possible rate.

• Chokes are one of the most important flow controllers in oil and gas producing wells.
• Accurate modeling of choke performance and selection of optimum choke size is vitally important for a petroleum engineer in production from reservoirs due to high sensitivity of oil and gas production to choke size.
• Flow through a surface choke can be described as either critical or sub-critical. Critical flow occurs when the velocity through the choke is greater than the sonic velocity of the fluid.
• Chokes are classified as nozzle-type and orifice-type with fixed (or adjustable) diameters.
Sonic and Subsonic Flow

• Pressure drop across well chokes is usually very significant. There is no universal equation for predicting pressure drop across the chokes for all types of production fluids.
• *Subsonic* or *sonic* flow is based on the gas fraction in the fluid and flow regimes;
• When the fluid flow velocity in a choke reaches the traveling velocity of sound in the fluid under the in situ condition, the flow is called sonic flow;
• Whether or not a sonic flow exists at a choke depends on a downstream to upstream pressure ratio. If this pressure ratio is less than a critical pressure ratio, sonic (critical) flow exists. If this pressure ratio is greater or equal to the critical pressure ratio, subsonic (subcritical) flow exists.

\[
\left( \frac{P_{\text{outlet}}}{P_{\text{up}}} \right)_c = \left( \frac{2}{k + 1} \right)^\frac{k}{k - 1}
\]

The value of the \(k\) is 1.4 for air and 1.28 for natural gas. Thus, the critical pressure ratio is 0.528 for air and 0.549 for natural gas.

\[k = \frac{C_p}{C_v}\]
Subsonic Flow

\[ Q_{sc} = 1,248 C A p_{up} \sqrt{\frac{k}{(k-1) \gamma g T_{up}}} \left[ \left( \frac{p_{dn}}{p_{up}} \right)^{\frac{2}{k}} - \left( \frac{p_{dn}}{p_{up}} \right)^{\frac{k+1}{k}} \right] \]

\[ C = \frac{d}{D} + \frac{0.3167}{\left( \frac{d}{D} \right)^{0.6}} + 0.025[\log(N_{Re}) - 4] \]

\[ v = \sqrt{v_{up}^2 + 2 g_c C_p T_{up}} \left[ 1 - \frac{z_{up}}{z_{dn}} \left( \frac{p_{down}}{p_{up}} \right)^{\frac{k-1}{k}} \right] \]
Figure 5-1  Choke flow coefficient for nozzle-type chokes.
Figure 5–2  Choke flow coefficient for orifice-type chokes.
Sonic Flow

\[ Q_{sc} = 879 C A P_{up} \sqrt{\left( \frac{k}{\gamma_{g} T_{up}} \right)^{k+1} \left( \frac{2}{k+1} \right)^{k-1}} \]

\[ v = \sqrt{v_{up}^2 + 2 g_c C_p T_{up} \left[ 1 - \frac{z_{up}}{z_{outlet}} \left( \frac{2}{k+1} \right) \right]} \]

\[ v \approx 44.76 \sqrt{T_{up}} \]

Temperature at Choke

\[ T_{dn} = T_{up} \frac{z_{up}}{z_{outlet}} \left( \frac{P_{outlet}}{P_{up}} \right)^{\frac{k-1}{k}} \]
Example Problem 5.1

A 0.6 specific gravity gas flows from a 2-in pipe through a 1-in orifice-type choke. The upstream pressure and temperature are 800 psia and 75 °F, respectively. The downstream pressure is 200 psia (measured 2 ft from the orifice). The gas-specific heat ratio is 1.3. (a) What is the expected daily flow rate? (b) Does heating need to be applied to assure that the frost does not clog the orifice? (c) What is the expected pressure at the orifice outlet?

\[
\left( \frac{P_{\text{outlet}}}{P_{\text{up}}} \right) = \left( \frac{2}{k+1} \right)^{k-1} \left( \frac{2}{1.3+1} \right)^{1.3-1} = 0.5459
\]

\[
\frac{P_{\text{dn}}}{P_{\text{up}}} = \frac{200}{800} = 0.25 < 0.5459 \text{ Sonic flow exists.}
\]

\[
\frac{d}{D} = \frac{1''}{2''} = 0.5
\]
(a) Assuming $N_{Re} > 10^6$, Figure 5–2 gives \( C = 0.62 \).

\[
Q_{sc} = 879 C A P_{up} \sqrt{\left( \frac{k}{\gamma_g T_{up}} \right) \left( \frac{2}{k+1} \right)^{\frac{k+1}{k-1}}} \]

\[
Q_{sc} = (879)(0.62)(\pi/4*1^2)(800) \sqrt{\left( \frac{1.3}{(0.6)(75+460)} \right) \left( \frac{2}{1.3+1} \right)^{\frac{1.3+1}{1.3-1}}} \]

\( Q_{sc} = 12,743 \text{ Mscf/d} \)

(b) \[
T_{dn} = T_{up} \frac{z_{up}}{z_{outlet}} \left( \frac{P_{outlet}}{P_{up}} \right)^{\frac{k-1}{k}} = (75+460)(1)(0.5459)^{\frac{1.3-1}{1.3}} = 465 \text{ °R} = 5 \text{ °F} \]

Therefore, heating is needed to prevent icing.

(c) \[
P_{outlet} = P_{up} \left( \frac{P_{outlet}}{P_{up}} \right) = (800)(0.5459) = 437 \text{ psia} \]
**Example Problem 5.2**

A 0.65 specific gravity natural gas flows from a 2-in pipe through a 1.5-in nozzle-type choke. The upstream pressure and temperature are 100 psia and 70 °F, respectively. The downstream pressure is 80 psia (measured 2 ft from the nozzle). The gas specific heat ratio is 1.25. (a) What is the expected daily flow rate? (b) Is icing a potential problem? (c) What is the expected pressure at the nozzle outlet?

\[
\left( \frac{P_{\text{outlet}}}{P_{\text{up}}} \right)_c = \left( \frac{2}{k+1} \right)^{\frac{k}{k-1}} = \left( \frac{2}{1.25+1} \right)^{1.25-1} = 0.5549
\]

\[
P_{dn} = \frac{80}{100} = 0.8 \quad > 0.5549 \quad \text{Subsonic flow exists.}
\]

\[
\frac{d}{D} = \frac{1.5''}{2''} = 0.75
\]

Assuming \(N_{Re} > 10^6\), Figure 5–1 gives \( C = 1.2 \).
Example Problem 5.3

For the following given data, estimate upstream pressure at choke:

- Downstream pressure: 300 psia
- Choke size: 32 1/64 in
- Flowline ID: 2 in
- Gas production rate: 5,000 Mscf/d
- Gas-specific gravity: 0.75 (1 for air)
- Gas-specific heat ratio: 1.3
- Upstream temperature: 110 °F
- Choke discharge coefficient: 0.99
Assignment 6

5-1 A 0.66 specific gravity gas flows from a 2-in pipe through a 1.5-in orifice-type choke. The upstream pressure and temperature are 600 psia and 75 °F, respectively. The downstream pressure is 200 psia (measured 2 ft from the orifice). The gas-specific heat ratio is 1.3. (a) What is the expected daily flow rate? (b) Does heating need to be applied to assure that the frost does not clog the orifice? (c) What is the expected pressure at the orifice outlet?

5-3 For the following given data, estimate upstream pressure at choke:

   Downstream pressure: 500 psia
   Choke size: 48 1/64 in
   Flowline ID: 2 in
   Gas production rate: 4,000 Mscf/d
   Gas-specific gravity: 0.70 1 for air
   Gas-specific heat ratio: 1.3
   Upstream temperature: 100 °F
   Choke discharge coefficient: 1.05
PRTT 2323- Natural Gas Production

Chapter 6:
Well Deliverability

Nodal Analysis with bottom hole
Nodal Analysis with well head
Nodal Analysis

• Fluid properties change with the location-dependent pressure and temperature in the gas production system. To simulate the fluid flow in the system, it is necessary to "break“ the system into discrete nodes that separate system elements. Fluid properties at the elements are evaluated locally.
• The system analysis for determination of fluid production rate and pressure at a specified node is called *Nodal analysis*.
• Nodal analysis is performed on the principle of pressure continuity, that is, there is only one unique pressure value at a given node no matter whether the pressure is evaluated from the performance of upstream equipment or downstream equipment.
• The performance curve (pressure rate relation) of upstream equipment is called *inflow performance* curve; the performance curve of downstream equipment is called *outflow performance* curve. The intersection of the two performance curves defines the operating point, i.e., operating flow rate and pressure, at the specified node.
• For the convenience of using pressure data measured normally at either bottom hole or wellhead, Nodal analysis is usually conducted using the bottom hole or wellhead as the solution node.
**Bottom hole Node**

When the bottom hole is used as a solution node in Nodal analysis,

- inflow performance is the well IPR
- outflow performance is the TPR

  the tubing shoe is set to the top of the pay zone.

IPR is constructed from

\[ q_{sc} = C \left( \bar{p}^2 - p_{wf}^2 \right)^n \]

TPR is constructed from

\[ p_{wf}^2 = \text{Exp}(s)p_{h}^2 + \frac{6.67 \times 10^{-4} [\text{Exp}(s) - 1] f q_{sc}^2 \bar{z}^2 T^2}{d_i^5 \cos \theta} \]

then the operating flow rate \( q_{sc} \) and pressure \( p_{wf} \) at the bottom hole node can be determined graphically by plotting the equations and finding the intersection point.
The operating point can also be solved numerically by combining the equations. The intersection can be calculated from

\[ \bar{p}^2 - \left( \frac{q_{sc}}{C} \right)^n - \text{Exp}(s)p_{hf}^2 - \frac{6.67 \times 10^{-4} [\text{Exp}(s) - 1] f q_{sc}^2 \bar{z}^2 \bar{T}^2}{D_i^5 \cos \theta} = 0 \]

which can be solved with a numerical technique such as the Newton-Raphson iteration for gas flow rate \( q_{sc} \).

**Example Problem 6.1**

Suppose that a vertical well produces 0.71 specific gravity gas through a 2 7/8-in tubing set to the top of a gas reservoir at a depth of 10,000 ft. At tubing head, the pressure is 800 psia and the temperature is 150 F, the bottom hole temperature is 200 F. The relative roughness of tubing is about 0.0006. Calculate the expected gas production rate of the well using the following data for IPR:

- Reservoir pressure: 2,000 psia
- IPR model parameter \( C: 0.01 \text{ Mscf/d-psi}^2n \)
- IPR model parameter \( n: 0.8 \)

**Solution**

This example problem is solved with the spreadsheet program BottomHoleNodal.xls. The spreadsheet for the data input and result sections indicates that the expected gas flow rate is 1,478 Mscf/d at a bottom hole pressure of 1,050 psia. The inflow and outflow performance curves plotted in Figure 6-1 confirm this operating point.
Wellhead Node

When the wellhead is used as a solution node in Nodal analysis,

- inflow performance curve is the Wellhead Performance Relationship (WPR) that is obtained by transforming the IPR to wellhead through TPR.
- outflow performance curve is the wellhead Choke Performance Relationship (CPR)

WPR is constructed from

$$\bar{p}^2 - \left( \frac{q_{sc}}{C} \right)^{1/n} - \text{Exp}(s) p_{hf}^2 - \frac{6.67 \times 10^{-4}[\text{Exp}(s) - 1]f q_{sc}^2 \bar{z}^2 \bar{T}^2}{D_i^5 \cos \theta} = 0$$

CPR is constructed from

$$q_{sc} = 879CAp_{hf} \sqrt{\left( \frac{k}{\gamma_g T_{up}} \right) \left( \frac{2}{k+1} \right)^{k+1/k-1}}$$

Nodal analysis with wellhead being a solution node is carried out by plotting the WPR and CPR curves and finding the solution at the intersection point of the two curves.
The operating point can also be solved numerically by combining the equations. The intersection can be calculated from

\[
q_{sc} = C \bar{p}^2 - \left( \frac{\text{Exp}(s)}{879CA \left( \frac{k}{\gamma_g T_{un}} \right) \left( \frac{2}{k+1} \right)^{k+1}} \right)^2 + \frac{6.67 \times 10^{-4} [\text{Exp}(s) - 1] f q_{sc}^2 \bar{z}^2 \bar{T}^2}{d_i^5 \cos \theta}
\]

which can be solved with a numerical technique for gas flow rate \( q_{sc} \).
Example Problem 6.2

Use the following given data to estimate gas production rate of the well:

Gas-specific gravity: 0.71
Tubing inside diameter: 2.259 in
Tubing wall relative roughness: 0.0006
Measured depth at tubing shoe: 10,000 ft
Inclination angle: 0°
Wellhead choke size: 16 1/64 in
Flowline diameter: 2 in
Gas-specific heat ratio: 1.3
Gas viscosity at wellhead: 0.01 cp
Wellhead temperature: 150 °F
Bottom hole temperature: 200 °F
Reservoir pressure: 2,000 psia

C-constant in backpressure IPR model: 0.01 Mscf/dpsi$^{2n}$
n-exponent in backpressure IPR model: 0.8

Solution:

This example problem is solved with the spreadsheet program WellheadNodal.xls. The spreadsheet for the data input and result sections indicates that the expected gas flow rate is 1,478 Mscf/d at a bottom hole pressure of 1,050 psia. The inflow and outflow performance curves plotted in to confirm this operating point.
Problem

I have a flowing vertical gas well. Is it worth my while to change the tubing from 2 3/8" to 2 7/8"? How much increased production will I get?

Data

Unit system: Field units (ft, lb, degrees F, etc)
Gas gravity: 0.65 (0% N2, CO₂, H₂S)
Tubing A : 2.375 in OD (1.995 in ID, 4.7lb/ft)
Tubing B : 2.875 in OD (2.441 in ID, 6.5lb/ft)
Tubing depth: 6350 ft
Flow path: Tubing
Casing: 5.250 in OD, 4.886 in ID
PBTD: 6500ft
Perforations: Top 3445ft, bottom 3543 ft (MPP = 3494ft)
Temperature: 77F (wellhead), 158F (sandface)
Reservoir pressure: 5000 psi
Reservoir parameters: C = 4.0e-7, n = 1)
Flowing Wellhead Pressure: 500 psi
Well Deliverability

- Both the IPR and the TPR or VFP (vertical flow performance) relate the wellbore flowing pressure to the surface production rate.

- The IPR represents what the reservoir can deliver to the bottomhole and the TPR (or VFP) represents what the well can deliver to the surface.

- Combined, the intersection of the IPR with the VFP yields the well deliverability, an expression of what a well will actually produce for a given operating condition. The role of a petroleum production engineer is to maximize the well deliverability in a cost-effective manner. Understanding and measuring the variables that control these relationships (well diagnosis) becomes imperative.

Combining IPR with TPR identifies the operating point.
Improving deliverability
The diagram illustrates the relationship between wellbore flowing pressure ($p_{wf}$) and flow rate ($q$). The original and improved IPR curves, as well as the original and improved VFP curves, are shown. The existing performance point is marked, and the potential performance point is indicated. Various methods to improve performance, such as skin removal (acidize), perforation (add to $h$), hydraulic fracture (stimulate), and horizontal well sidetracking, are listed. The performance gap is also highlighted.