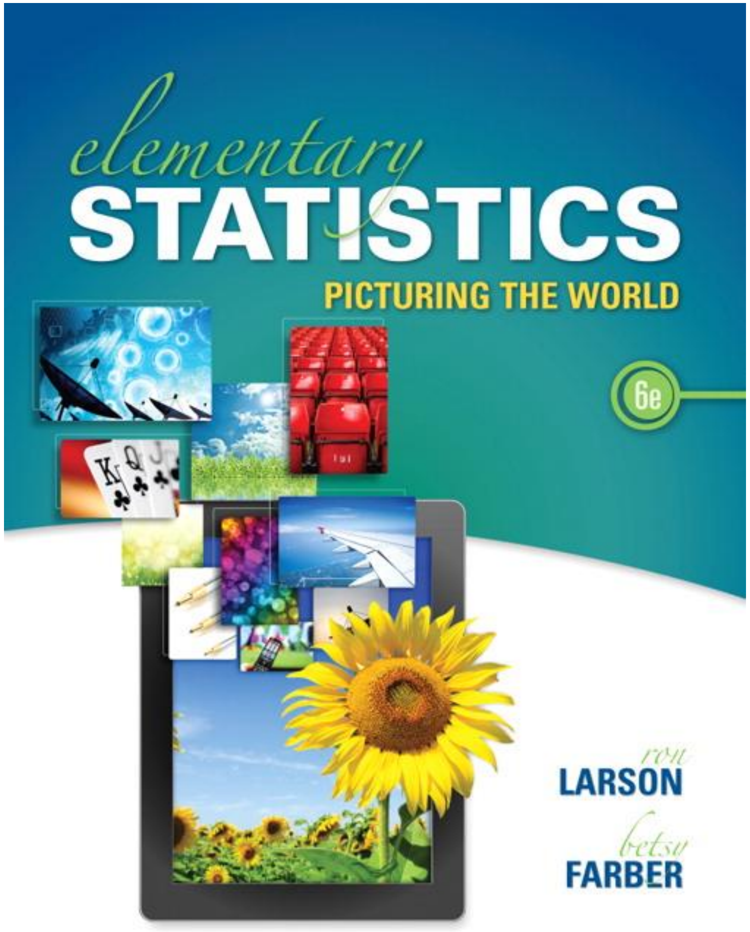
Elementary Statistics: Picturing The World

Sixth Edition



Chapter 3

Probability

Chapter Outline

3.1 Basic Concepts of Probability

3.2 Conditional Probability and the Multiplication Rule

3.3 The Addition Rule

3.4 Additional Topics in Probability and Counting

Section 3.4

Additional Topics in Probability and Counting

Section 3.4 Objectives

- How to find the number of ways a group of objects can be arranged in order
- How to find the number of ways to choose several objects from a group without regard to order
- How to use the counting principles to find probabilities

Permutations (1 of 2)

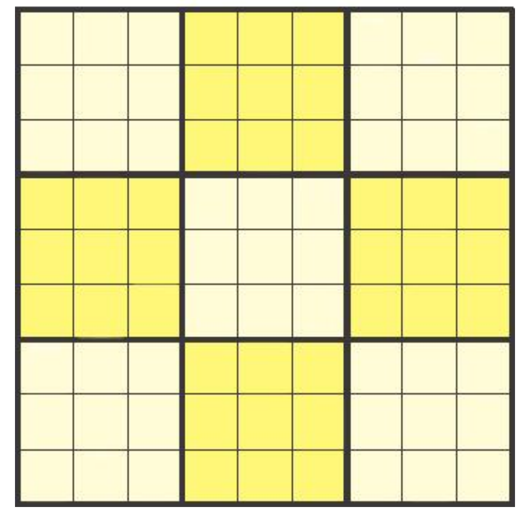
Permutation

- An ordered arrangement of objects
- The number of different permutations of n distinct objects is $n!$ (***n* factorial**)
  - $n! = n \cdot (n-1) \cdot (n-2) \cdot (n-3) \cdot \cdot \cdot 3 \cdot 2 \cdot 1$
  - $0! = 1$
  - Examples:
    - $6! = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720$
    - $4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$

Example: Permutation of *n* Objects

The objective of a 9 x 9 Sudoku number puzzle is to fill the grid so that each row, each column, and each 3 x 3 grid contain the digits 1 to 9. How many different ways can the first row of a blank 9 x 9 Sudoku grid be filled?

Sudoku Number Puzzle



Solution

The number of permutations is

$$9! = 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 362{,}880 \text{ ways}$$

Permutations (2 of 2)

Permutation of *n* objects taken *r* at a time

- The number of different permutations of *n* distinct objects taken *r* at a time
  - $_nP_r = \frac{n!}{(n-r)!}$, where $r \le n$

Example 1: Finding ${}_nP_r$

Find the number of ways of forming four-digit codes in which no digit is repeated.

Solution

- You need to select 4 digits from a group of 10
- $n = 10,\ r = 4$

$$
\begin{aligned}
{}_{10}P_4 &= \frac{10!}{(10-4)!} = \frac{10!}{6!} \\
&= \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot \cancel{6} \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{\cancel{6} \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}} \\
&= 5040 \text{ ways}
\end{aligned}
$$

Example 2: Finding $_nP_r$

Forty-three race cars started the 2007 Daytona 500. How many ways can the cars finish first, second, and third?

Solution

- You need to select 3 cars from a group of 43
- $n = 43,\ r = 3$

$$_{43}P_3 = \frac{43!}{(43-3)!} = \frac{43!}{40!}$$
$$= 43 \cdot 42 \cdot 41$$
$$= 74{,}046 \text{ ways}$$

Distinguishable Permutations

Distinguishable Permutations

- The number of distinguishable permutations of n objects where n_1 are of one type, n_2 are of another type, and so on
  - $$\frac{n!}{n_1! \cdot n_2! \cdot n_3! \cdots n_k!}, \text{ where } n_1 + n_2 + n_3 + \cdots + n_k = n$$

Example: Distinguishable Permutations

A building contractor is planning to develop a subdivision that consists of 6 one-story houses, 4 two-story houses, and 2 split-level houses. In how many distinguishable ways can the houses be arranged?

Solution

- There are 12 houses in the subdivision
- $n = 12$, $n_1 = 6$, $n_2 = 4$, $n_3 = 2$

$$\frac{12!}{6! \cdot 4! \cdot 2!}$$

$= 13{,}860$ distinguishable ways

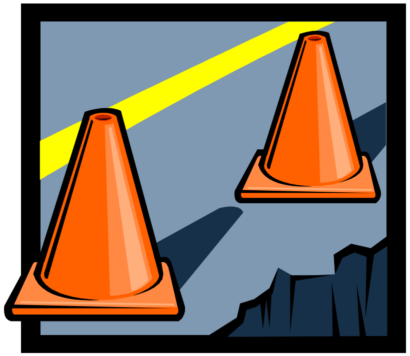
Combinations

Combination of *n* objects taken *r* at a time

- A selection of *r* objects from a group of *n* objects without regard to order
  - $_nC_r = \dfrac{n!}{(n-r)!r!}$, where $r \leq n$

Example: Combinations (1 of 2)

A state's department of transportation plans to develop a new section of interstate highway and receives 16 bids for the project. The state plans to hire four of the bidding companies. How many different combinations of four companies can be selected from the 16 bidding companies?

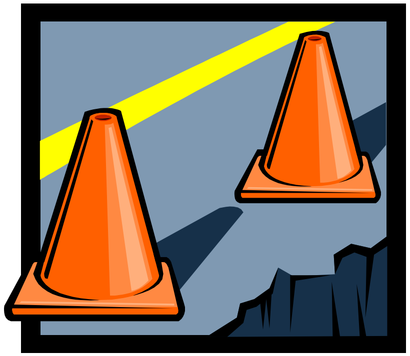


Solution

- You need to select 4 companies from a group of 16
- $n = 16$, $r = 4$
- Order is not important

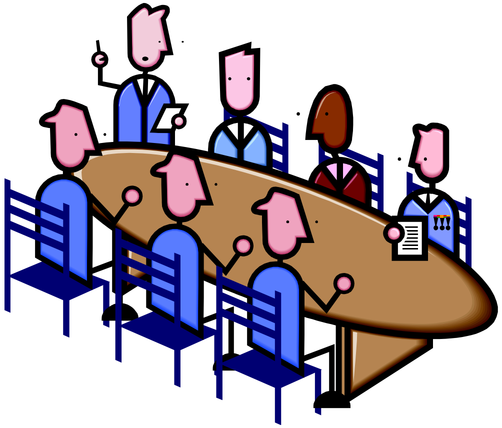
Example: Combinations (2 of 2)

$$
\begin{aligned}
{}_{16}C_4 &= \frac{16!}{(16-4)!4!} \\
&= \frac{16!}{12!4!} \\
&= \frac{16 \times 15 \times 14 \times 13 \times 12!}{12! \times 4 \times 3 \times 2 \times 1} \\
&= 1820 \text{ different combinations}
\end{aligned}
$$



Example 1: Finding Probabilities (1 of 2)

A student advisory board consists of 17 members. Three members serve as the board's chair, secretary, and webmaster. Each member is equally likely to serve any of the positions. What is the probability of selecting at random the three members that hold each position?



Example 1: Finding Probabilities (2 of 2)

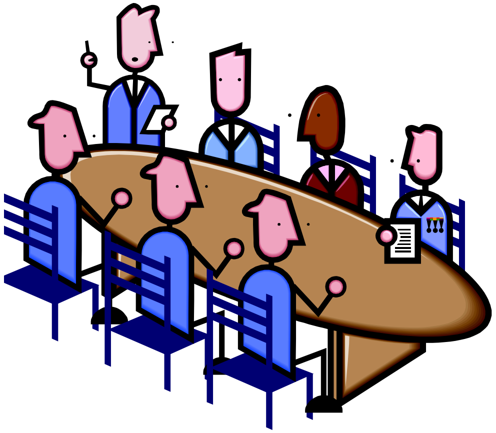
Solution

- There is only one favorable outcome
- There are

$$
{}_{17}P_3 = \frac{17!}{(17-3)!}
= \frac{17!}{14!} = 17 \cdot 16 \cdot 15 = 4080
$$

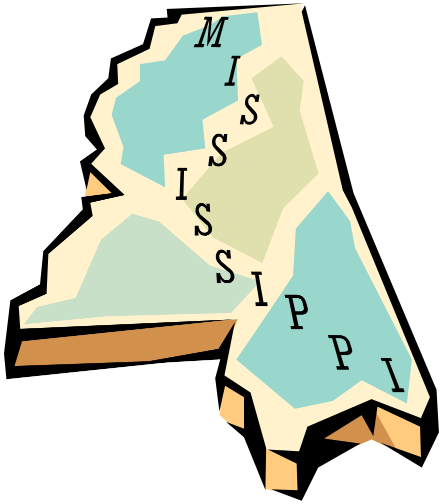
ways the three positions can be filled

$$
P(\text{selecting the 3 members}) = \frac{1}{4080} \approx 0.0002
$$

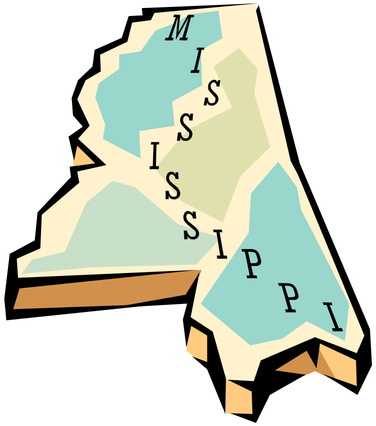


Example 2: Finding Probabilities (1 of 2)

You have 11 letters consisting of one M, four Is, four Ss, and two Ps. If the letters are randomly arranged in order, what is the probability that the arrangement spells the word **Mississippi**?



Example 2: Finding Probabilities (2 of 2)



Solution

- There is only one favorable outcome
- There are

$$\frac{11!}{1! \cdot 4! \cdot 4! \cdot 2!} = 34,650$$

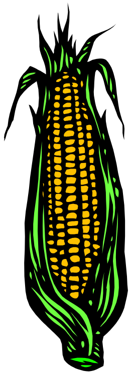
11 letters with 1,4,4, and 2 like letters

distinguishable permutations of the given letters

$$P(\text{Mississippi}) = \frac{1}{34650} \approx 0.000029$$

Example 3: Finding Probabilities (1 of 3)

A food manufacturer is analyzing a sample of 400 corn kernels for the presence of a toxin. In this sample, three kernels have dangerously high levels of the toxin. If four kernels are randomly selected from the sample, what is the probability that exactly one kernel contains a dangerously high level of the toxin?



Example 3: Finding Probabilities (2 of 3)

Solution

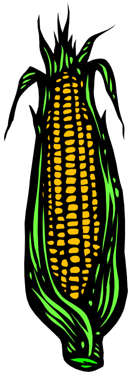
- The possible number of ways of choosing one toxic kernel out of three toxic kernels is

$${}_{3}C_{1} = 3$$

- The possible number of ways of choosing three nontoxic kernels from 397 nontoxic kernels is

$${}_{397}C_{3} = 10{,}349{,}790$$

- Using the Multiplication Rule, the number of ways of choosing one toxic kernel and three nontoxic kernels is



$${}_{3}C_{1} \cdot {}_{397}C_{3} = 3 \cdot 10{,}349{,}790 = 31{,}049{,}370$$

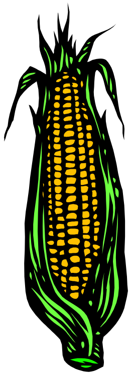
Example 3: Finding Probabilities (3 of 3)

- The number of possible ways of choosing 4 kernels from 400 kernels is

$$_{400}C_4 = 1{,}050{,}739{,}900$$

- The probability of selecting exactly 1 toxic kernel is

$$P(1\text{ toxic kernel}) = \frac{_{3}C_{1} \cdot {_{397}C_{3}}}{_{400}C_{4}}$$
$$= \frac{31{,}049{,}370}{1{,}050{,}739{,}900} \approx 0.0296$$



Section 3.4 Summary

- Found the number of ways a group of objects can be arranged in order
- Found the number of ways to choose several objects from a group without regard to order
- Used the counting principles to find probabilities