Questions

□ denotes answer available in Student Solutions Manual/Study Guide; O denotes objective question

1. What is the angular speed of the second hand of a clock? What is the direction of \( \vec{\omega} \) as you view a clock hanging on a vertical wall? What is the magnitude of the angular acceleration vector \( \vec{a} \) of the second hand?

2. One blade of a pair of scissors rotates counterclockwise in the \( xy \) plane. What is the direction of \( \vec{a} \)? What is the magnitude of the angular velocity is decreasing in time?

3. O A wheel is moving with constant angular acceleration 3 rad/s\(^2\). At different moments its angular speed is \(-2 \text{ rad/s}, 0, +2 \text{ rad/s}\). At these moments, analyze the magnitude of the tangential component of acceleration and the magnitude of the radial component of acceleration for a point on the rim of the wheel. Rank the following six items from largest to smallest: (a) \( |a_t| \) when \( \omega = -2 \text{ rad/s} \) (b) \( |a_t| \) when \( \omega = 0 \) (c) \( |a_t| \) when \( \omega = +2 \text{ rad/s} \) (d) \( |a_t| \) when \( \omega = 0 \) (e) \( |a_t| \) when \( \omega = +2 \text{ rad/s} \) (f) \( |a_t| \) when \( \omega = -2 \text{ rad/s} \). If two items are equal, show them as equal in your ranking. If a quantity is equal to zero, show that in your ranking.

4. O (i) Suppose a car’s standard tires are replaced with tires 1.30 times larger in diameter. Then what will the speedometer reading be? (a) 1.69 times too high (b) 1.30 times too high (c) accurate (d) 1.30 times too low (e) 1.69 times too low (e) inaccurate by an unpredictable factor (ii) What will be the car’s fuel economy in miles per gallon or km/L? (a) 1.69 times better (b) 1.30 times better (c) essentially the same (d) 1.30 times worse (e) 1.69 times worse

5. O Figure 10.8 shows a system of four particles joined by light, rigid rods. Assume \( a = b \) and \( M \) is somewhat larger than \( m \). (i) About which of the coordinate axes does the system have the smallest moment of inertia? (a) the \( x \) axis (b) the \( y \) axis (c) the \( z \) axis (d) About which axis does the moment of inertia have the smallest value for two axes. (e) The moment of inertia is the same for all axes. (ii) About which axis does the system have the largest moment of inertia? (a) the \( x \) axis (b) the \( y \) axis (c) the \( z \) axis (d) The moment of inertia has the same large value for two axes. (e) The moment of inertia is the same for all axes.

6. Suppose just two external forces act on a stationary rigid object and the two forces are equal in magnitude and opposite in direction. Under what condition does the object start to rotate?

7. O As shown in Figure 10.19, a cord is wrapped onto a cylindrical reel mounted on a fixed, frictionless, horizontal axle. Two experiments are conducted. (a) The cord is pulled down with a constant force of 50 N. (b) An object of weight 50 N is hung from the cord and released. Are the angular accelerations equal in the two experiments? If not, in which experiment is the angular acceleration greater in magnitude?

8. Explain how you might use the apparatus described in Example 10.10 to determine the moment of inertia of the wheel. (If the wheel does not have a uniform mass density, the moment of inertia is not necessarily equal to \( \frac{1}{2}MR^2 \).)

9. O A constant nonzero net torque is exerted on an object. Which of the following can not be constant? Choose all that apply: (a) angular position (b) angular velocity (c) angular acceleration (d) moment of inertia (e) kinetic energy (f) location of center of mass

10. Using the results from Example 10.10, how would you calculate the angular speed of the wheel and the linear speed of the suspended counterweight at \( t = 2 \text{ s} \), assuming the system is released from rest at \( t = 0 \)? Is the expression \( v = R\omega \) valid in this situation?

11. If a small sphere of mass \( M \) were placed at the end of the rod in Figure 10.21, would the result for \( \omega \) be greater than, less than, or equal to the value obtained in Example 10.11?

12. O A solid aluminum sphere of radius \( R \) has moment of inertia \( I \) about an axis through its center. What is the moment of inertia about a central axis of a solid aluminum sphere of radius \( 2R \)? (a) \( I \) (b) \( 2I \) (c) \( 4I \) (d) \( 8I \) (e) \( 16I \) (f) \( 32I \)

13. Explain why changing the axis of rotation of an object changes its moment of inertia.

14. Suppose you remove two eggs from the refrigerator, one hard-boiled and the other uncooked. You wish to determine which is the hard-boiled egg without breaking the eggs. This determination can be made by spinning the two eggs on the floor and comparing the rotational motions. Which egg spins faster? Which egg rotates more uniformly? Explain.

15. Which of the entries in Table 10.2 applies to finding the moment of inertia of a long, straight sewer pipe rotating about its axis of symmetry? Of an embroidery hoop rotating about an axis through its center and perpendicular to its plane? Of a uniform door turning on its hinges? Of a coin turning about an axis through its center and perpendicular to its faces?

16. Is it possible to change the translational kinetic energy of an object without changing its rotational energy?

17. Must an object be rotating to have a nonzero moment of inertia?

18. If you see an object rotating, is there necessarily a net torque acting on it?

19. O A decoration hangs from the ceiling of your room at the bottom end of a string. Your bored roommate turns the decoration clockwise several times to wind up the string. When your roommate releases it, the decoration starts to spin counterclockwise, slowly at first and then faster and faster. Take counterclockwise as the positive sense and assume friction is negligible. When the string is entirely unwound, the ornament has its maximum rate of rotation. (i) At this moment, is its angular acceleration (a) positive, (b) negative, or (c) zero? (ii) The decoration continues to spin, winding the string counterclockwise as it slows down. At the moment it finally stops, is its angular acceleration (a) positive, (b) negative, or (c) zero?

20. The polar diameter of the Earth is slightly less than the equatorial diameter. How would the moment of inertia of
the Earth about its axis of rotation change if some material from near the equator were removed and transferred to the polar regions to make the Earth a perfect sphere?

21. O A basketball rolls across a floor without slipping, with its center of mass moving at a certain velocity. A block of ice of the same mass is set sliding across the floor with the same speed along a parallel line. (i) How do their energies compare? (a) The basketball has more kinetic energy. (b) The ice has more kinetic energy. (c) They have equal kinetic energies. (ii) How do their momenta compare? (a) The basketball has more momentum. (b) The ice has more momentum. (c) They have equal momenta. (d) Their momenta have equal magnitudes but are different vectors. (iii) The two objects encounter a ramp sloping upward. (a) The basketball will travel farther up the ramp. (b) The ice will travel farther up the ramp. (c) They will travel equally far up the ramp.

22. Suppose you set your textbook sliding across a gymnasium floor with a certain initial speed. It quickly stops moving because of a friction force exerted on it by the floor. Next, you start a basketball rolling with the same initial speed. It keeps rolling from one end of the gym to the other. Why does the basketball roll so far? Does friction significantly affect its motion?

23. Three objects of uniform density—a solid sphere, a solid cylinder, and a hollow cylinder—are placed at the top of an incline (Fig. Q10.23). They are all released from rest at the same elevation and roll without slipping. Which object reaches the bottom first? Which reaches it last? Try this experiment at home and notice that the result is independent of the masses and the radii of the objects.

24. Figure Q10.24 shows a side view of a child’s tricycle with rubber tires on a horizontal concrete sidewalk. If a string is attached to the upper pedal on the far side and pulled forward horizontally, the tricycle rolls forward. Instead, assume a string is attached to the lower pedal on the near side and pulled forward horizontally as shown by A. Does the tricycle start to roll? If so, which way? Answer the same questions if (b) the string is pulled forward and upward as shown by B, (c) the string is pulled straight down as shown by C, and (d) the string is pulled forward and downward as shown by D. (e) What if the string is instead attached to the rim of the front wheel and pulled upward and backward as shown by E? (f) Explain a pattern of reasoning, based on the diagram, that makes it easy to answer questions such as all of these. What physical quantity must you evaluate?
through 50.0 revolutions before coming to rest. Find the constant angular acceleration of the centrifuge.

5. ▲ An electric motor rotating a grinding wheel at 100 rev/min is switched off. The wheel then moves with constant negative angular acceleration of magnitude 2.00 rad/s². (a) During what time interval does the wheel come to rest? (b) Through how many radians does it turn while it is slowing down?

6. A rotating wheel requires 3.00 s to rotate through 37.0 revolutions. Its angular speed at the end of the 3.00-s interval is 98.0 rad/s. What is the constant angular acceleration of the wheel?

7. (a) Find the angular speed of the Earth’s rotation on its axis. As the Earth turns toward the east, we see the sky turning toward the west at this same rate. (b) The rainy Pleiads wester
And seek beyond the sea
The head that I shall dream of
That shall not dream of me.

—A. E. Housman (© Robert E. Symons)

Cambridge, England is at longitude 0°, and Saskatoon, Saskatchewan, Canada is at longitude 107° west. How much time elapses after the Pleiades set in Cambridge until these stars fall below the western horizon in Saskatoon?

8. A merry-go-round is stationary. A dog is running on the ground just outside the merry-go-round’s circumference, moving with a constant angular speed of 0.750 rad/s. The dog does not change his pace when he sees what he has been looking for: a bone resting on the edge of the merry-go-round one third of a revolution in front of him. At the instant the dog sees the bone (t = 0), the merry-go-round begins to move in the direction the dog is running, with a constant angular acceleration equal to 0.0150 rad/s². (a) At what time will the dog reach the bone? (b) The confused dog keeps running and passes the bone. How long after the merry-go-round starts to turn do the dog and the bone draw even with each other for the second time?

9. The tub of a washing machine goes into its spin cycle, starting from rest and gaining angular speed steadily for 8.00 s, at which time it is turning at 5.00 rev/s. At this point, the person doing the laundry opens the lid and a safety switch turns off the machine. The tub smoothly slows to rest in 12.0 s. Through how many revolutions does the tub turn while it is in motion?

Section 10.3 Angular and Translational Quantities

10. A racing car travels on a circular track of radius 250 m. Assuming the car moves with a constant speed of 45.0 m/s, find (a) its angular speed and (b) the magnitude and direction of its acceleration.

11. Make an order-of-magnitude estimate of the number of revolutions through which a typical automobile tire turns in 1 yr. State the quantities you measure or estimate and their values.

12. ● Figure P10.12 shows the drive train of a bicycle that has wheels 67.3 cm in diameter and pedal cranks 17.5 cm long. The cyclist pedals at a steady cadence of 76.0 rev/min. The chain engages with a front sprocket 15.2 cm in diameter and a rear sprocket 7.00 cm in diameter. (a) Calculate the speed of a link of the chain relative to the bicycle frame. (b) Calculate the angular speed of the bicycle wheels. (c) Calculate the speed of the bicycle relative to the road. (d) What pieces of data, if any, are not necessary for the calculations?

13. A wheel 2.00 m in diameter lies in a vertical plane and rotates with a constant angular acceleration of 4.00 rad/s². The wheel starts at rest at t = 0, and the radius vector of a certain point P on the rim makes an angle of 57.3° with the horizontal at this time. At t = 2.00 s, find (a) the angular speed of the wheel, (b) the tangential speed and the total acceleration of the point P, and (c) the angular position of the point P.

14. A discus thrower (Fig. P10.14) accelerates a discus from rest to a speed of 25.0 m/s by whirling it through 1.25 rev. Assume the discus moves on the arc of a circle 1.00 m in radius. (a) Calculate the final angular speed of the discus. (b) Determine the magnitude of the angular acceleration of the discus, assuming it to be constant. (c) Calculate the time interval required for the discus to accelerate from rest to 25.0 m/s.

15. A small object with mass 4.00 kg moves counterclockwise with constant speed 4.50 m/s in a circle of radius 3.00 m centered at the origin. It starts at the point with position vector (3.00î + 0ĵ) m. Then it undergoes an angular displacement of 9.00 rad. (a) What is its position vector? Use unit-vector notation for all vector answers. (b) In what quadrant is the particle located, and what angle does its position vector make with the positive x axis? (c) What is its velocity? (d) In what direction is it moving? Make a sketch of its position, velocity, and acceleration vectors. (e) What is its acceleration? (f) What total force is exerted on the object?
16. A car accelerates uniformly from rest and reaches a speed of 22.0 m/s in 9.00 s. The tires have diameter 58.0 cm and do not slip on the pavement. (a) Find the number of revolutions each tire makes during this motion. (b) What is the final angular speed of a tire in revolutions per second?

17. ▲ A disk 8.00 cm in radius rotates at a constant rate of 1 200 rev/min about its central axis. Determine (a) its angular speed, (b) the tangential speed at a point 3.00 cm from its center, (c) the radial acceleration of a point on the rim, and (d) the total distance a point on the rim moves in 2.00 s.

18. ● A straight ladder is leaning against the wall of a house. The ladder has rails 4.90 m long, joined by rungs 0.410 m long. Its bottom end is on solid but sloping ground so that the top of the ladder is 0.690 m to the left of where it should be, and the ladder is unsafe to climb. You want to put a rock under one foot of the ladder to compensate for the slope of the ground. (a) What should be the thickness of the flat rock? (b) Does using ideas from this chapter make it easier to explain the solution to part (a)? Explain your answer.

19. A car traveling on a flat (unbanked) circular track accelerates uniformly from rest with a tangential acceleration of 1.70 m/s². The car makes it one-quarter of the way around the circle before it skids off the track. Determine the coefficient of static friction between the car and track from these data.

20. In part (B) of Example 10.2, the compact disc was modeled as a rigid object under constant angular acceleration to find the total angular displacement during the playing time of the disc. In reality, the angular acceleration of a disc is not constant. In this problem, let us explore the actual time dependence of the angular acceleration. (a) Assume the track on the disc is a spiral such that adjacent loops of the track are separated by a small distance h. Show that the radius r of a given portion of the track is given by

\[ r = r_i + \frac{h}{2\pi} \theta \]

where \( r_i \) is the radius of the innermost portion of the track and \( \theta \) is the angle through which the disc turns to arrive at the location of the track of radius \( r \). (b) Show that the rate of change of the angle \( \theta \) is given by

\[ \frac{d\theta}{dt} = \frac{v}{r_i + (h/2\pi)\theta} \]

where \( v \) is the constant speed with which the disc surface passes the laser. (c) From the result in part (b), use integration to find an expression for the angle \( \theta \) as a function of time. (d) From the result in part (c), use differentiation to find the angular acceleration of the disc as a function of time.

Section 10.4 Rotational Kinetic Energy

21. ▲ The four particles in Figure P10.21 are connected by rigid rods of negligible mass. The origin is at the center of the rectangle. The system rotates in the \( xy \) plane about the \( z \) axis with an angular speed of 6.00 rad/s. Calculate (a) the moment of inertia of the system about the \( z \) axis and (b) the rotational kinetic energy of the system.

22. ● Rigid rods of negligible mass lying along the \( y \) axis connect three particles (Fig. P10.22). The system rotates about the \( x \) axis with an angular speed of 2.00 rad/s. Find (a) the moment of inertia about the \( x \) axis and the total rotational kinetic energy evaluated from \( \frac{1}{2}I\omega^2 \) and (b) the tangential speed of each particle and the total kinetic energy evaluated from \( \Sigma \frac{1}{2}m(v_i)^2 \). (c) Compare the answers for kinetic energy in parts (a) and (b).

23. Two balls with masses \( M \) and \( m \) are connected by a rigid rod of length \( L \) and negligible mass as shown in Figure P10.23. For an axis perpendicular to the rod, show that the system has the minimum moment of inertia when the axis passes through the center of mass. Show that this moment of inertia is \( I = \mu L^2 \), where \( \mu = mM/(m + M) \).

24. As a gasoline engine operates, a flywheel turning with the crankshaft stores energy after each fuel explosion to provide the energy required to compress the next charge of fuel and air. In the engine of a certain lawn tractor, suppose a flywheel must be no more than 18.0 cm in diameter. Its thickness, measured along its axis of rotation, must be no larger than 8.00 cm. The flywheel must release 60.0 J of energy when its angular speed drops from 800 rev/min to 600 rev/min. Design a sturdy steel flywheel to meet these requirements with the smallest mass you can reasonably attain. Assume the material has the density listed for iron in Table 14.1. Specify the shape and mass of the flywheel.
25. A war-wolf or trebuchet is a device used during the Middle Ages to throw rocks at castles and now sometimes used to fling large vegetables and pianos as a sport. A simple trebuchet is shown in Figure P10.25. Model it as a stiff rod of negligible mass, 3.00 m long, joining particles of mass 60.0 kg and 0.120 kg at its ends. It can turn on a frictionless, horizontal axle perpendicular to the rod and 14.0 cm from the large-mass particle. The rod is released from rest in a horizontal orientation. (a) Find the maximum speed that the 0.120-kg object attains. (b) While the 0.120-kg object is gaining speed, does it move with constant acceleration? Does it move with constant tangential acceleration? Does the trebuchet move with constant angular acceleration? Does it have constant momentum? Does the trebuchet-Earth system have constant mechanical energy?

Section 10.5 Calculation of Moments of Inertia

26. Three identical thin rods, each of length $L$ and mass $m$, are welded perpendicular to one another as shown in Figure P10.26. The assembly is rotated about an axis that passes through the end of one rod and is parallel to another. Determine the moment of inertia of this structure.

27. Figure P10.27 shows a side view of a car tire. Model it as having two sidewalls of uniform thickness 0.635 cm and a tread wall of uniform thickness 2.50 cm and width 20.0 cm. Assume the rubber has uniform density equal to $1.10 \times 10^3$ kg/m$^3$. Find its moment of inertia about an axis through its center.

28. A uniform, thin solid door has height 2.20 m, width 0.870 m, and mass 23.0 kg. Find its moment of inertia for rotation on its hinges. Is any piece of data unnecessary?

29. Attention! About face! Compute an order-of-magnitude estimate for the moment of inertia of your body as you stand tall and turn about a vertical axis through the top of your head and the point halfway between your ankles. In your solution, state the quantities you measure or estimate and their values.

30. Many machines employ cams for various purposes such as opening and closing valves. In Figure P10.30, the cam is a circular disk rotating on a shaft that does not pass through the center of the disk. In the manufacture of the cam, a uniform solid cylinder of radius $R$ is first machined. Then an off-center hole of radius $R/2$ is drilled, parallel to the axis of the cylinder, and centered at a point a distance $R/2$ from the cylinder’s center. The cam, of mass $M$, is then slipped onto the circular shaft and welded into place. What is the kinetic energy of the cam when it is rotating with angular speed $\omega$ about the axis of the shaft?

31. Following the procedure used in Example 10.4, prove that the moment of inertia about the $y'$ axis of the rigid rod in Figure 10.9 is $\frac{1}{12}ML^2$.

Section 10.6 Torque

32. The fishing pole in Figure P10.32 makes an angle of 20.0° with the horizontal. What is the torque exerted by the fish about an axis perpendicular to the page and passing through the angler’s hand?

33. Find the net torque on the wheel in Figure P10.33 about the axle through $O$, taking $a = 10.0$ cm and $b = 25.0$ cm.
Section 10.7 The Rigid Object Under a Net Torque

34. A grinding wheel is in the form of a uniform solid disk of radius 7.00 cm and mass 2.00 kg. It starts from rest and accelerates uniformly under the action of the constant torque of 0.600 N·m that the motor exerts on the wheel. (a) How long does the wheel take to reach its final operating speed of 1 200 rev/min? (b) Through how many revolutions does it turn while accelerating?

35. A model airplane with mass 0.750 kg is tethered by a wire so that it flies in a circle 30.0 m in radius. The airplane engine provides a net thrust of 0.800 N perpendicular to the tethering wire. (a) Find the torque the net thrust produces about the center of the circle. (b) Find the angular acceleration of the airplane when it is in level flight. (c) Find the translational acceleration of the airplane tangent to its flight path.

36. The combination of an applied force and a friction force produces a constant total torque of 36.0 N·m on a wheel rotating about a fixed axis. The applied force acts for 6.00 s. During this time, the angular speed of the wheel increases from 0 to 10.0 rad/s. The applied force is then removed, and the wheel comes to rest in 60.0 s. Find (a) the moment of inertia of the wheel, (b) the magnitude of the frictional torque, and (c) the total number of revolutions of the wheel.

37. A block of mass \( m_1 = 2.00 \text{ kg} \) and a block of mass \( m_2 = 6.00 \text{ kg} \) are connected by a massless string over a pulley in the shape of a solid disk having radius \( R = 0.250 \text{ m} \) and mass \( M = 10.0 \text{ kg} \). These blocks are allowed to move on a fixed wedge of angle \( \theta = 30.0^\circ \) as shown in Figure P10.37. The coefficient of kinetic friction is 0.360 for both blocks. Draw free-body diagrams of both blocks and of the pulley. Determine (a) the acceleration of the two blocks and (b) the tensions in the string on both sides of the pulley.

38. A potter’s wheel—a thick stone disk of radius 0.500 m and mass 100 kg—is freely rotating at 50.0 rev/min. The potter can stop the wheel in 6.00 s by pressing a wet rag against the rim and exerting a radially inward force of 70.0 N. Find the effective coefficient of kinetic friction between wheel and rag.

39. An electric motor turns a flywheel through a drive belt that joins a pulley on the motor and a pulley that is rigidly attached to the flywheel as shown in Figure P10.39. The flywheel is a solid disk with a mass of 80.0 kg and a diameter of 1.25 m. It turns on a frictionless axle. Its pulley has much smaller mass and a radius of 0.230 m. The tension in the upper (taut) segment of the belt is 135 N, and the flywheel has a clockwise angular acceleration of 1.67 rad/s\(^2\). Find the tension in the lower (slack) segment of the belt.

40. A disk having moment of inertia 100 kg·m\(^2\) is free to rotate without friction, starting from rest, about a fixed axis through its center as shown at the top of Figure 10.19. A tangential force whose magnitude can range from \( F = 0 \) to \( F = 50.0 \text{ N} \) can be applied at any distance ranging from \( R = 0 \) to \( R = 3.00 \text{ m} \) from the axis of rotation. Find a pair of values of \( F \) and \( R \) that cause the disk to complete 2.00 revolutions in 10.0 s. Does one answer exist, or no answer, or two answers, or more than two, or many, or an infinite number?

Section 10.8 Energy Considerations in Rotational Motion

41. In a city with an air-pollution problem, a bus has no combustion engine. It runs on energy drawn from a large, rapidly rotating flywheel under the floor of the bus. At the bus terminal, the flywheel is spun up to its maximum rotation rate of 4 000 rev/min by an electric motor. Every time the bus speeds up, the flywheel slows down slightly. The bus is equipped with regenerative braking so that the flywheel can speed up when the bus slows down. The flywheel is a uniform solid cylinder with mass 1 600 kg and radius 0.650 m. The bus body does work against air resistance and rolling resistance at the average rate of 18.0 hp as it travels with an average speed of 40.0 km/h. How far can the bus travel before the flywheel has to be spun up to speed again?

42. Big Ben, the Parliament tower clock in London, has an hour hand 2.70 m long with a mass of 60.0 kg and a minute hand 4.50 m long with a mass of 100 kg (Fig.
P10.42). Calculate the total rotational kinetic energy of the two hands about the axis of rotation. (You may model the hands as long, thin rods.)

43. The top in Figure P10.43 has a moment of inertia equal to $4.00 \times 10^{-4}$ kg·m² and is initially at rest. It is free to rotate about the stationary axis $AA'$. A string, wrapped around a peg along the axis of the top, is pulled in such a manner as to maintain a constant tension of 5.57 N. If the string does not slip while it is unwound from the peg, what is the angular speed of the top after 80.0 cm of string has been pulled off the peg?

44. Consider the system shown in Figure P10.44 with $m_1 = 20.0$ kg, $m_2 = 12.5$ kg, $R = 0.200$ m, and the mass of the uniform pulley $M = 5.00$ kg. Object $m_2$ is resting on the floor, and object $m_1$ is 4.00 m above the floor when it is released from rest. The pulley axis is frictionless. The cord is light, does not stretch, and does not slip on the pulley. Calculate the time interval required for $m_1$ to hit the floor. How would your answer change if the pulley were massless?

45. In Figure P10.45, the sliding block has a mass of 0.850 kg, the counterweight has a mass of 0.420 kg, and the pulley is a hollow cylinder with a mass of 0.350 kg, an inner radius of 0.0200 m, and an outer radius of 0.0300 m. The coefficient of kinetic friction between the block and the horizontal surface is 0.250. The pulley turns without friction on its axle. The light cord does not stretch and does not slip on the pulley. The block has a velocity of 0.820 m/s toward the pulley when it passes through a photogate. (a) Use energy methods to predict its speed after it has moved to a second photogate, 0.700 m away. (b) Find the angular speed of the pulley at the same moment.

46. A cylindrical rod 24.0 cm long with mass 1.20 kg and radius 1.50 cm has a ball of diameter 8.00 cm and mass 2.00 kg attached to one end. The arrangement is originally vertical and stationary, with the ball at the top. The system is free to pivot about the bottom end of the rod after being given a slight nudge. (a) After the rod rotates through 90°, what is its rotational kinetic energy? (b) What is the angular speed of the rod and ball? (c) What is the linear speed of the ball? (d) How does this speed compare with the speed if the ball had fallen freely through the same distance of 28 cm?

47. An object with a weight of 50.0 N is attached to the free end of a light string wrapped around a reel of radius 0.250 m and mass 3.00 kg. The reel is a solid disk, free to rotate in a vertical plane about the horizontal axis passing through its center. The suspended object is released 6.00 m above the floor. (a) Determine the tension in the string, the acceleration of the object, and the speed with which the object hits the floor. (b) Verify your last answer by using the principle of conservation of energy to find the speed with which the object hits the floor.

48. A horizontal 800-N merry-go-round is a solid disk of radius 1.50 m, started from rest by a constant horizontal force of 50.0 N applied tangentially to the edge of the disk. Find the kinetic energy of the disk after 3.00 s.

49. This problem describes one experimental method for determining the moment of inertia of an irregularly shaped object such as the payload for a satellite. Figure P10.49 shows a counterweight of mass $m$ suspended by a cord wound around a spool of radius $r$, forming part of a turntable supporting the object. The turntable can rotate without friction. When the counterweight is released from rest, it descends through a distance $h$, acquiring a speed $v$. Show that the moment of inertia $I$ of the rotating apparatus (including the turntable) is $mr^2(2gh/v^2 - 1)$.
50. The head of a grass string trimmer has 100 g of cord wound in a light cylindrical spool with inside diameter 3.00 cm and outside diameter 18.0 cm, as shown in Figure P10.50. The cord has a linear density of 10.0 g/m. A single strand of the cord extends 16.0 cm from the outer edge of the spool. (a) When switched on, the trimmer speeds up from 0 to 2 500 rev/min in 0.215 s. (a) What average power is delivered to the head by the trimmer motor while it is accelerating? (b) When the trimmer is cutting grass, it spins at 2 000 rev/min and the grass exerts an average tangential force of 7.65 N on the outer end of the cord, which is still at a radial distance of 16.0 cm from the outer edge of the spool. What is the power delivered to the head under load?

![Figure P10.50](image)

51. (a) A uniform solid disk of radius $R$ and mass $M$ is free to rotate on a frictionless pivot through a point on its rim (Fig. P10.51). If the disk is released from rest in the position shown by the blue circle, what is the speed of its center of mass when the disk reaches the position indicated by the dashed circle? (b) What is the speed of the lowest point on the disk in the dashed position? (c) What If? Repeat part (a) using a uniform hoop.

![Figure P10.51](image)

Section 10.9 Rolling Motion of a Rigid Object

52. ● A solid sphere is released from height $h$ from the top of an incline making an angle $\theta$ with the horizontal. Calculate the speed of the sphere when it reaches the bottom of the incline (a) in the case that it rolls without slipping and (b) in the case that it slides frictionlessly without rolling. (c) Compare the time intervals required to reach the bottom in cases (a) and (b).

53. ▲ A cylinder of mass 10.0 kg rolls without slipping on a horizontal surface. At a certain instant its center of mass has a speed of 10.0 m/s. Determine (a) the translational kinetic energy of its center of mass, (b) the rotational kinetic energy about its center of mass, and (c) its total energy.

54. ● A smooth cube of mass $m$ and edge length $r$ slides with speed $v$ on a horizontal surface with negligible friction. The cube then moves up a smooth incline that makes an angle $\theta$ with the horizontal. A cylinder of mass $m$ and radius $r$ rolls without slipping with its center of mass moving with speed $v$ and encounters an incline of the same angle of inclination but with sufficient friction that the cylinder continues to roll without slipping. (a) Which object will go the greater distance up the incline? (b) Find the difference between the maximum distances the objects travel up the incline. (c) Explain what accounts for this difference in distances traveled.

55. (a) Determine the acceleration of the center of mass of a uniform solid disk rolling down an incline making angle $\theta$ with the horizontal. Compare this acceleration with that of a uniform hoop. (b) What is the minimum coefficient of friction required to maintain pure rolling motion for the disk?

56. A uniform solid disk and a uniform hoop are placed side by side at the top of an incline of height $h$. If they are released from rest at the same time and roll without slipping, which object reaches the bottom first? Verify your answer by calculating their speeds when they reach the bottom in terms of $h$.

57. ● A metal can containing condensed mushroom soup has mass 215 g, height 10.8 cm, and diameter 6.38 cm. It is placed at rest on its side at the top of a 3.00-m-long incline that is at 25.0° to the horizontal and is then released to roll straight down. It reaches the bottom of the incline after 1.50 s. Assuming mechanical energy conservation, calculate the moment of inertia of the can. Which pieces of data, if any, are unnecessary for calculating the solution?

58. ● A tennis ball is a hollow sphere with a thin wall. It is set rolling without slipping at 4.05 m/s on a horizontal section of a track as shown in Figure P10.58. It rolls around the inside of a vertical circular loop 90.0 cm in diameter and finally leaves the track at a point 20.0 cm below the horizontal section. (a) Find the speed of the ball at the top of the loop. Demonstrate that it will not fall from the track. (b) Find its speed as it leaves the track. What If? (c) Suppose static friction between ball and track were negligible so that the ball slid instead of rolling. Would its speed then be higher, lower, or the same at the top of the loop? Explain.

![Figure P10.58](image)

Additional Problems

59. As shown in Figure P10.59, toppling chimneys often break apart in midfall because the mortar between the bricks cannot withstand much shear stress. As the chimney begins to fall, shear forces must act on the topmost sections to accelerate them tangentially so that they can keep up with the rotation of the lower part of the stack. For simplicity, let us model the chimney as a uniform rod of
length $\ell$ pivoted at the lower end. The rod starts at rest in a vertical position (with the frictionless pivot at the bottom) and falls over under the influence of gravity. What fraction of the length of the rod has a tangential acceleration greater than $g \sin \theta$, where $\theta$ is the angle the chimney makes with the vertical axis?

**Figure P10.59** A building demolition site in Baltimore, Maryland. At the left is a chimney, mostly concealed by the building, that has broken apart on its way down. Compare with Figure 10.18.

60. **Review problem.** A mixing beater consists of three thin rods, each 10.0 cm long. The rods diverge from a central hub, separated from one another by $120^\circ$, and all turn in the same plane. A ball is attached to the end of each rod. Each ball has cross-sectional area 4.00 cm$^2$ and is so shaped that it has a drag coefficient of 0.600. Calculate the power input required to spin the beater at 1000 rev/min (a) in air and (b) in water.

61. A 4.00-m length of light nylon cord is wound around a uniform cylindrical spool of radius 0.500 m and mass 1.00 kg. The spool is mounted on a frictionless axle and is initially at rest. The cord is pulled from the spool with a constant acceleration of magnitude 2.50 m/s$^2$. (a) How much work has been done on the spool when it reaches an angular speed of 8.00 rad/s? (b) Assuming there is enough cord on the spool, how long does it take the spool to reach this angular speed? (c) Is there enough cord on the spool?

62. **An elevator system in a tall building consists of an 800-kg car and a 950-kg counterweight, joined by a cable that passes over a pulley of mass 280 kg. The pulley, called a sheave, is a solid cylinder of radius 0.700 m turning on a horizontal axle. The cable has comparatively small mass and constant length. It does not slip on the sheave. The car and the counterweight move vertically, next to each other inside the same shaft. A number $n$ of people, each of mass 80.0 kg, are riding in the elevator car, moving upward at 3.00 m/s and approaching the floor where the car should stop. As an energy-conservation measure, a computer disconnects the electric elevator motor at just the right moment so that the sheave-car-counterweight system then coasts freely without friction and comes to rest at the floor desired. There it is caught by a simple latch rather than by a massive brake. (a) Determine the distance $d$ the car coasts upward as a function of $n$. Evaluate the distance for (b) $n = 2$, (c) $n = 12$, and (d) $n = 0$.

63. **Figure P10.63** is a photograph of a lawn sprinkler. Its rotor consists of three metal tubes that fill with water when a hose is connected to the base. As water sprays out of the holes at the ends of the arms and the hole near the center of each arm, the assembly with the three arms rotates. To analyze this situation, let us make the following assumptions: (1) The arms can be modeled as thin, straight rods, each of length $L$. (2) The water coming from the hole at distance $\ell$ from the center sprays out horizontally, parallel to the ground and perpendicular to the arm. (3) The water emitted from the holes at the ends of the arms sprays out radially away from the center of the rotor. When filled with water, each arm has mass $m$. The center of the assembly is massless. The water ejected from a hole at distance $\ell$ from the center causes a thrust force $F$ on the arm containing the hole. The mounting for the three-arm rotor assembly exerts a frictional torque that is described by $\tau = -b\omega$, where $\omega$ is the angular speed of the assembly. (a) Imagine that the sprinkler is in operation. Find an expression for the constant angular speed with which the assembly rotates after it completes an initial period of angular acceleration. Your expression should be in terms of $F$, $\ell$, and $b$. (b) Imagine that the sprinkler has been at rest and is just turned on. Find an expression for the initial angular acceleration of the rotor, that is, the angular acceleration when the arms are filled with water and the assembly just begins to move from rest. Your expression should be in terms of $F$, $\ell$, $m$, and $L$. (c) Now, take a step toward reality from the simplified model. The arms are actually bent as shown in the photograph. Therefore, the water from the ends of the arms is not actually sprayed radially. How will this fact affect the constant angular speed with which the assembly rotates in part (a)? In reality, will it be larger, smaller, or unchanged? Provide a convincing argument for your response. (d) How will the bend in the arms, described in part (c), affect the angular acceleration in part (b)? In reality, will it be larger, smaller, or unchanged? Provide a convincing argument for your response.
64. A shaft is turning at 65.0 rad/s at time $t = 0$. Thereafter, its angular acceleration is given by
\[ \alpha = -10.0 \text{ rad/s}^2 - 5.00t \text{ rad/s}^3 \]
where $t$ is the elapsed time. (a) Find its angular speed at $t = 3.00$ s. (b) How far does it turn in these 3 s?

65. A long, uniform rod of length $L$ and mass $M$ is pivoted about a horizontal, frictionless pin through one end. The rod is released, almost from rest in a vertical position as shown in Figure P10.65. At the instant the rod is horizontal, find (a) its angular speed, (b) the magnitude of its angular acceleration, (c) the $x$ and $y$ components of the acceleration of its center of mass, and (d) the components of the reaction force at the pivot.

66. A cord is wrapped around a pulley of mass $m$ and radius $r$. The free end of the cord is connected to a block of mass $M$. The block starts from rest and then slides down an incline that makes an angle $\theta$ with the horizontal. The coefficient of kinetic friction between block and incline is $\mu$. (a) Use energy methods to show that the block’s speed as a function of position $d$ down the incline is
\[ v = \sqrt{\frac{4gdM(\sin \theta - \mu \cos \theta)}{m + 2M}} \]
(b) Find the magnitude of the acceleration of the block in terms of $\mu$, $m$, $M$, $g$, and $\theta$.

67. A bicycle is turned upside down while its owner repairs a flat tire. A friend spins the other wheel, of radius $R$, and observes that drops of water fly off tangentially. She measures the height reached by drops moving vertically (Fig. P10.67). A drop that breaks loose from the tire on one turn rises a distance $h_1$ above the tangent point. A drop that breaks loose on the next turn rises a distance $h_2 < h_1$ above the tangent point. The height to which the drops rise decreases because the angular speed of the wheel decreases. From this information, determine the magnitude of the average angular acceleration of the wheel.

68. A bicycle is turned upside down while its owner repairs a flat tire. A friend spins the other wheel, of radius $R$, and measures the height reached by drops moving vertically (Fig. P10.67). A drop that breaks loose from the tire on one turn rises a distance $h_1$ above the tangent point. A drop that breaks loose on the next turn rises a distance $h_2 < h_1$ above the tangent point. The height to which the drops rise decreases because the angular speed of the wheel decreases. From this information, determine the magnitude of the average angular acceleration of the wheel.

69. A uniform, hollow, cylindrical spool has inside radius $R/2$, outside radius $R$, and mass $M$ (Fig. P10.69). It is mounted so that it rotates on a fixed, horizontal axle. A counterweight of mass $m$ is connected to the end of a string wound around the spool. The counterweight falls from rest at $t = 0$ to a position $y$ at time $t$. Show that the torque due to the friction forces between spool and axle is
\[ \tau_j = R \left[ m \left( g - \frac{2y}{r^2} \right) - M \frac{5y}{4r^2} \right] \]

70. (a) What is the rotational kinetic energy of the Earth about its spin axis? Model the Earth as a uniform sphere and use data from the endpapers. (b) The rotational kinetic energy of the Earth is decreasing steadily because of tidal friction. Find the change in one day, assuming the rotational period increases by 10.0 $\mu$s each year.

71. Two blocks as shown in Figure P10.71 are connected by a string of negligible mass passing over a pulley of radius 0.250 m and moment of inertia $I$. The block on the frictionless incline is moving up with a constant acceleration of 2.00 m/s$^2$. (a) Determine $T_1$ and $T_2$, the tensions in the two parts of the string. (b) Find the moment of inertia of the pulley.

72. The reel shown in Figure P10.72 has radius $R$ and moment of inertia $I$. One end of the block of mass $m$ is connected to a spring of force constant $k$, and the other end is fastened to a cord wrapped around the reel. The reel axle and the incline are frictionless. The reel is
wound counterclockwise so that the spring stretches a distance \( d \) from its unstretched position and the reel is then released from rest. (a) Find the angular speed of the reel when the spring is again unstretched. (b) Evaluate the angular speed numerically at this point, taking \( I = 1.00 \, \text{kg} \cdot \text{m}^2 \), \( R = 0.300 \, \text{m} \), \( k = 50.0 \, \text{N/m} \), \( m = 0.500 \, \text{kg} \), \( d = 0.200 \, \text{m} \), and \( \theta = 37.0^\circ \).

Express your answer in terms of \( h \), \( g \), and the angular speed \( \omega \) of the Earth. Ignore air resistance, and assume the free-fall acceleration is constant over this range of heights. (b) Evaluate the eastward displacement for \( h = 50.0 \, \text{m} \). (c) In your judgment, were we justified in ignoring this aspect of the Coriolis effect in our previous study of free fall?

76. The hour hand and the minute hand of Big Ben, the Parliament tower clock in London, are 2.70 m and 4.50 m long and have masses of 60.0 kg and 100 kg, respectively (see Figure P10.42). (i) Determine the total torque due to the weight of these hands about the axis of rotation when the time reads (a) 3:00, (b) 5:15, (c) 6:00, (d) 8:20, and (e) 9:45. (You may model the hands as long, thin uniform rods.) (ii) Determine all times when the total torque about the axis of rotation is zero. Determine the times to the nearest second, solving a transcendental equation numerically.

77. A string is wound around a uniform disk of radius \( R \) and mass \( M \). The disk is released from rest with the string vertical and its top end tied to a fixed bar (Fig. P10.77). Show that (a) the tension in the string is one-third of the weight of the disk, (b) the magnitude of the acceleration of the center of mass is \( 2g/3 \), and (c) the speed of the center of mass is \((4gh/3)^{1/2}\) after the disk has descended through distance \( h \). Verify your answer to part (c) using the energy approach.

78. A uniform solid sphere of radius \( r \) is placed on the inside surface of a hemispherical bowl with much larger radius \( R \). The sphere is released from rest at an angle \( \theta \) to the vertical and rolls without slipping (Fig. P10.78). Determine the angular speed of the sphere when it reaches the bottom of the bowl.

79. A solid sphere of mass \( m \) and radius \( r \) rolls without slipping along the track shown in Figure P10.79. It starts from rest with the lowest point of the sphere at height \( h \)
above the bottom of the loop of radius \( R \), much larger than \( r \). (a) What is the minimum value of \( h \) (in terms of \( R \)) such that the sphere completes the loop? (b) What are the components of the net force on the sphere at the point \( P \) if \( h = 3R \)?

![Figure P10.79](image1)

80. A thin rod of mass 0.630 kg and length 1.24 m is at rest, hanging vertically from a strong fixed hinge at its top end. Suddenly a horizontal impulsive force \((14.7 \hat{i}) \text{ N}\) is applied to it. (a) Suppose the force acts at the bottom end of the rod. Find the acceleration of its center of mass and the horizontal force the hinge exerts. (b) Suppose the force acts at the midpoint of the rod. Find the acceleration of this point and the horizontal hinge reaction. (c) Where can the impulse be applied so that the hinge will exert no horizontal force? This point is called the center of percussion.

81. (a) A thin rod of length \( h \) and mass \( M \) is held vertically with its lower end resting on a frictionless horizontal surface. The rod is then released to fall freely. Determine the speed of its center of mass just before it hits the horizontal surface. (b) What If? Now suppose the rod has a fixed pivot at its lower end. Determine the speed of the rod’s center of mass just before it hits the surface.

82. Following Thanksgiving dinner your uncle falls into a deep sleep, sitting straight up facing the television set. A naughty grandchild balances a small spherical grape at the top of his bald head, which itself has the shape of a sphere. After all the children have had time to giggle, the grape loses contact with your uncle’s scalp when the radial line joining it to the center of curvature makes what angle with the vertical?

83. A spool of wire of mass \( M \) and radius \( R \) is unwound under a constant force \( \vec{F} \) (Fig. P10.83). Assuming the spool is a uniform solid cylinder that doesn’t slip, show that (a) the acceleration of the center of mass is \( 4F/3M \) and (b) the force of friction is to the right and equal in magnitude to \( F/3 \). (c) If the cylinder starts from rest and rolls without slipping, what is the speed of its center of mass after it has rolled through a distance \( d \)?

84. A plank with a mass \( M = 6.00 \text{ kg} \) rides on top of two identical solid cylindrical rollers that have \( R = 5.00 \text{ cm} \) and \( m = 2.00 \text{ kg} \) (Fig. P10.84). The plank is pulled by a constant horizontal force \( \vec{F} \) of magnitude 6.00 N applied to the end of the plank and perpendicular to the axes of the cylinders (which are parallel). The cylinders roll without slipping on a flat surface. There is also no slipping between the cylinders and the plank. (a) Find the acceleration of the plank and of the rollers. (b) What friction forces are acting?

85. A spool of thread consists of a cylinder of radius \( R_1 \) with end caps of radius \( R_2 \) as shown in the end view illustrated in Figure P10.85. The mass of the spool, including the thread, is \( m \), and its moment of inertia about an axis through its center is \( I \). The spool is placed on a rough horizontal surface so that it rolls without slipping when a force \( \vec{T} \) acting to the right is applied to the free end of the thread. Show that the magnitude of the friction force exerted by the surface on the spool is given by

\[
f = \left( \frac{I + mR_1^2}{I + mR_2^2} \right) T
\]

Determine the direction of the force of friction.

86. A large, cylindrical roll of tissue paper of initial radius \( R \) lies on a long, horizontal surface with the outside end of the paper nailed to the surface. The roll is given a slight shove \((v_i = 0)\) and commences to unroll. Assume the roll has a uniform density and that mechanical energy is conserved in the process. (a) Determine the speed of the center of mass of the roll when its radius has diminished to \( r \). (b) Calculate a numerical value for this speed at \( r = 1.00 \text{ mm} \), assuming \( R = 6.00 \text{ m} \). (c) What If? What happens to the energy of the system when the paper is completely unrolled?
Chapter 10  Rotation of a Rigid Object About a Fixed Axis

Answers to Quick Quizzes

10.1 (i), (c). For a rotation of more than 180°, the angular displacement must be larger than \( \pi = 3.14 \) rad. The angular displacements in the three choices are (a) 6 rad \(- 3\) rad = 3 rad, (b) 1 rad \(- (-1)\) rad = 2 rad, and (c) 5 rad \(- 1\) rad = 4 rad. (ii), (b). Because all angular displacements occur in the same time interval, the displacement with the lowest value will be associated with the lowest average angular speed.

10.2 (b). In Equation 10.8, both the initial and final angular speeds are the same in all three cases. As a result, the angular acceleration is inversely proportional to the angular displacement. Therefore, the highest angular acceleration is associated with the lowest angular displacement.

10.3 (i), (b). The system of the platform, Alex, and Brian is a rigid object, so all points on the rigid object have the same angular speed. (ii), (a). The tangential speed is proportional to the radial distance from the rotation axis.

10.4 (a). Almost all the mass of the pipe is at the same distance from the rotation axis, so it has a larger moment of inertia than the solid cylinder.

10.5 (i), (b). The fatter handle of the screwdriver gives you a larger moment arm and increases the torque you can apply with a given force from your hand. (ii), (a). The longer handle of the wrench gives you a larger moment arm and increases the torque you can apply with a given force from your hand.

10.6 (b). With twice the moment of inertia and the same frictional torque, there is half the angular acceleration. With half the angular acceleration, it will require twice as long to change the speed to zero.

10.7 (b). All the gravitational potential energy of the box–Earth system is transformed to kinetic energy of translation. For the ball, some of the gravitational potential energy of the ball–Earth system is transformed to rotational kinetic energy, leaving less for translational kinetic energy, so the ball moves downhill more slowly than the box does.