Questions

☐ denotes answer available in Student Solutions Manual/Study Guide; O denotes objective question

1. O What creates a magnetic field? Choose every correct answer. (a) a stationary object with electric charge (b) a moving object with electric charge (c) a stationary conductor carrying electric current (d) a difference in electric potential (e) an electric resistor. Note: In Chapter 34, we will see that a changing electric field also creates a magnetic field.

2. O A long, vertical, metallic wire carries downward electric current. (i) What is the direction of the magnetic field it creates at a point 2 cm horizontally east of the center of the wire? (a) north (b) south (c) east (d) west (e) up (f) down (ii) What would be the direction of the field if the current consisted of positive charges moving downward, instead of electrons moving upward? Choose from the same possibilities.

3. O Suppose you are facing a tall makeup mirror on a vertical wall. Fluorescent tubes framing the mirror carry a clockwise electric current. (i) What is the direction of the magnetic field created by that current at a point slightly to the right of the center of the mirror? (a) up (b) down (c) left (d) right (e) horizontally toward you (f) away from you (ii) What is the direction of the field the current creates at a point on the wall outside the frame to the right? Choose from the same possibilities.

4. Explain why two parallel wires carrying currents in opposite directions repel each other.

5. O In Active Figure 30.8, assume \( I_1 = 2 \) A and \( I_2 = 6 \) A. What is the relationship between the magnitude \( F_1 \) of the force exerted on wire 1 and the magnitude \( F_2 \) of the force exerted on wire 2? (a) \( F_1 = 6F_2 \) (b) \( F_1 = 3F_2 \) (c) \( F_1 = F_2 \) (d) \( F_1 = F_2/3 \) (e) \( F_1 = F_2/6 \)

6. O Answer each question yes or no. (a) Is it possible for each of three stationary charged particles to exert a force of attraction on the other two? (b) Is it possible for each of three stationary charged particles to repel both of the other particles? (c) Is it possible for each of three current-carrying metal wires to attract the other two? (d) Is it possible for each of three current-carrying metal wires to repel both of the other wires? (e) Andre-Marie Ampere’s experiments on electromagnetism are models of logical precision and included observation of the phenomena referred to in this question.

7. Is Ampere’s law valid for all closed paths surrounding a conductor? Why is it not useful for calculating \( \mathbf{B} \) for all such paths?

8. Compare Ampere’s law with the Biot-Savart law. Which is more generally useful for calculating \( \mathbf{B} \) for a current-carrying conductor?

9. A hollow copper tube carries a current along its length. Why is \( \mathbf{B} = 0 \) inside the tube? Is \( \mathbf{B} \) nonzero outside the tube?

10. O (i) What happens to the magnitude of the magnetic field inside a long solenoid if the current is doubled? (a) It becomes 4 times larger. (b) It becomes twice as large. (c) It is unchanged. (d) It becomes one-half as large. (e) It becomes one-fourth as large. (ii) What happens to the field if instead the length of the solenoid is doubled, with the number of turns remaining the same? Choose from the same possibilities. (iii) What happens to the field if the number of turns is doubled, with the length remaining the same? Choose from the same possibilities. (iv) What happens to the field if the radius is doubled? Choose from the same possibilities.

11. O A long solenoid with closely spaced turns carries electric current. Does each turn of wire exert (a) an attractive force on the next adjacent turn, (b) a repulsive force on the next adjacent turn, (c) zero force on the next adjacent turn, (d) either an attractive or a repulsive force on the next turn, depending on the direction of current in the solenoid?

12. O A uniform magnetic field is directed along the x axis. For what orientation of a flat, rectangular coil is the flux through the rectangle a maximum? (a) It is a maximum in the xy plane. (b) It is a maximum in the xz plane. (c) It is a maximum in the yz plane. (d) The flux has the same nonzero value for all these orientations. (e) The flux is zero in all cases.

13. The quantity \( \oint \mathbf{B} \cdot d\mathbf{s} \) in Ampere’s law is called magnetic circulation. Active Figure 30.10 and Figure 30.13 show paths around which the magnetic circulation was evaluated. Each of these paths encloses an area. What is the magnetic flux through each area? Explain your answer.

14. O (a) Two stationary charged particles exert forces of attraction on each other. One of the particles has negative charge. Is the other positive or negative? (b) Is the net electric field at a point halfway between the particles larger, smaller, or the same in magnitude as the field due to one charge by itself? (c) Two straight, vertical, current-carrying wires exert forces of attraction on each other. One of them carries downward current. Does the other wire carry upward or downward current? (d) Is the net magnetic field at a point halfway between the wires larger, smaller, or the same in magnitude as the field due to one wire by itself?

15. O Rank the magnitudes of the following magnetic fields from the largest to the smallest, noting any cases of equality. (a) the field 2 cm away from a long, straight wire carrying a current of 3 A (b) the field at the center of a flat, compact, circular coil, 2 cm in radius, with 10 turns, carrying a current of 0.3 A (c) the field at the center of a solenoid 2 cm in radius and 200 cm long, with 1 000 turns, carrying a current of 0.3 A (d) the field at the center of a long, straight metal bar, 2 cm in radius, carrying a current of 300 A (e) a field of 1 mT

16. One pole of a magnet attracts a nail. Will the other pole of the magnet attract the nail? Explain. Explain how a magnet sticks to a refrigerator door.

17. A magnet attracts a piece of iron. The iron can then attract another piece of iron. On the basis of domain alignment, explain what happens in each piece of iron.

18. Why does hitting a magnet with a hammer cause the magnetism to be reduced?

19. Which way would a compass point if you were at the north magnetic pole of the Earth?
20. Figure Q30.20 shows four permanent magnets, each having a hole through its center. Notice that the blue and yellow magnets are levitated above the red ones. (a) How does this levitation occur? (b) What purpose do the rods serve? (c) What can you say about the poles of the magnets from this observation? (d) If the upper magnet were inverted, what do you suppose would happen?
of a circle of radius $r$ as shown in Figure P30.8. Determine the magnetic field at the center of the arc.

**Figure P30.8**

9. One long wire carries current 30.0 A to the left along the $x$ axis. A second long wire carries current 50.0 A to the right along the line $(y = 0.280 \text{ m}, z = 0)$. (a) Where in the plane of the two wires is the total magnetic field equal to zero? (b) A particle with a charge of $-2.00 \mu \text{C}$ is moving with a velocity of 150 Mm/s along the line $(y = 0.100 \text{ m}, z = 0)$. Calculate the vector magnetic force acting on the particle. (c) What If? A uniform electric field is applied to allow this particle to pass through this region undeflected. Calculate the required vector electric field.

10. A current path shaped as shown in Figure P30.10 produces a magnetic field at $P$, the center of the arc. If the arc subtends an angle of 30.0° and the radius of the arc is 0.600 m, what are the magnitude and direction of the field produced at $P$ if the current is 3.00 A?

**Figure P30.10**

11. Three long, parallel conductors carry currents of $I = 2.00 \text{ A}$. Figure P30.11 is an end view of the conductors, with each current coming out of the page. Taking $a = 1.00 \text{ cm}$, determine the magnitude and direction of the magnetic field at points A, B, and C.

**Figure P30.11**

12. In a long, straight, vertical lightning stroke, electrons move downward and positive ions move upward, to constitute a current of magnitude 20.0 kA. At a location 50.0 m east of the middle of the stroke, a free electron drifts through the air toward the west with a speed of 300 m/s. (a) Find the vector force the lightning stroke exerts on the electron. Make a sketch showing the various vectors involved. Ignore the effect of the Earth’s magnetic field. (b) Find the radius of the electron’s path. Is it a good approximation to model the electron as moving in a uniform field? Explain your answer. (c) If it does not collide with any obstacles, how many revolutions will the electron complete during the 60.0-μs duration of the lightning stroke?

13. A wire carrying a current $I$ is bent into the shape of an equilateral triangle of side $L$. (a) Find the magnitude of the magnetic field at the center of the triangle. (b) At a point halfway between the center and any vertex, is the field stronger or weaker than at the center? Give a qualitative argument for your answer.

14. Determine the magnetic field (in terms of $I$, $a$, and $d$) at the origin due to the current loop in Figure P30.14.

**Figure P30.14**

15. Two long, parallel conductors carry currents $I_1 = 3.00 \text{ A}$ and $I_2 = 3.00 \text{ A}$, both directed into the page in Figure P30.15. Determine the magnitude and direction of the resultant magnetic field at $P$.

**Figure P30.15**

16. The idea that a magnetic field can have therapeutic value has been around for centuries. A rare-earth magnet sold to relieve joint pain is a disk 1.20 mm thick and 3.50 mm in diameter. Its circular flat faces are its north and south poles. Assume it is accurately modeled as a magnetic dipole. Also assume Equation 30.10 describes the magnetic field it produces at all points along its axis. The field is strongest, with the value 40.0 mT, at the center of each flat face. At what distance from the surface is the magnitude of the magnetic field like that of the Earth, with a value of 50.0 mT?

**Section 30.2 The Magnetic Force Between Two Parallel Conductors**

17. In Figure P30.17, the current in the long, straight wire is $I_1 = 5.00 \text{ A}$ and the wire lies in the plane of the rectangular loop, which carries the current $I_2 = 10.0 \text{ A}$. The dimensions are $c = 0.100 \text{ m}$, $a = 0.150 \text{ m}$, and $\ell = 0.450 \text{ m}$. Find the magnitude and direction of the net force exerted on the loop by the magnetic field created by the wire.
18. Two long, parallel conductors, separated by 10.0 cm, carry currents in the same direction. The first wire carries current \( I_1 = 5.00 \, \text{A} \), and the second carries \( I_2 = 8.00 \, \text{A} \). (a) What is the magnitude of the magnetic field created by \( I_1 \) at the location of \( I_2 \)? (b) What is the force per unit length exerted by \( I_1 \) on \( I_2 \)? (c) What is the magnitude of the magnetic field created by \( I_2 \) at the location of \( I_1 \)? (d) What is the force per length exerted by \( I_2 \) on \( I_1 \)?

19. Two long, parallel wires are attracted to each other by a force per unit length of 320 \( \mu \text{N/m} \) when they are separated by a vertical distance of 0.500 m. The current in the upper wire is 20.0 A to the right. Determine the location of the line in the plane of the two wires along which the total magnetic field is zero.

20. Three long wires (wire 1, wire 2, and wire 3) hang vertically. The distance between wire 1 and wire 2 is 20.0 cm. On the left, wire 1 carries an upward current of 1.50 A. To the right, wire 2 carries a downward current of 4.00 A. Wire 3 is to be located such that when it carries a certain current, each wire experiences no net force. (a) Is this situation possible? Is it possible in more than one way? Describe (b) the position of wire 3 and (c) the magnitude and direction of the current in wire 3.

21. The unit of magnetic flux is named for Wilhelm Weber. A practical-size unit of magnetic field is named for Johann Karl Friedrich Gauss. Both were scientists at Göttingen, Germany. Along with their individual accomplishments, together they built a telegraph in 1833. It consisted of a battery and switch, at one end of a transmission line 3 km long, operating an electromagnet at the other end. (André Ampère suggested electrical signaling in 1821; Samuel Morse built a telegraph line between Baltimore and Washington, D.C., in 1844.) Suppose Weber and Gauss’s transmission line was as diagrammed in Figure P30.21. Two long, parallel wires, each having a mass per unit length of 40.0 g/m, are supported in a horizontal plane by strings 6.00 cm long. When both wires carry the same current \( I \), the wires repel each other so that the angle \( \theta \) between the supporting strings is 16.0°. (a) Are the currents in the same direction or in opposite directions? (b) Find the magnitude of the current. (c) If this apparatus were taken to Mars, would the current required to separate the wires by 16° be larger or smaller than on Earth? Why?

22. Two parallel copper conductors are each 0.500 m long. They carry 10.0-A currents in opposite directions. (a) What center-to-center separation must the conductors have if they are to repel each other with a force of 1.00 N? (b) Is this situation physically possible? Explain.

### Section 30.3 Ampère’s Law

23. Four long, parallel conductors carry equal currents of \( I = 5.00 \, \text{A} \). Figure P30.23 is an end view of the conductors. The current direction is into the page at points \( A \) and \( B \) (indicated by the crosses) and out of the page at \( C \) and \( D \) (indicated by the dots). Calculate the magnitude and direction of the magnetic field at point \( P \), located at the center of the square of edge length 0.200 m.

24. A long, straight wire lies on a horizontal table and carries a current of 1.20 \( \mu \text{A} \). In a vacuum, a proton moves parallel to the wire (opposite the current) with a constant speed of \( 2.30 \times 10^4 \, \text{m/s} \) at a distance \( d \) above the wire. Determine the value of \( d \). You may ignore the magnetic field due to the Earth.

25. Figure P30.25 is a cross-sectional view of a coaxial cable. The center conductor is surrounded by a rubber layer, which is surrounded by an outer conductor, which is surrounded by another rubber layer. In a particular application, the current in the inner conductor is 1.00 A out of the page and the current in the outer conductor is 3.00 A into the page. Determine the magnitude and direction of the magnetic field at points \( a \) and \( b \).

26. The magnetic field 40.0 cm away from a long, straight wire carrying current 2.00 A is 1.00 \( \mu \text{T} \). (a) At what distance is it 0.100 \( \mu \text{T} \)? (b) What IF? At one instant, the two conductors in a long household extension cord carry equal 2.00-A currents in opposite directions. The two wires are 3.00 mm apart. Find the magnetic field 40.0 cm away.
from the middle of the straight cord, in the plane of the two wires. (c) At what distance is it one-tenth as large? (d) The center wire in a coaxial cable carries current 2.00 A in one direction, and the sheath around it carries current 2.00 A in the opposite direction. What magnetic field does the cable create at points outside?

27. A packed bundle of 100 long, straight, insulated wires forms a cylinder of radius $R = 0.500$ cm. (a) If each wire carries 2.00 A, what are the magnitude and direction of the magnetic force per unit length acting on a wire located 0.200 cm from the center of the bundle? (b) What If? Would a wire on the outer edge of the bundle experience a force greater or smaller than the value calculated in part (a)? Give a qualitative argument for your answer.

28. The magnetic coils of a tokamak fusion reactor are in the shape of a toroid having an inner radius of 0.700 m and an outer radius of 1.30 m. The toroid has 900 turns of large-diameter wire, each of which carries a current of 14.0 kA. Find the magnitude of the magnetic field inside the toroid along (a) the inner radius and (b) the outer radius.

29. Consider a column of electric current passing through plasma (ionized gas). Filaments of current within the column are magnetically attracted to one another. They can crowd together to yield a very great current density and a very strong magnetic field in a small region. Sometimes the current can be cut off momentarily by this pinch effect. (In a metallic wire, a pinch effect is not important because the current-carrying electrons repel one another with electric forces.) The pinch effect can be demonstrated by making an empty aluminum can carry a large current parallel to its axis. Let $R$ represent the radius of the can and $J$ the upward current, uniformly distributed over its curved wall. Determine the magnetic field (a) just inside the wall and (b) just outside. (c) Determine the pressure on the wall.

30. Niobium metal becomes a superconductor when cooled below 9 K. Its superconductivity is destroyed when the surface magnetic field exceeds 0.100 T. Determine the maximum current a 2.00-mm-diameter niobium wire can carry and remain superconducting, in the absence of any external magnetic field.

31. A long, cylindrical conductor of radius $R$ carries a current $I$ as shown in Figure P30.31. The current density $J$, however, is not uniform over the cross section of the conductor but is a function of the radius according to $J = br$, where $b$ is a constant. Find an expression for the magnetic field magnitude $B$ (a) at a distance $r_1 < R$ and (b) at a distance $r_2 > R$, measured from the axis.

32. In Figure P30.32, both currents in the infinitely long wires are 8.00 A in the negative $x$ direction. The wires are separated by the distance $2a = 6.00$ cm. (a) Sketch the magnetic field pattern in the $yz$ plane. (b) What is the value of the magnetic field at the origin? At ($y = 0$, $z \rightarrow \infty$)? (c) Find the magnetic field at points along the $z$ axis as a function of $z$. (d) At what distance $d$ along the positive $z$ axis is the magnetic field a maximum? (e) What is this maximum value?

33. An infinite sheet of current lying in the $yz$ plane carries a surface current of linear density $J_s$. The current is in the $y$ direction, and $J_s$ represents the current per unit length measured along the $z$ axis. Figure P30.33 is an edge view of the sheet. Prove that the magnetic field near the sheet is parallel to the sheet and perpendicular to the current direction, with magnitude $\mu_0 J_s/2$. Suggestion: Use Ampère’s law and evaluate the line integral for a rectangular path around the sheet, represented by the dashed line in Figure P30.33.

34. You are given a certain volume of copper from which you can make copper wire. To insulate the wire, you can have as much enamel as you like. You will use the wire to make a tightly wound solenoid 20 cm long having the greatest possible magnetic field at the center and using a power supply that can deliver a current of 5 A. The solenoid can be wrapped with wire in one or more layers. (a) Should you make the wire long and thin or shorter and thick? Explain. (b) Should you make the solenoid radius small or large? Explain.

35. What current is required in the windings of a long solenoid that has 1 000 turns uniformly distributed over a length of 0.400 m to produce at the center of the solenoid a magnetic field of magnitude $1.00 \times 10^{-4}$ T?

36. Consider a solenoid of length $\ell$ and radius $R$, containing $N$ closely spaced turns and carrying a steady current $I$. (a) In terms of these parameters, find the magnetic field at a point along the axis as a function of distance $a$ from the end of the solenoid. (b) Show that as $\ell$ becomes very long, $B$ approaches $\mu_0 NI/2\ell$ at each end of the solenoid.
37. A single-turn square loop of wire, 2.00 cm on each edge, carries a clockwise current of 0.200 A. The loop is inside a solenoid, with the plane of the loop perpendicular to the magnetic field of the solenoid. The solenoid has 30 turns/cm and carries a clockwise current of 15.0 A. Find the force on each side of the loop and the torque acting on the loop.

38. A solenoid 10.0 cm in diameter and 75.0 cm long is made from copper wire of diameter 0.100 cm, with very thin insulation. The wire is wound onto a cardboard tube in a single layer, with adjacent turns touching each other. What power must be delivered to the solenoid if it is to produce a field of 8.00 mT at its center?

Section 30.5 Gauss’s Law in Magnetism

39. A cube of edge length \( \ell = 2.50 \) cm is positioned as shown in Figure P30.39. A uniform magnetic field given by \( \mathbf{B} = (5\mathbf{i} + 4\mathbf{j} + 3\mathbf{k}) \) T exists throughout the region. (a) Calculate the magnetic flux through the shaded face. (b) What is the total flux through the six faces?

![Figure P30.39](image)

40. Consider the hemispherical closed surface in Figure P30.40. The hemisphere is in a uniform magnetic field that makes an angle \( \theta \) with the vertical. Calculate the magnetic flux through (a) the flat surface \( S_1 \) and (b) the hemispherical surface \( S_2 \).

![Figure P30.40](image)

41. A solenoid 2.50 cm in diameter and 30.0 cm long has 300 turns and carries 12.0 A. (a) Calculate the flux through the surface of a disk of radius 5.00 cm that is positioned perpendicular to and centered on the axis of the solenoid, as shown in Figure P30.41a. (b) Figure P30.41b shows an enlarged end view of the same solenoid. Calculate the flux through the blue area, which is an annulus with an inner radius of 0.400 cm and an outer radius of 0.800 cm.

![Figure P30.41](image)

42. Compare this problem with Problem 65 in Chapter 24. Consider a magnetic field that is uniform in direction throughout a certain volume. Can it be uniform in magnitude? Must it be uniform in magnitude? Give evidence for your answers.

Section 30.6 Magnetism in Matter

43. At saturation, when nearly all the atoms have their magnetic moments aligned, the magnetic field in a sample of iron can be 2.00 T. If each electron contributes a magnetic moment of \( 9.27 \times 10^{-24} \) A · m\(^2\) (one Bohr magneton), how many electrons per atom contribute to the saturated field of iron? The number density of atoms in iron is approximately \( 8.50 \times 10^{28} \) atoms/m\(^3\).

Section 30.7 The Magnetic Field of the Earth

44. A circular coil of 5 turns and a diameter of 30.0 cm is oriented in a vertical plane with its axis perpendicular to the horizontal component of the Earth’s magnetic field. A horizontal compass placed at the center of the coil is made to deflect 45.0° from magnetic north by a current of 0.600 A in the coil. (a) What is the horizontal component of the Earth’s magnetic field? (b) The current in the coil is switched off. A “dip needle” is a magnetic compass mounted so that it can rotate in a vertical north–south plane. At this location, a dip needle makes an angle of 13.0° from the vertical. What is the total magnitude of the Earth’s magnetic field at this location?

45. The magnetic moment of the Earth is approximately \( 8.00 \times 10^{22} \) A · m\(^2\). (a) Imagine that the planetary magnetic field were caused by the complete magnetization of a huge iron deposit. How many unpaired electrons would participate? (b) At two unpaired electrons per iron atom, how many kilograms of iron would compose the deposit? Iron has a density of 7.900 kg/m\(^3\) and approximately \( 8.50 \times 10^{28} \) iron atoms/m\(^3\).

46. A particular location on the Earth’s surface is characterized by a value of gravitational field, a value of magnetic field, and a value of atmospheric pressure. (a) Which of these quantities are vectors and which are scalars? (b) Determine a value for each quantity at your current location. Include the direction of each vector quantity. State your sources. (c) Which of the quantities have separate causes from which of the others?

Additional Problems

47. A very long, thin strip of metal of width \( w \) carries a current \( I \) along its length as shown in Figure P30.47. Find the magnetic field at the point \( P \) in the diagram. The point \( P \) is in the plane of the strip at distance \( b \) away from it.
48. The magnitude of the Earth’s magnetic field at either pole is approximately $7.00 \times 10^{-5}$ T. Suppose the field fades away, before its next reversal. Scouts, sailors, and conservative politicians around the world join together in a program to replace the field. One plan is to use a current loop around the equator, without relying on magnetization of any materials inside the Earth. Determine the current that would generate such a field if this plan were carried out. Take the radius of the Earth as $R_E = 6.37 \times 10^6$ m.

49. A thin copper bar of length $\ell = 10.0$ cm is supported horizontally by two (nonmagnetic) contacts. The bar carries current $I_1 = 100$ A in the $-x$ direction as shown in Figure P30.49. At a distance $h = 0.500$ cm below one end of the bar, a long, straight wire carries a current $I_2 = 200$ A in the $z$ direction. Determine the magnetic force exerted on the bar.

50. Suppose you install a compass on the center of the dashboard of a car. Compute an order-of-magnitude estimate for the magnetic field at this location produced by the current when you switch on the headlights. How does this estimate compare with the Earth’s magnetic field? You may suppose the dashboard is made mostly of plastic.

51. ▲ A nonconducting ring of radius 10.0 cm is uniformly charged with a total positive charge 10.0 $\mu$C. The ring rotates at a constant angular speed 20.0 rad/s about an axis through its center, perpendicular to the plane of the ring. What is the magnitude of the magnetic field on the axis of the ring 5.00 cm from its center?

52. A nonconducting ring of radius $R$ is uniformly charged with a total positive charge $q$. The ring rotates at a constant angular speed $\omega$ about an axis through its center, perpendicular to the plane of the ring. What is the magnitude of the magnetic field on the axis of the ring a distance $R/2$ from its center?

53. Two circular coils of radius $R$, each with $N$ turns, are perpendicular to a common axis. The coil centers are a distance $R$ apart. Each coil carries a steady current $I$ in the same direction as shown in Figure P30.53. (a) Show that the magnetic field on the axis at a distance $x$ from the center of one coil is

$$B = \frac{N\mu_0R^2}{2} \left[ \frac{1}{(R^2 + x^2)^{3/2}} + \frac{1}{(2R^2 + x^2 - 2Rx)^{3/2}} \right]$$

(b) Show that $dB/dx$ and $d^2B/dx^2$ are both zero at the point midway between the coils. We may then conclude that the magnetic field in the region midway between the coils is uniform. Coils in this configuration are called Helmholtz coils.

54. Two identical, flat, circular coils of wire each have 100 turns and a radius of 0.500 m. The coils are arranged as a set of Helmholtz coils (see Fig. P30.53), parallel and with separation 0.500 m. Each coil carries a current of 10.0 A. Determine the magnitude of the magnetic field at a point on the common axis of the coils and halfway between them.

55. We have seen that a long solenoid produces a uniform magnetic field directed along the axis of a cylindrical region. To produce a uniform magnetic field directed parallel to a diameter of a cylindrical region, however, one can use the saddle coils illustrated in Figure P30.55. The loops are wrapped over a somewhat flattened tube. Assume the straight sections of wire are very long. The end view of the tube shows how the windings are applied. The overall current distribution is the superposition of two overlapping, circular cylinders of uniformly distributed current, one toward you and one away from you. The current density $j$ is the same for each cylinder. The position of the axis of one cylinder is described by a position vector $\mathbf{a}$ relative to the other cylinder. Prove that the magnetic field inside the hollow tube is $\mu_0 j a/2$ downward. Suggestion: The use of vector methods simplifies the calculation.

56. You may use the result of Problem 33 in solving this problem. A very large parallel-plate capacitor carries charge with uni-
form charge per unit area $+\sigma$ on the upper plate and $-\sigma$ on the lower plate. The plates are horizontal and both move horizontally with speed $v$ to the right. (a) What is the magnetic field between the plates? (b) What is the magnetic field close to the plates but outside of the capacitor? (c) What is the magnitude and direction of the magnetic field per unit area on the upper plate? (d) At what extrapolated speed $v$ will the magnetic force on a plate balance the electric force on the plate? Calculate this speed numerically.

57. Two circular loops are parallel, coaxial, and almost in contact, 1.00 mm apart (Fig. P30.57). Each loop is 10.0 cm in radius. The top loop carries a clockwise current of 140 A. The bottom loop carries a counterclockwise current of 140 A. (a) Calculate the magnetic force exerted by the bottom loop on the top loop. (b) Suppose a student thinks the first step in solving part (a) is to use Equation 30.7 to find the magnetic field created by one of the loops. How would you argue for or against this idea? Suggestion: Think about how one loop looks to a bug perched on the other loop. (c) The upper loop has a mass of 0.021 kg. Calculate its acceleration, assuming the only forces acting on it are the force in part (a) and the gravitational force.

![Figure P30.57](Image)

58. What objects experience a force in an electric field? Chapter 23 gives the answer: any object with electric charge, stationary or moving, other than the charged object that created the field. What creates an electric field? Any object with electric charge, stationary or moving, as you studied in Chapter 23. What objects experience a force in a magnetic field? An electric current or a moving electric charge, other than the current or charge that created the field, as discussed in Chapter 29. What creates a magnetic field? An electric current, as you studied in Section 30.1, or a moving electric charge, as shown in this problem. (a) To understand how a moving charge creates a magnetic field, consider a particle with charge $q$ moving with velocity $\mathbf{v}$. Define the position vector $\mathbf{r} = r\mathbf{\hat{r}}$ leading from the particle to some location. Show that the magnetic field at that location is

$$\mathbf{B} = \frac{\mu_0}{4\pi} \frac{q\mathbf{v} \times \mathbf{r}}{r^2}$$

(b) Find the magnitude of the magnetic field 1.00 mm to the side of a proton moving at $2.00 \times 10^7$ m/s. (c) Find the magnetic force on a second proton at this point, moving with the same speed in the opposite direction. (d) Find the electric force on the second proton.

59. The chapter-opening photograph shows a rail gun. Rail guns have been suggested for launching projectiles into space without chemical rockets and for ground-to-air antiaircraft weapons of war. A tabletop model rail gun (Fig. P30.59) consists of two long, parallel, horizontal rails 3.50 cm apart, bridged by a bar $BD$ of mass 3.00 g. The bar is originally at rest at the midpoint of the rails and is free to slide without friction. When the switch is closed, electric current is quickly established in the circuit $ABCDEA$. The rails and bar have low electric resistance, and the current is limited to a constant 24.0 A by the power supply. (a) Find the magnitude of the magnetic field 1.75 cm from a single very long, straight wire carrying current 24.0 A. (b) Find the magnitude and direction of the magnetic field at point $C$ in the diagram, the midpoint of the bar, immediately after the switch is closed. Suggestion: Consider what conclusions you can draw from the Biot–Savart law. (c) At other points along the bar $BD$, the field is in the same direction as at point $C$, but is larger in magnitude. Assume the average effective magnetic field along $BD$ is five times larger than the field at $C$. With this assumption, find the magnitude and direction of the force on the bar. (d) Find the acceleration of the bar when it is in motion. (e) Does the bar move with constant acceleration? (f) Find the velocity of the bar after it has traveled 130 cm to the end of the rails.

![Figure P30.59](Image)

60. Fifty turns of insulated wire 0.100 cm in diameter are tightly wound to form a flat spiral. The spiral fills a disk surrounding a circle of radius 5.00 cm and extending to a radius 10.00 cm at the outer edge. Assume the wire carries current $I$ at the center of its cross section. Approximate each turn of wire as a circle. Then a loop of current exists at radius 5.05 cm, another at 5.15 cm, and so on. Numerically calculate the magnetic field at the center of the coil.

61. An infinitely long, straight wire carrying a current $I_1$ is partially surrounded by a loop as shown in Figure P30.61. The loop has a length $L$ and radius $R$, and it carries a current $I_2$. The axis of the loop coincides with the wire. Calculate the force exerted on the loop.

![Figure P30.61](Image)

62. The magnitude of the force on a magnetic dipole $\mathbf{\mu}$ aligned with a nonuniform magnetic field in the $x$ direction is $F_x = |\mathbf{\mu}| dB/dx$. Suppose two flat loops of wire each have radius $R$ and carry current $I$. (a) The loops are arranged coaxially and separated by a variable distance $x$, large compared with $R$. Show that the magnetic force
between them varies as $1/x^4$. (b) Evaluate the magnitude of this force, taking $I = 10.0$ A, $R = 0.500$ cm, and $x = 5.00$ cm.

A wire is formed into the shape of a square of edge length $L$ (Fig. P30.63). Show that when the current in the loop is $I$, the magnetic field at point $P$, a distance $x$ from the center of the square along its axis is

$$B = \frac{\mu_0 I L^2}{2\pi(x^2 + L^2/4)\sqrt{x^2 + L^2/2}}$$

### Answers to Quick Quizzes

30.1 $B$, $C$, $A$. Point $B$ is closest to the current element. Point $C$ is farther away, and the field is further reduced by the $\sin \theta$ factor in the cross product $\vec{A} \times \hat{r}$. The field at $A$ is zero because $\theta = 0$.

30.2 (a). The coils act like wires carrying parallel currents in the same direction and hence attract one another.

30.3 $b$, $d$, $a$, $c$. Equation 30.13 indicates that the value of the line integral depends only on the net current through each closed path. Path $b$ encloses 1 A, path $d$ encloses 3 A, path $a$ encloses 4 A, and path $c$ encloses 6 A.

30.4 $b$, then $a = c = d$. Paths $a$, $c$, and $d$ all give the same nonzero value $\mu_0 I$ because the size and shape of the paths do not matter. Path $b$ does not enclose the current; hence, its line integral is zero.

30.5 (c). The magnetic field in a very long solenoid is independent of its length or radius. Overwrapping with an additional layer of wire increases the number of turns per unit length.

**Figure P30.63**

**Figure P30.64**

**Figure P30.65**

**Figure P30.67**