MINI-LECTURES
AND
ANSWERS

Basic Concepts

Learning Objectives:

1. Write sets using set notation.
2. Use number lines.
3. Know the common sets of numbers.
4. Find additive inverses.
5. Use absolute value.
6. Use inequality symbols.

Examples:

1. List all numbers from the set $\left\{-10, \sqrt{5}, -2\frac{3}{4}, 0, -5, 10\right\}$ that are:

   a) natural numbers b) whole numbers
   c) integers d) rational numbers
   e) irrational numbers f) real numbers

2. Graph each of the numbers on a number line: $-\sqrt{5}, 5, -4\frac{1}{3}, 1\frac{1}{2}, \pi$.

3. Select the lesser number in each pair.

   a) $-3, 8$ b) $-\frac{4}{5}, -\frac{1}{3}$ c) $10, -12$

4. For each number, find the additive inverse.
   a) 6 b) –5 c) 2.7

5. Simplify.
   a) $|-3|$ b) $-|-6|$ c) $|13-16|$

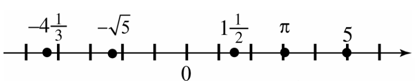
6. Decide whether the statement is true or false.
   a) $-5 \geq -11$ b) $|-3| \leq |-5|$ c) $-|-7| > 1$

7. Write the following in interval notation and graph the interval.
   a) $\{x \mid x \leq 5\}$ b) $\{x \mid x > -8\}$ c) $\{x \mid -1 < x \leq 3\}$

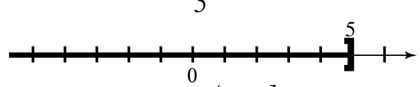
Teaching Notes:

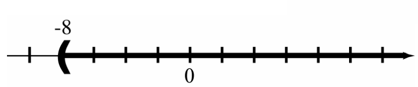
- Many students need to be reminded of set notation.
- Remind students that integers are rational numbers; any integer can be written as the ratio of itself and 1.
- Decimal numbers that terminate or repeat in a fixed block are examples of rational numbers; ask students to give examples of both.
- The decimal form of an irrational number neither terminates nor repeats.
- The number line is a good way to illustrate opposite numbers/additive inverses

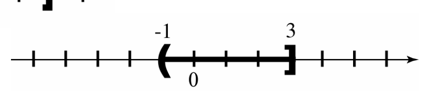
Answers: *1a) 10, b) 0, 10, c) –10, –5, 0, 10, d) –10, –5, $-2\frac{3}{4}$, 0, 10, e) $\sqrt{5}$, f) –10, –5, $-2\frac{3}{4}$, 0, $\sqrt{5}$, 10;*

2) 

; 3a) –3, b) $-\frac{4}{5}$, c) –12; 4a) –6, b) 5, c) –2.7; 5a) 3, b) –6, c) 3;

6a) true, b) true, c) false; 7a) $(-\infty, 5]$ 

b) $(-8, \infty)$ 

c) $(-1, 3]$ 

Operations on Real Numbers

Learning Objectives:

1. Add real numbers.
2. Subtract real numbers.
3. Find the distance between two points on a number line.
4. Multiply real numbers.
5. Find reciprocals and divide real numbers.

Examples:

1. Perform the operations indicated.

   a) $-5+(-3)$ b) $5+3$ c) $5+(-3)$ d) $-5+3$

   e) $3-5$ f) $3-(-5)$ g) $-3-5$ h) $-3-(-5)$

   i) $(3)(5)$ j) $(-3)(-5)$ k) $(-3)(5)$ l) $(3)(-5)$

2. Find the distance between each pair of points.

   a) 7 and -6 b) -3 and -9

3. Give the reciprocal of each number.

   a) 12 b) $\frac{5}{6}$ c) $-\frac{1}{2}$ d) 0

4. Find each quotient.

   a) $8 \div 4$ b) $-8 \div -4$ c) $-8 \div 4$ d) $8 \div -4$

5. Perform each of the following operations, if possible.

   a) $|-2.1+(-7.3)|$ b) $-\frac{3}{5} \div \frac{15}{20}$ c) $-\frac{2}{12}+\frac{3}{24}$ d) $|5.4-9.2|$ e) $0 \div -4$

   f) $3.2 \div 0$ g) $\frac{1}{2}+\left(-\frac{1}{2}\right)$ h) $\frac{-4.2-4.2}{-8.4}$ i) $13.32+9.75-5.21$ j) $\frac{-\frac{3}{8}}{4}$

Teaching Notes:

- Refer students to the charts for ***Adding***, ***Subtracting***, ***Multiplying***, and ***Dividing Real Numbers*** .
- Many students find working with fractions difficult. It may be useful to review finding the least common denominator.
- Remind students that only nonzero numbers have inverses.
- The fraction is undefined when zero is "underneath".

Answers: 1a) –8, b) 8, c) 2, d) –2, e) –2, f) 8, g) –8, h) 2, i) 15, j) 15, k) –15, l) –15; 2a) 13 , b) 6; 3a) 1/12, b) 6/5, c) –2, d) zero does not have a reciprocal, or multiplicative inverse; 4a) 2, b) 2, c) –2, d) –2; 5a) 9.4, b) –4/5, c) –1/24, d) 3.8, e) 0, f) undefined, g) 0, h) 1, i) 17.86, j) –3/32

Exponents, Roots, and Order of Operations

Learning Objectives:

1. Use exponents.
2. Find square roots.
3. Use the order of operations.
4. Evaluate algebraic expressions for given values of variables.

Examples:

1. Write in exponential form.

   a) $(4)(4)(4)(4)(4)$ b) $(-2)(-2)(-2)(-2)$ c) $x \cdot x \cdot x \cdot y \cdot y$

2. Evaluate.

   a) 5^2 b) 2^3 c) 3^4 d) $(-5)^2$ e) -5^2

   f) $(-2)^3$ g) -2^3 h) $(-3)^4$ i) $\left(-\frac{1}{2}\right)^2$ j) $(-1.2)^3$

3. Find each square root, if it exists.

   a) $\sqrt{4}$ b) $-\sqrt{4}$ c) $\sqrt{-4}$ d) $-\sqrt{-25}$

   e) $\sqrt{0.25}$ f) $-\sqrt{\frac{49}{36}}$ g) $\sqrt{30+6}$ h) $-\sqrt{\frac{1}{16}+\frac{3}{16}}$

4. Simplify using the order of operations.

   a) $5(2-4)+6$ b) $-7^2-3(2-4)$ c) $(9-6-1)^3 \div 4 - 9$

   d) $(-9)(\sqrt{36})-(-2)(-9)$ e) $\frac{3+(-5)^2+8\cdot 3}{4(6-5)}$ f) $\frac{|-2|+|-6+9|}{-3^2-6}$

5. Evaluate each expression for the values given.

   a) $x^2+5x;\ x=-3$ b) $4y-(2x)^2;\ x=-2,\ y=4$

Teaching Notes:

- In 1b) some students answer -2^4 instead of $(-2)^4$. Many students think these are equal.
- Many students do not understand that the square root symbol indicates a principal square root, which is never negative.
- Order of Operations – introduce PEMDAS or Please Excuse My Dear Aunt Sally.

Answers: 1a) 4^5, b) $(-2)^4$, c) x^3y^2; 2a) 25, b) 8, c) 81, d) 25, e) –25, f) –8, g) –8, h) 81, i) ¼, j) –1.728; 3a) 2, b) –2, c) DNE, d) DNE, e) 0.5, f) –7/6, g) 6, h) –1/2; 4a) –4, b) –43, c) –7, d) –72, e) 13, f) –1/3; 5a) –6, b) 0

Properties of Real Numbers

Learning Objectives:

1. Use the distributive property.
2. Use the identity properties.
3. Use the inverse properties.
4. Use the commutative and associative properties.

Examples:

1. Name the property that justifies each statement.

a) $(-5) + 3 = 3 + (-5)$ b) $(5 + 2) + 4 = 5 + (2 + 4)$ c) $\frac{1}{3}\cdot 3 = 1$

d) $7 + (-7) = 0$ e) $4(5 + 2) = 4\cdot 5 + 4\cdot 2$ f) $\frac{3}{5}\cdot 1 = \frac{3}{5}$

g) $6.3 + 0 = 6.3$ h) $-5\cdot(4\cdot 2) = (-5\cdot 4)\cdot 2$ i) $4 \cdot 3 = 3 \cdot 4$

2. Simplify.

a) $4x+5x$ b) $1+12y-y$ c) $3x-9-12x$

d) $2.5y-8.6+3.4y-12.3$ e) $3(6y+9)$ f) $2k-(5k-4)$

g) $7.3b+8.1-2(3.2b-0.4)$ h) $\frac{1}{2}(12x-4)-\frac{1}{6}(30x-y)$ i) $-(9-t)+(3t-6)$

Teaching Notes:

- Remind students that the ***commutative property*** deals with the *order* of the operands, while, the ***associative property*** deals with the *grouping* of the operands.
- Discuss difference between the ***identity properties*** and the ***inverse properties.***
- Remind students that when we use the identity property the result is identical to the quantity we begin with.
- Remind students that parentheses do not always imply that the ***distributive property*** is to be used (e.g., $(3a)(5)(6b)=90ab$, not $(3a)(5)+(3a)(6b)$).

Answers: 1a) commutative property of addition, b) associative property of addition, c) inverse property of multiplication, d) inverse property of addition, e) distributive property of multiplication with respect to addition, f) identity property of multiplication, g) identity property of addition, h) associative property of multiplication, i) commutative property of multiplication; 2a) $9x$, b) $1+11y$, c) $-9x-9$, d) $5.9y-20.9$, e) $18y+27$, f) $-3k+4$, g), $0.9b+8.9$ h) $x+\frac{1}{6}y-2$, i) $4t-15$

Linear Equations in One Variable

Learning Objectives:

1. Distinguish between expressions and equations.
2. Identify linear equations.
3. Solve linear equations using the addition and multiplication properties of equality.
4. Solve linear equations using the distributive property.
5. Solve linear equations with fractions or decimals.
6. Identify conditional equations, contradictions, and identities.

Examples:

1. Decide whether each of the following is an expression or an equation.
   a) $4x+1$ b) $4x+1=0$

2. Determine whether the given value is a solution to the equation.
   a) Is $x = 3$ a solution to $4x+2=18$? Why or why not?
   b) Is $x = 12$ a solution to $\frac{5}{6}x=10$? Why or why not?

3. Solve. Check your solutions.
   a) $-12+x=-2$ b) $13+x=-35$ c) $5x=45$
   d) $-5x=70$ e) $10x+5=17$ f) $6x+3=2x-9$
   g) $-14x-10=8-5x$ h) $6x-2+5x=4x+12$ i) $5-x=8-2(x-1)$

4. Solve. Check your solutions.
   a) $\frac{1}{2}x=8$ b) $-\frac{5}{6}x=10$ c) $\frac{x}{3}+5=\frac{3}{5}$
   d) $\frac{4x}{5}+\frac{5}{2}=3x$ e) $3-\frac{2}{3}(x+2)=1$ f) $0.1x+0.5=0.4x-0.9$
   g) $0.5x-0.2=0.3x+0.8$ h) $0.4(3x+1)=1$ i) $0.4(x+3)-2=0.4x$

5. Classify each of the following equations as a *conditional equation*, a *contradiction*, or an *identity*.
   a) $3(x+2)=5x+6-2x$ b) $2x-3=7$ c) $-5x+7=3-9x+4x$

Teaching Notes:

- Encourage students to check their solutions.
- Encourage students to simplify each side of the equation as a first step.
- Some students prefer to always end with the variable on the left, while others prefer to arrive at a positive coefficient of the variable.
- Some students try to subtract the coefficient from the variable instead of eliminating it by dividing.

Answers: 1a) expression, b) equation; 2a) no because 4·3+2≠18, b) yes because (5/6)·12=10; 3a) {10}, b) {–48}, c) {9}, d) {–14}, e) {6/5}, f) {–3}, g) {–2}, h) {2}, i) {5}; 4a) {16}, b) {–12}, c){–66/5}, d) {25/22}, e) {1}, f) { $4.\overline{6}$ *}, g) {5}, h) {0.5}, i)∅ ; 5a) identity, b) conditional equation, c) contradiction*

Formulas and Percent

Learning Objectives:

1. Solve a formula for a specified variable.
2. Solve applied problems using formulas.
3. Solve percent problems.
4. Solve problems involving percent increase or decrease.

Examples:

1. Solve each formula for the specified variable.

   a) $6x+3y=5$; for y b) $y=-\frac{2}{3}x+3$; for x c) $V=lwh$; for h

   d) $A=\frac{h}{2}(B+b)$; for b e) $A=2\pi rh$; for r f) $3(4ax+y)=3ax-y$; for x

2. Solve as indicated.

   a) Solve $F=\frac{9}{5}C+32$ for C; then evaluate for $F = 25°$. Round to the nearest tenth.

   b) Solve $V=\frac{1}{3}\pi r^2 h$ for h; then evaluate for $V = 5.35$, $r = 2$, $\pi \approx 3.14$. Round to the nearest hundredth.

3. Application problems.

   a) Suppose economists use $C=0.644D+5.822$ as a model of the country's economy, where C and D are in billions of dollars. Solve the equation for D, and use this result to determine the disposable income D if the consumption C is \$9.44 billion. Round your answer to the nearest tenth of a billion.

   b) Some doctors use the formula $ND = 1.1T$ to relate N (the number of appointments scheduled in one day), D (the duration of each appointment), and T (the total number of minutes the doctor can use to see patients in one day). Solve the formula for D, and use this result to find the duration of each appointment if the doctor has 6 hours available for appointments and must see 25 patients per day.

   c) A car salesman earned \$6750 in commission on sales of automobiles amounting to \$45,000. What is his rate of commission?

   d) In July, there were 14,150 visitors to the *Dunes State Park*. In August, there were 9644 visitors. What was the percent increase/decrease in visitors from July to August?

Teaching Notes:

- For questions like number 1, encourage students to underline or circle the specified variable.
- When calculating a percent increase or decrease, be sure to use the original number as the denominator.

<u>Answers</u>: 1a) y=–2x+(5/3), b) x=(–3/2)y+(9/2), c) $h=\frac{V}{lw}$*, d)* $b=\frac{2A}{h}-B$*, e)* $r=\frac{A}{2\pi h}$*, f)* $x=\frac{-4y}{9a}$*;*
2a) C=(5/9)(F–32), $C\approx -3.9°$*, b)* $h=\frac{3V}{\pi r^2}$*, h≈1.28; 3a)* $D=\frac{C-5.822}{0.644}$*, D ≈ \$5.6 billion, b) D=1.1T/N, D=15.84 min, c) 15%, d) 31.8% decrease*

Applications of Linear Equations

Learning Objectives:

1. Translate from words to mathematical expressions.
2. Write equations from given information.
3. Distinguish between simplifying expressions and solving equations.
4. Use the six steps in solving an applied problem.
5. Solve percent problems.
6. Solve investment problems.
7. Solve mixture problems.

Examples:

1. Translate into a mathematical expression, then simplify.
   a) The sum of three consecutive integers if the first integer is x
   b) The perimeter of a rectangle with length x and width $x - 7$
   c) The total amount of money (in cents) in x quarters, $5x$ dimes, and $(3x - 1)$ nickels

2. Use the *six-step approach* to solving applied problems to solve each of the following.

   a) ***Geometry*** The length of a rectangular room is 6 feet longer than twice the width. If the room's perimeter is 168 feet, what are the room's dimensions?

   b) ***Rental*** It costs $20 plus $0.25 per mile to rent a truck from U-Rent. How many miles were traveled if the final bill was $36.25?

   c) ***Revenue*** The revenue of Company X quadrupled. Then it increased by $1.6 million. The present revenue is $24.4 million. What was the original revenue?

   d) ***Satellite Phone*** Shaun's satellite phone provider charges its customers $15 per month plus 30 cents per minute of on-line usage. Shaun received a bill from the provider covering a month period and was charged a total of $52.50. How many minutes did he spend using this phone service that period, to the nearest whole minute?

   e) ***Balloon Altitude*** A hot air balloon spent several minutes ascending. It then stayed at a level altitude for three times as long as it had ascended. It took 5 minutes less to descend than it had to ascend. The entire trip took one hour and 30 minutes. For how long was the balloon at a level altitude?

   f) ***Investment*** Joe invested $12,000 in certificates of deposit paying 4%. How much additional money should he invest in certificates paying 6% so that his total return for the investments will be $792?

   g) ***Mixture*** How much water must be added to 10 quarts of an 8% concentrated detergent to reduce the concentration to 5%?

Teaching Notes:

- Have students create a list of key terms and associated mathematical operations.
- Some students need to see many of the number problems in order to understand how an English sentence can be represented as an algebraic equation.
- Encourage students to draw and label diagrams when appropriate.
- Refer students to the ***Solving an Applied Problem*** and the ***Translating from Words to Mathematical Expressions*** charts in the textbook.

Answers: *1a)* $x+x+1+x+2,\ 3x+3$, *b)* $2x+2(x-7),\ 4x-14$ *c)* $25x+10(5x)+5(3x-1),\ 90x-5$; *2a) l=58 ft, w=26 ft, b) 65 miles, c) $5.7 million, d) 125 minutes, e) 57 minutes, f) $5200, g) 6 quarts*

Further Applications of Linear Equations

Learning Objectives:

1. Solve problems about different denominations of money.
2. Solve problems about uniform motion.
3. Solve problems about angles.

Examples:

1. Mary has a piggy bank in which she has saved only dimes and quarters. She has 17 coins in the bank worth \$2.45. How many coins of each type does she have?

2. Angela has 37 coins, consisting of only dimes and quarters, worth \$7.45. How many coins of each type does she have?

3. A collection of 70 coins consisting of dimes, quarters, and half-dollars has a value of \$17.75. There are three times as many quarters as dimes. Find the number of each kind of coin.

4. Two cars are traveling on opposite directions on a highway. One car is driving 4 miles per hour faster than the other car. After 3 hours the cars are 414 miles apart. Find the speed of both cars.

5. Lori starts jogging at 5 miles per hour. One half hour later, Debbie starts jogging on the same route at 7 miles per hour. How long will it take Debbie to catch Lori?

6. One angle of a triangle measures 4 more degrees than three times the measure of the second angle. The third angle measures 40°. What are the measures of the two unknown angles of the triangle?

7. Two angles are complementary if the sum of their measures is 90º. If the measure of the first angle is xº, and the measure of the second angle is $(3x - 2)$º, find the measure of each angle.

Teaching Notes:

- Encourage students to draw and label diagrams when appropriate.
- Some students need to see several examples of each type of word problem.
- Refer students to the ***six-step plan*** for solving applied problems in the text.

Answers: 1) 12 dimes and 5 quarters; 2) 12 dimes and 25 quarters; 3) 15 dimes, 45 quarters, and 10 half-dollars; 4) 67 mph, 71 mph; 5) 1 hour 15 minutes; 6) 34º and 106º; 7) 23º and 67º

Linear Inequalities in One Variable

Learning Objectives:

1. Solve Graph intervals on a number line.
2. Solve linear inequalities using the addition property.
3. Solve linear inequalities using the multiplication property.
4. Solve linear inequalities with three parts.
5. Solve applied problems by using linear inequalities.

Examples:

1. Write each inequality in interval notation and graph the interval.

   a) $1 < x < 4$ b) $x \leq 0$ c) $x \geq 5$

2. Solve each inequality. Give the solution set in interval form.

   a) $x - 1 < 4$ b) $6x \leq -30$ c) $-5x > 25$

   d) $\frac{1}{2}x \geq 3$ e) $2x - 6 \leq 4$ f) $4 - 4x > 16$

   g) $-6x + 8 > -7x + 4$ h) $-25x + 25 \leq -5(4x - 1)$ i) $3(2x + 4) > 2(x + 2)$

   j) $\frac{1}{3}(x+2) < \frac{2}{5}(x+1)$

3. Solve each three-part inequality and express your answer in interval notation.

   a) $3 \leq 2x - 1 < 5$ b) $6 < -2(x - 1) < 12$

4. A student has test scores of 68%, 76%, and 78% in a biology class. What must she score on the last exam to earn a B (80% or better) in the course?

Teaching Notes:

- Many students forget to reverse the direction of the inequality symbol when necessary.
- Some students do better if they move the variable in such a way that it has a positive coefficient whenever possible.
- Some students need frequent reminders that solving inequalities is the same as solving equations, with one important difference: reverse the direction of the inequality symbol when multiplying or dividing by a negative number.
- It is a good idea to have students rewrite an inequality such as $5 \leq x$ as $x \geq 5$ so that the variable is on the left.

<u>Answers</u>: 1a) (1,4), b) $(-\infty, 0]$, c) $[5, \infty)$ 2) $(-\infty, 5)$, b) $(-\infty, -5]$, c) $(-\infty, -5)$, d) $[6, \infty)$, e) $(-\infty, 5]$, f) $(-\infty, -3)$, g) $(-4, \infty)$, h) $[4, \infty)$, i) $(-2, \infty)$, j) $(4, \infty)$; 3) $[2, 3)$, b) $(-5, -2)$; 4 98% or better

Set Operations and Compound Inequalities

Learning Objectives:

1. Recognize set intersection and union.
2. Find the intersection of two sets.
3. Solve compound inequalities with the word *and*.
4. Find the union of two sets.
5. Solve compound inequalities with the word *or*.

Examples:

1. If $A = \{x|x \text{ is an even integer}\}$, $B = \{x|x \text{ is an odd integer}\}$, $C = \{1,2,3,4\}$, and $D = \{3,4,5,6\}$, list the elements of each of the following sets.

   a) $C \cap D$ b) $C \cup D$ c) $A \cup D$ d) $B \cap C$ e) $A \cap B$

2. Graph the values of x that satisfy the conditions given.

   a) $3 < x$ and $x < 8$ b) $-5 < x$ and $x \le 2$ c) $-2 \le x \le 5$

3. Graph the values of x that satisfy the conditions given.

   a) $x \ge 1$ or $x \le 0$ b) $x < 4$ or $x > \frac{13}{2}$ c) $x \le -4$ or $x \ge 3$

4. For each compound inequality, give the solution set in both interval and graph forms.

   a) $x+1<7$ and $x>-2$ b) $x-1 \ge 5$ or $x-1<2.5$ c) $x<5$ and $x>8$

5. For each compound inequality, give the solution set in interval form.

   a) $2x-7 \ge -1$ and $3x-5 \le 4$ b) $\frac{5x}{3}-5<\frac{5}{3}$ and $3x-\frac{1}{2}<-\frac{7}{2}$

   c) $4x+5<1$ or $2x-1>-9$ d) $6x-7<11$ and $2x+4>12$

   e) $-0.7x+1.5>0.8x$ or $0.9x+0.6 \le 5.1$

Teaching Notes:

- Mention that the intersection of two streets is the region common to both streets.
- Mention that the union of two sets is “combine” elements from both sets.
- Some students need a lot of practice transitioning from the statement $x < 3$ and $x > -1$ to the statement $-1 < x < 3$.

Answers: (graph answers at end of mini–lectures) 1a) $\{3,4\}$, *b)* $\{1,2,3,4,5,6\}$, *c)* $\{x|x \text{ is an even integer}, x=3, x=5\}$, *d)* $\{1,3\}$, *e)* $\varnothing$; *4a)* $(-2,6)$, *b)* $(-\infty,3.5)\cup[6,\infty)$, *c)* $\varnothing$; *5a)* $\{3\}$, *b)* $(\infty,-1)$, *c)* $(-\infty,\infty)$, *d)* $\varnothing$, *e)* $(-\infty,5]$

Absolute Value Equations and Inequalities

Learning Objectives:

1. Use the distance definition of absolute value.
2. Solve equations of the form $|ax+b|=k$, for $k>0$.
3. Solve inequalities of the form $|ax+b|<k$ and of the form $|ax+b|>k$ for $k>0$.
4. Solve absolute value equations that involve rewriting.
5. Solve equations of the form $|ax+b|=|cx+d|$.
6. Solve special cases of absolute value equations and inequalities.
7. Solve an application involving relative error.

Examples:

1. Solve.

   a) $|x|=5$ b) $|x|=-5$ c) $|3m|=9.3$ d) $6|x|-7=5$

   e) $|x+4|=9$ f) $\left|\frac{x}{3}-2\right|=1$ g) $|5x|=0$ h) $|2n+3|+9=4$

   i) $2|x-1|+15=20$ j) $|5x+9|=|x+4|$ k) $\left|\frac{1}{2}x+3\right|=\left|\frac{2}{3}x-1\right|$

2. Solve each inequality, and give the solution set in interval and graph forms.

   a) $|x|<2$ b) $|x+3|\le 4$ c) $|x-2.5|<3.5$

   d) $|x|>2$ e) $|x+3|\ge 4$ f) $|x-2.5|>3.5$

3. Solve each inequality and write the answer in interval form.

   a) $|x|\le 8$ b) $|x|\ge 8$ c) $|x|<-3$ d) $|x|>-3$

   e) $|x+3|<7$ f) $|x|+4\le 8$ g) $\left|\frac{x-3}{5}\right|<1$ h) $|6-3x|<4$

   i) $|x-5|\ge 8$ j) $|x|+6>7$ k) $|9+4x|-3>-2$

4. Suppose a machine that makes computer parts is set for a relative error that is no greater than 0.02 mm. How many millimeters may a 3 mm part be?

Teaching Notes:

- Most students need to see the solutions to 3a–d) on a number line in order to visualize the solution set.
- $|x|\ge 0$ for all real numbers x, $|x|\le 0$ when $x=0$, and there is no solution to $|x|\le k$ when $k<0$.

Answers: *(graph answers at end of mini-lectures) 1a)* $\{-5,5\}$, *b)* $\varnothing$, *c)* $\{-3.1,3.1\}$, *d)* $\{-2,2\}$, *e)* $\{-13,5\}$, *f)* $\{3,9\}$, *g)* $\{0\}$, *h)* $\varnothing$, *i)* $\left\{-\frac{3}{2},\frac{7}{2}\right\}$, *j)* $\left\{-\frac{13}{6},-\frac{5}{4}\right\}$, *k)* $\left\{24,-\frac{12}{7}\right\}$; *2a)* $(-2,2)$, *b)* $[-7,1]$, *c)* $(-1,6)$, *d)* $(-\infty,-2)\cup(2,\infty)$, *e)* $(-\infty,-7]\cup[1,\infty)$, *f)* $(-\infty,-1)\cup(6,\infty)$; *3a)* $[-8,8]$, *b)* $(-\infty,-8]\cup[8,\infty)$, *c)* $\varnothing$, *d)* $(-\infty,\infty)$, *e)* $(-10,4)$, *f)* $[-4,4]$, *g)* $(-2,8)$, *h)* $\left(\frac{2}{3},\frac{10}{3}\right)$, *i)* $(\infty,-3]\cup[13,\infty)$, *j)* $(-\infty,-1)\cup(1,\infty)$, *k)* $\left(-\infty,-\frac{5}{2}\right)\cup(-2,\infty)$; *4) [2.94, 3.06]*

Linear Equations in Two Variables

Learning Objectives:

1. Interpret a line graph.
2. Plot ordered pairs.
3. Find ordered pairs that satisfy a given equation.
4. Graph lines.
5. Find *x*– and *y*–intercepts.
6. Recognize equations of horizontal and vertical lines.
7. Use the midpoint formula.

Examples:

1. Fill in the missing information.
   a) The standard form of a linear equation in two variables is __________.
   b) The ordered pair (*x*, *y*) that makes an equation true is called a __________ to that equation.
   c) (3, 4) is a solution to $x+y=7$ because _________________.
   d) Three more solutions to $x+y=7$ are ________________.
   e) When the solutions to $x+y=7$ are graphed on a rectangular coordinate system the result is a __________ line.

2. Find the missing coordinate.
   a) (–1, __) is a solution of $y=2x+4$ b) (__ , 3) is a solution of $x-y=9$

3. Graph each equation.
   a) $y=x+1$ b) $y=-2x+3$ c) $y=\dfrac{2}{3}x-3$

4. Simplify the equation if possible. Find the *x*-intercept, the *y*-intercept, and one or two additional ordered pairs that are solutions to the equation. Then graph the equation.
   a) $2x+4y=8$ b) $-3x+5y=6$ c) $2x+6y+3=3$

5. Simplify the equation if possible. Then graph the horizontal or vertical line.
   a) $x=3$ b) $2x-10=0$ c) $6-3y=0$

6. Find the coordinate of the midpoint of the line segment with the given endpoints.
   a) $P(3,2)$ and $Q(-2,1)$ b) $P(-2,4)$ and $Q(3,-8)$ c) $P(-1,0)$ and $Q(0,5)$

Teaching Notes:

- Some students do not realize they can choose any *x*–value at all, and solve for *y*.
- A linear equation with both *x* and *y* variables will have both *x*- and *y*- intercepts.
- Point out the difference between x^2 and x_2. Explain how to read them correctly.

<u>Answers</u>: *(graph answers at end of mini–lectures) 1a) Ax+By=C, b) solution, c) 3+4=7, d) answers vary, e) straight; 2a) 2, b) 12; 4a) x+2y=4, (4,0), (0,2), answers vary, b) (–2,0), (0,6/5), answers vary, c) x+3y=0, (0,0), (0,0), answers vary;5a) x=3, 5b) x=5, c) y=2; 6a)* $\left(\frac{1}{2},\frac{3}{2}\right)$*, b)* $\left(\frac{1}{2},-2\right)$*, c)* $\left(-\frac{1}{2},\frac{5}{2}\right)$

The Slope of a Line

Learning Objectives:

1. Find the slope of a line given two points on the line.
2. Find the slope of a line given an equation of the line.
3. Graph a line given its slope and a point on the line.
4. Use slopes to determine whether two lines are parallel, perpendicular, or neither.
5. Solve problems involving average rate of change.

Examples:

1. Fill in the missing information and draw an example of each line.

   a) If the height of a line increases as the *x*–value increases, the slope is __________.
   b) If the height of a line decreases as the *x*–value decreases, the slope is __________.

2. Find the slope, if possible, of the line passing through each pair of points.

   a) (3, 4) and (5, 8) b) (–2, 4) and (–4, 7) c) (–1.5, 4.5) and (4.5, 0)

   d) $\left(2,\frac{3}{5}\right)$ and $\left(-7,\frac{3}{5}\right)$ e) $\left(\frac{5}{3},-2\right)$ and $\left(\frac{5}{3},-\frac{1}{6}\right)$

3. Find the slope of each line.

   a) $3x-y=8$ b) $2x+4y=14$ c) $y=4$

4. Graph the line given its slope and a point on the line.

   a) $(-5,-4)$; $m=\frac{3}{2}$ b) $(2,-3)$, $m=-\frac{1}{2}$

5. Given the equations of two lines, determine whether they are *parallel*, *perpendicular*, or *neither*.

   a) $2x-y=6$ & $x+2y=-5$ b) $x+2y=5$ & $x+2y=9$ c) $4x-3y=11$ & $6x-y=13$

6. A river has a slope of 0.14. How many feet does it rise vertically over a horizontal distance of 400 feet?

Teaching Notes:

- Some students need to see many numeric examples of $m=\frac{rise}{run}$ shown on a graph before trying to use the slope formula.
- Many students make sign errors with the slope formula.
- Point out that no slope is the same as undefined slope. No slope does not mean a slope of 0.

<u>Answers</u>: (graph answers at end of mini–lectures) 1a) positive, b) negative; 2a) 2, b) –3/2, c) –0.75, d) 0, e) undefined; 3a) 3, b) $-\frac{1}{2}$ *c) 0; 5a) perpendicular, b) parallel, c) neither; 6) 56 ft*

Writing Equations of Lines

Learning Objectives:

1. Write an equation of a line given its slope and y–intercept.
2. Graph a line using its slope and y–intercept.
3. Write an equation of a line given its slope and a point on the line.
4. Write an equation of a line given two points on the line.
5. Write equations of horizontal and vertical lines
6. Write an equation of a line parallel or perpendicular to a given line.
7. Write an equation of a line that models real data.

Examples:

1. Write the slope-intercept form for the equation of a line given a slope and a y–intercept.

   a) m = 2, b = –5 b) slope $-\frac{3}{4}$, y-intercept (0, 2)

2. Graph the line, using its slope and y-intercept.

   a) slope = 2, (0, –2) b) slope = $-\frac{2}{3}$, (0, 3)

3. Write each equation in slope–intercept form.

   a) $3x-4y=12$ b) $2x+\frac{2}{3}y=-4$

4. For each equation, find the slope and the y-intercept.

   a) $y=2x+3$ b) $3x+5y=20$

5. Find the equation of the line that satisfies the given conditions. Write the equation in standard form and write the equation in slope-intercept form, if possible.

   a) (3, 4), m = –2 b) (2, 3) and (–1, 5) c) $\left(\frac{1}{2},-4\right)$ and $\left(\frac{7}{2},-6\right)$

   d) parallel to $3x-y=4$, through (0, –3) e) perpendicular to $2y=-5x$, through (2, 4)

   f) $(4,-5)$, $m=0$ g) $(6,2)$, slope undefined.

6. A lawn aerator rents for \$35, plus \$12 per hour. Let x represent the number of hours the aerator was rented and y the total rental cost. Write an equation in the form $y=mx+b$ to represent this situation. How many hours was the aerator rented if the total cost was \$89.00?

Teaching Notes:

- Many students have trouble understanding all of the information in this section and need to spend extra time working on it.
- Point out the differences and similarities between the point- slope and slope- intercept forms.
- Consider starting with real- life examples of graphing as this helps show the relevance of graphing.

Answers: (graph answers at end of mini–lectures) 1a) y=2x–5, b) y=(–3/4)x+2; 3a) y=(3/4)x–3, b) y=–3x–6; 4a) m=2, (0,3), b) m=–3/5, (0,4); 5a) 2x+y=10, y=–2x+10, b) 2x+3y =13, y=(–2/3)x+13/3, c) 2x+3y = –11, y=(–2/3)x–11/3, d) 3x–y =3, y=3x–3, e) 2x–5y= –16, y=(2/5)x+16/5 f) $y=-5$, $y=-5$ *g)* $x=6$ *, not possible; 6) y=12x + 35, 4 ½ hours*

Linear Inequalities in Two Variables

Learning Objectives:

1. Graph linear inequalities in two variables.
2. Graph the intersection of two linear inequalities.

Examples:

1. Determine whether the ordered pair satisfies $y \geq x+2$.

   a) (0, 2) b) (1, 4) c) (3, 1) d) (–4, 2) e) (–1, –2)

2. Graph each linear inequality in two variables.

   a) $y < x$ b) $y \geq x+2$ c) $y \leq -x-3$

   d) $y > -5x+3$ e) $x+2y > -2$ f) $-2x-5y \leq 10$

   g) $y > \frac{1}{2}x$ h) $y \leq 2$ i) $x \geq -2$

   j) $2x > -3y$ k) $x > -3y$ l) $-2x < 8$

3. Graph each compound inequality:

   a) $y \leq x+1$ and $y \geq 5x-3$ b) $y \leq x+1$ or $y \leq 5x-3$

Teaching Notes:

- Most students who are good at graphing linear equations find this section easy.
- Although students do not fully understand the region they are testing in problem 1 until they graph it in 2b), many of them need to practice testing ordered pairs before they use it within a graphing problem.
- Remind students to always use a test point from the graphed region to check their work.
- Remind students to use a dashed line for < or > and a solid line for ≤ or ≥ inequalities.
- Refer students to the ***Graphing a Linear Inequality in Two Variables*** chart in the textbook.

Answers: (graph answers at end of min-lectures) 1a) yes, b) yes, c) no, d) yes, e) no

Introduction to Relations and Functions

Learning Objectives:

1. Define and identify relations and functions.
2. Find the domain and range.
3. Identify functions defined by graphs and equations.

Examples:

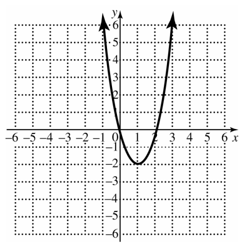
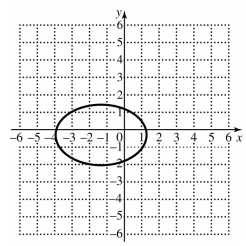
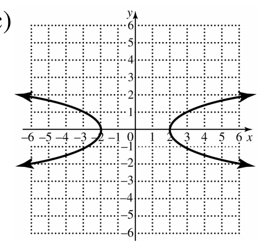
1. Find the domain and range of the relation. Determine whether the relation is a function.

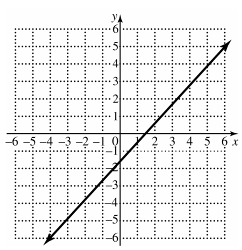
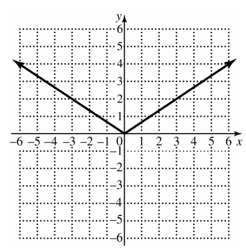
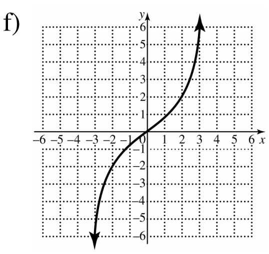
   a) $\{(1, 4), (1, 6)\}$ b) $\{(3, -4), (8, 9)\}$ c) $\{(-2, -6), (0, -6)\}$

   d) $\{(-4, 5), (-1, 2), (2, -6), (2, -9)\}$ e) $y > x - 3$ f) $y = \sqrt{x+1}$

   g) $\{(2000,1500),(2005,4000),(2010,6500)\}$

2. Determine whether each relation is a function.

   a) 
   b) 
   c) 

   d) 
   e) 
   f) 

Teaching Notes:

- Use real-life examples of functions and show how they are related to earlier applications.
- Some students find it helpful to calculate ordered pairs for a linear equation and discuss why a linear equation is a function. Also discuss the domain and range.
- Use graphs of many different types of relations to show how to find the domain and range.

<u>Answers</u>: 1a) d={1}, r={4,6}, not a function, b) d={3,8}, r={–4,9}, function, c) d={–2,0}, r={–6}, function, d) d={–4,–1,2}, r={–9,–6,2,5}, not a function, e) d= $(-\infty,\infty)$*, r=* $(-\infty,\infty)$*, not a function, f) d=* $[-1,\infty)$*, r=* $[0,\infty)$*, function, g) d={2000,2005,2010}, r = {1500,4000,6500}, function 2a) function, b) not a function, c) not a function, d) function, e) function, f) function;*

Function Notation and Linear Functions

Learning Objectives:

1. Use function notation.
2. Graph linear and constant functions.

Examples:

1. Given the following functions, find the indicated values.

   a) $f(x) = 4 - 2x$; find $f(-1)$, $f(4)$, $f\left(\frac{1}{2}\right)$

   b) $f(x) = \frac{2}{5}x + 2$; find $f(-5)$, $f(2)$, $f\left(\frac{1}{2}\right)$

   c) $f(x) = -x^2 + 2x - 3$; find $f(-1)$, $f(2)$

   d) $f(x) = x^3 - 4$; find $f(-1)$, $f(10)$, $f(-3)$

2. Graph the following functions.

   a) $f(x) = x + 3$

   b) $f(x) = 5$

3. Renting a car for x days costs $f(x)$ dollars, where $f(x) = 17.45x$. Evaluate $f(4)$.

Teaching Notes:

- For the function $y = f(x)$ emphasize that y and $f(x)$ are equivalent.
- Have students evaluate $f(a)$, $f(a+b)$, and/or $f(\square)$ and interpret the result as a coordinate.

Answers: 1a) 6, –4, 3, b) 0, 14/5, 11/5, c) –6, –3, d) –5, 996, –31; 2a-b) (graph answers at end of mini-lectures) 3) $69.80

Systems of Linear Equations in Two Variables

Learning Objectives:

1. Decide whether an ordered pair is a solution of a linear system.
2. Solve linear systems by graphing.
3. Solve linear systems (with two equations and two variables) by substitution.
4. Solve linear systems (with two equations and two variables) by elimination.
5. Solve special systems.

Examples:

1. Determine whether the given ordered pair is a solution to the system of equations.

a) $\begin{aligned} x+y&=4 \\ x-y&=2 \end{aligned}$; (3, 1) b) $\begin{aligned} x+y&=4 \\ x-y&=2 \end{aligned}$; (2, 2) c) $\begin{aligned} 2x-3(y-6)&=8 \\ 6y-2&=x+6 \end{aligned}$; $\left(-4, \frac{2}{3}\right)$

2. Solve the system of equations by graphing. If a system is inconsistent or has dependent equations, say so.

a) $\begin{aligned} x+y&=4 \\ x-y&=2 \end{aligned}$ b) $\begin{aligned} 6x+2y&=10 \\ 2x-y&=5 \end{aligned}$ c) $\begin{aligned} y&=-x+3 \\ 2x+2y&=-1 \end{aligned}$

3. Solve the system of equations by the substitution method. If a system is inconsistent or has dependent equations, say so.

a) $\begin{aligned} x+y&=4 \\ x-y&=2 \end{aligned}$ b) $\begin{aligned} 3x+y&=5 \\ y&=2x-5 \end{aligned}$ c) $\begin{aligned} x-\frac{1}{2}y&=2 \\ x-3y&=3 \end{aligned}$

4. Solve the system of equations by the addition method. If a system is inconsistent or has dependent equations, say so.

a) $\begin{aligned} x+y&=4 \\ x-y&=2 \end{aligned}$ b) $\begin{aligned} 5x-2y&=8 \\ 3x-y&=7 \end{aligned}$ c) $\begin{aligned} 0.4x+0.4y&=0.3 \\ 0.2x-0.8y&=0.4 \end{aligned}$

5. Solve each system of equations by any method. If a system is inconsistent or has dependent equations, say so.

a) $\begin{aligned} x+y&=4 \\ 2x+2y&=8 \end{aligned}$ b) $\begin{aligned} y&=-3x+4 \\ 6x+2y-12&=0 \end{aligned}$ c) $\begin{aligned} 4x+6y&=32 \\ 5x-\frac{3}{4}y&=7 \end{aligned}$

Teaching Notes:

- Review how to graph lines by determining *x*- and *y*- intercepts and use a straight edge to solve graphically.
- Emphasize that the solution found by substitution or elimination will be the same as the solution found by graphing.
- Some students have trouble drawing a conclusion of "no solution" or "infinite number of solutions" from the results of the non-graphing methods.

Answers: (graph answers at end of mini–lectures) 1a) yes, b) no, c) yes; 2a) {(3,1)}, b) {(2,–1)}, c) $\varnothing$ *, inconsistent; 3a) {(3,1)}, b) {(2,–1)}, c) {(9/5,–2/5)}; 4a) {(3,1)}, b) {(6,11)}, c) {(1,–0.25)}; 5a)* $\{(x, y) \mid x+y=4\}$ *, dependent equations, b)* $\varnothing$ *, inconsistent, c) {(2,4)}*

Systems of Linear Equations in Three Variables

Learning Objectives:

1. Understand the geometry of systems of three equations in three variables.
2. Solve linear systems (with three equations and three variables) by elimination.
3. Solve linear systems (with three equations and three variables) in which some of the equations have missing terms.
4. Solve special systems.

Examples:

1. Determine whether the ordered triple is a solution to the system.

a) $\begin{aligned} x+y+z&=4 \\ 2x-y-z&=2 \\ -x-y+2z&=-7 \end{aligned}$; (2, 3, –1)

b) $\begin{aligned} 2x+2y+4z&=12 \\ -x-y+4z&=-3 \\ 2x-3y-6z&=16 \end{aligned}$; $\left(5,0,\frac{1}{2}\right)$

2. Solve each system. If the system is inconsistent or has dependent equations, say so.

a) $\begin{aligned} x+y+z&=7 \\ x-y+2z&=7 \\ 5x+y+z&=11 \end{aligned}$

b) $\begin{aligned} 5x+2y+z&=-18 \\ 2x-3y-z&=-4 \\ 3x+y+5z&=-33 \end{aligned}$

c) $\begin{aligned} \frac{2}{3}x+2y-2z&=8 \\ 3x+\frac{1}{3}y+z&=-6 \\ x-3y+2z&=-19 \end{aligned}$

d) $\begin{aligned} x-3y-3z&=-5.701 \\ -x-4y+z&=-7.602 \\ 2x+y-z&=5.654 \end{aligned}$

e) $\begin{aligned} x-y+3z&=9 \\ 5x+\quad z&=4 \\ x+3y+z&=13 \end{aligned}$

f) $\begin{aligned} x+5y+4z&=-3 \\ 3y+5z&=8 \\ z&=4 \end{aligned}$

g) $\begin{aligned} x+y+z&=-3 \\ x-y+4z&=-3 \\ 3x+3y+3z&=1 \end{aligned}$

h) $\begin{aligned} x-y+2z&=3 \\ -3x+3y-6z&=-9 \\ 2x-2y+4z&=6 \end{aligned}$

Teaching Notes:

- Students need to be extremely neat and organized to succeed with these problems. Try numbering each equation when showing examples.
- Use the walls and floors of the classroom as a visualization of the three types of solutions for three planes.
- Some students prefer to use the substitution method to eliminate the first variable whenever it is possible to do so without generating fractions.
- Remind students to write the values of *x, y* and z in the appropriate order in the solution.

Answers: 1a) yes, b) no; 2a) {(1,2,4)}, b) {(–3,1,–5)}, c) {(–3,6,1)}, d) {(2.681,1.543,1.251)}, e) {(0,3,4)}, f) {(1,–4,4)}; g) $\varnothing$ *, inconsistent system, h)* $\{(x,y,z)\,|\,x-y+2z=3\}$

Applications of Systems of Linear Equations

Learning Objectives:

1. Solve geometry problems using two variables.
2. Solve money problems using two variables.
3. Solve mixture problems using two variables.
4. Solve distance-rate-time problems using two variables.
5. Solve problems with three variables using a system of three equations.

Examples:

1. Solve each problem

   a) The manager of Kings Movie theatre reported a given night receipts from both the 3D and non - 3D showing of *Godzilla* as $4845. A 3D ticket costs $12 and a non - 3D ticket costs $9 and the total number of tickets sold was 465. How many of each type of ticket were sold on this night?

   b) The difference of the measures of two complementary angles is 4º. Find the measure of each angle.

   c) A car travels 140 miles in the same time that a truck travels 120 miles. If the speed of the car is 8 mph faster than the speed of the truck, find both speeds.

   d) Shawn wants to make 5 liters of a 40% hydrochloric acid solution by adding a 60% solution to distilled water. How much water should he use?

2. Solve each system by using three variables.

   a) A basketball player scored 50 points in a game. The number of three-point field goals the player made was 14 less than three times the number of free throws made (each worth 1 point). Twice the number of two-point field goals the player made was 15 more than the number of three-point field goals made. Find the number of free-throws, two-point field goals, and three-point field goals that the player made in the game.

   b) A real estate investor is examining a triangular plot of land. She measures each angle of the field. The sum of the first and second angles is 80° more than the measure of the third angle. If the measure of the third angle is subtracted from the measure of the second angle, the result is three times the measure of the first angle. Find the measure of each angle. (Note: The sum of the measures of the angles of a triangle is 180°.)

Teaching Notes:

- Many students find these word problems difficult. Provide many examples for the students.
- Encourage students to draw and label a diagram whenever possible.
- Remind students to verify that their answers seem reasonable.

Answers: 1a) 3D: 220, non-3D: 245, b) 43º and 47º, c) car speed is 56 mph and truck speed is 48 mph, d) $1\frac{2}{3}$ liters; 2a) 7 free throws, 11 2-pt shots, 7 3-pt shots, b) 20°, 110°, 50°

Integer Exponents and Scientific Notation

Learning Objectives:

1. Use the product rule for exponents.
2. Define 0 and negative exponents.
3. Use the quotient rule for exponents.
4. Use the power rules for exponents.
5. Simplify exponential expressions.
6. Use the rules for exponents with scientific notation.

Examples:

1. Simplify. Write final answers with positive exponents only. Assume that all variables represent nonzero real numbers.

   a) 2^{-3} b) $(-4)^{-2}$ c) $\left(\frac{1}{8}\right)^{-1}$

   d) $\left(-\frac{1}{3}\right)^{-3}$ e) x^{-5} f) $2x^{-3}$

2. Simplify. Write final answers with positive exponents only. Assume that all variables represent nonzero real numbers.

   a) $x^3 \cdot x^5$ b) $9^5 \cdot 9^{10}$ c) $(2x)(-3x^4)$

   d) $(-20x^2y^3)(6xy)$ e) $(2xy)^0(5x)$ f) $\dfrac{x^{12}}{x^3}$

   g) $\dfrac{x^5}{x^8}$ h) $\dfrac{4^{15}}{4^{19}}$ i) $\dfrac{15x^2y^5z}{-3x^2y^2}$

3. Simplify. Write final answers with positive exponents only. Assume that all variables represent nonzero real numbers.

   a) $(x^6)^5$ b) $(3x^2y^6)^4$ c) $\left(\dfrac{x^2}{y^4z^9}\right)^5$

   d) $\left(\dfrac{2x^{-3}y}{x^{-4}y^3}\right)^{-2}$ e) $\dfrac{(m^3n^4)^{-1}}{(-3m^{-2}n^5)^2}$

4. Write numbers in scientific or standard notation.

   a) Write in scientific notation: 125 3442 0.022 0.00000453
   b) Write in standard notation: 2.04×10^3 1.9902×10^7 9.311×10^{-4}

5. Evaluate. a) $(2.1\times10^{-3})(1.4\times10^{-5})$ b) $\dfrac{4.8\times10^{-6}}{1.2\times10^{-7}}$

Teaching Notes:

- Show the difference between examples such as -5^0 and $(-5)^0$.
- Students often move constants along with a variable that has a negative exponent. For example, in 1f) a common answer is $2x^{-3} = 1/(2x^3)$.

Answers: 1a) 1/8, b) 1/16, c) 8, d) –27, e) $1/x^5$, f) $2/x^3$; 2a) x^8, b) 9^{15}, c) $-6x^5$, d) $-120x^3y^4$, e) 5x, f) x^9, g) $1/x^3$, h) 1/256, i) $-5y^3z$; 3a) x^{30}, b) $81x^8y^{24}$, c) $\dfrac{x^{10}}{y^{20}z^{45}}$, d) $\dfrac{y^4}{4x^2}$, e) $\dfrac{m}{9n^{14}}$; 4a) $1.25\text{x}10^2$, $3.442\text{x}10^3$, $2.2\text{x}10^{-2}$, $4.53\text{x}10^{-6}$, b) 2040, 19,902,000, 0.0009311; 5a) $2.94\text{x}10^{-8}$, b) 40

Adding and Subtracting Polynomials

Learning Objectives:

1. Know the basic definitions for polynomials.
2. Add and subtract polynomials.

Examples:

1. State whether each expression is a polynomial. Label the polynomial(s) as a monomial/ binomial/ trinomial, give the leading coefficient and degree.

 a) $3x^2 - 2x - 1$ b) $2x - \frac{3}{x}$ c) $x^2 - y^2$

2. Simplify each polynomial by combining like terms.

 a) $2x + 3x$ b) $10y - 8y$ c) $xy + 3x - 2xy$

 d) $-x + 2x - 6x^2 - 3x^2$ e) $-9y + 8y + 2y^5$ f) $-2xy^2 + 3x - x + 8xy^2 - \frac{3}{5}$

3. Add or subtract.

 a) $\left(-3y^2 - 2y + 5\right) + \left(2y + 7\right)$ b) $\left(2x^2 - 3x\right) + \left(-6x^2 - 7x\right)$ c) $\begin{array}{r} 5x^2 + 3x - 2 \\ \underline{+\ 7x^2 - 5x - 3} \end{array}$

 d) $\left(2x - 2\right) - \left(-x - 2\right)$ e) $\begin{array}{r} -6x^2 - 3x + 9 \\ \underline{-\qquad\quad 7x + 10} \end{array}$ f) $\left(3x^2 + x + 2\right) - \left(4x^2 + x - 5\right)$

 g) $\left(7.2y^2 - y + 11.32\right) + \left(2.3y^2 + 4.8y - 1.2\right)$ h) $\left(\frac{2}{5}x^2 - \frac{1}{3}x + \frac{1}{5}\right) - \left(\frac{3}{5}x^2 + \frac{2}{3}x - \frac{4}{5}\right)$

Teaching Notes:

- Show students examples of expressions that are not polynomials.
- Remind students that this section is a review of distributing and collecting like terms.
- Some students forget to distribute the negative sign when aligning vertically.

Answers: *1a) polynomial, trinomial, 3, 2, b) not a polynomial, c) polynomial, binomial; 2a)* $5x$*, b)* $2y$*, c)* $-xy + 3x$*, d)* $-9x^2 + x$*, e)* $2y^5 - y$*, f)* $6xy^2 + 2x - \frac{3}{5}$*; 3a)* $-3y^2 + 12$*, b)* $-4x^2 - 10x$*, c)* $12x^2 - 2x - 5$*, d)* $3x$*, e)* $-6x^2 - 10x - 1$*, f)* $-x^2 + 7$*, g)* $9.5y^2 + 3.8y + 10.12$*, h)* $-\frac{1}{5}x^2 - x + 1$

Polynomial Functions, Graphs, and Composition

Learning Objectives:

1. Recognize and evaluate polynomial functions.
2. Use a polynomial function to model data.
3. Add and subtract polynomial functions.
4. Find the composition of functions.
5. Graph basic polynomial functions.

Examples:

1. If $f(x) = 2x^2 - 3x + 1$, find each function value.
   a) $f(1)$ b) $f(0)$ c) $f(-2)$ d) $f(-10)$

2. Evaluate the polynomial for the given value.
   a) $f(x) = 3x^2 - 5x - 10$; for $f(2)$ b) $f(x) = -x^3 + 3x^2 + 2x - 8$ for $f(-3)$

3. A car rental agency charges \$50 per day plus \$0.32 per mile. Therefore, the daily charge for renting a car is a function of the number of miles traveled m and can be expressed as $C(m) = 50 + 0.32m$. Compute each of the following:
   a) $C(75)$ b) $C(150)$ c) $C(225)$ d) $C(650)$

4. For $f(x) = 2x^2 - x$ and $g(x) = 3 - 5x$, find the following:
   a) $(f+g)(x)$ b) $(g-f)(x)$ c) $(f+g)(2)$ d) $(g-f)(-2)$
   e) $(f \circ g)(1)$ f) $(g \circ f)(x)$

5. Graph each function. Give the domain and range of each function.

   a) $f(x) = x^2 - 3$ b) $f(x) = -x^3 + 4$

Teaching Notes:

- Show how to evaluate $-x^2$ for $x = 2$ and $x = -2$.
- Remind students that this section is will include the distributive property and collecting like terms.
- Some students forget to distribute the negative sign when aligning vertically.
- Show that $(f \circ g)(x)$ is not necessarily equal to $(g \circ f)(x)$.

Answers: (graph answers at end of mini-lectures) 1a) 0, b) 1, c) 15, d) 231; 2a) –8, b) 40; 3a) \$74, b) \$98, c) \$122, d) \$258; 4a) $2x^2 - 6x + 3$, *b)* $-2x^2 - 4x + 3$, *c) –1, d) 3 e) 10, f)* $3 - 10x^2 + 5x$; *5a) domain* $(-\infty, \infty)$, *range* $[-3, \infty)$, *b) domain* $(-\infty, \infty)$, *range* $(-\infty, \infty)$

Multiplying Polynomials

Learning Objectives:

1. Multiply terms.
2. Multiply any two polynomials.
3. Multiply binomials.
4. Find the product of a sum and difference of two terms.
5. Find the square of a binomial.
6. Multiply polynomial functions.

Examples:

1. Find each product.

a) $(2x)(4x)$ b) $(-6a^2)(5a^3)$ c) $(4.1xy^2z^{10})(6xy^5z)$

d) $2x(3x-4)$ e) $-3y(5xy+2x)$ f) $-2b^2z(2z^2a+baz-b)$

g) $(x+3)(3x-4)$ h) $\begin{array}{r} 4y-3 \\ \underline{\times\ 2y-2} \end{array}$ i) $\left(3y-\frac{1}{4}\right)\left(4y-\frac{1}{6}\right)$

j) $(3x^2-4y^2)(x^2-6y^2)$ k) $(x+3)(2x^2-x+5)$ l) $(x+3)(x-2)(2x-1)$

2. Find each product.

a) $(x+2)^2$ b) $(x-4)^2$ c) $(x+3)(x-3)$

d) $(2xy-3b)(2yx+3b)$ e) $\left(5x-\frac{1}{2}\right)\left(5x+\frac{1}{2}\right)$ f) $[6-(2b-2)]^2$

3. Let $f(x)=3x$ and $g(x)=x+2$. Find each of the following.

a) $(fg)(2)$ b) $(gf)(-1)$ c) $(fg)(x)$

Teaching Notes:

- Encourage students to multiply binomials with FOIL mentally whenever possible. This will make factoring easier for them in future sections.
- In problem 2d), some students do not realize that $xy = yx$ and that they are therefore like terms.
- Many students distribute the exponent when squaring a binomial, even after repeated reminders to multiply the binomial by itself.
- Refer students to the ***Square of a Binomial*** and ***Product of the Sum and Difference of Two Terms*** charts in the text.

Answers: 1a) $8x^2$, b) $-30a^5$, c) $24.6x^2y^7z^{11}$, d) $6x^2-8x$, e) $-15xy^2-6xy$, f) $-4ab^2z^3-2ab^3z^2+2b^3z$, g) $3x^2+5x-12$, h) $8y^2-14y+6$, i) $12y^2-\frac{3}{2}y+\frac{1}{24}$, j) $3x^4-22x^2y^2+24y^4$, k) $2x^3+5x^2+2x+15$, l) $2x^3+x^2-13x+6$; 2a) x^2+4x+4, b) $x^2-8x+16$, c) x^2-9, d) $4x^2y^2-9b^2$, e) $25x^2-\frac{1}{4}$, f) $64-32b+4b^2$; 3a) 24, b) –3, c) $3x^2+6x$

Dividing Polynomials

Learning Objectives:

1. Divide a polynomial by a monomial.
2. Divide a polynomial by a polynomial of two or more terms.
3. Divide polynomial functions.

Examples:

1. Divide.

a) $\dfrac{20x^4 - 25x^2 + 20x}{5x}$ b) $\dfrac{27b^5 - 15b^3 - 9b^2}{3b^2}$ c) $\dfrac{64x^7 - 32x^5 + 16x^3}{8x^3}$

2. Divide. Check your answers by multiplication.

a) $\dfrac{x^2 + 18x + 81}{x + 9}$ b) $\dfrac{2m^2 + 11m - 40}{m + 8}$ c) $\dfrac{6m^3 + 49m^2 - 36m + 81}{m + 9}$

3. Divide. Check your answers by multiplication.

a) $\dfrac{x^2 + 2x - 5}{x + 4}$ b) $\dfrac{x^2 + 17x + 66}{x + 8}$ c) $\dfrac{4x^2 - 22x + 32}{2x + 3}$

d) $\dfrac{9x^2 + 6x + 10}{3x - 2}$ e) $\dfrac{3y^4 + y^2 + 2}{y + 1}$ f) $\dfrac{y^3 + 1}{y + 1}$

g) $\dfrac{8t^4 + 16t^3 + 24t^2 + 16t - 24}{2t^2 + 2t + 6}$ h) $\dfrac{-8t^4 + 18t^3 + 12t^2 + 63t + 27}{2t^2 - 6t - 3}$

4. For $f(x) = x^2 + 3x - 18$ and $g(x) = x + 6$, find each of the following.

a) $\left(\dfrac{f}{g}\right)(x)$ b) $\left(\dfrac{f}{g}\right)(2)$ c) $\left(\dfrac{g}{f}\right)(-1)$

Teaching Notes:

- Most students understand the algebraic long division process better if they are shown a numerical long division first.
- Remind students of how to check the result of long division.
- Remind students to include a zero term for a place holder as needed. This will help keep the process more organized.

Answers: *1a) $4x^3–5x+4$, b) $9b^3–5b–3$, c) $8x^4–4x^2+2$; 2a) $x+9$, b) $2m–5$, c) $6m^2–5m+9$; 3a) $x-2+\dfrac{3}{x+4}$, b) $x+9-\dfrac{6}{x+8}$, c) $2x-14+\dfrac{74}{2x+3}$, d) $3x+4+\dfrac{18}{3x-2}$, e) $3y^3-3y^2+4y-4+\dfrac{6}{y+1}$, f) $y^2–y+1$ g) $4t^2+4t–4$, h) $–4t^2–3t–9$; 4a) $x-3$, b) $–1$, c) $–1/4$*

Greatest Common Factors and Factoring by Grouping

Learning Objectives:

1. Factor out the greatest common factor.
2. Factor by grouping.

Examples:

1. Factor out the greatest common factor.

   a) $8x+28$ b) $6x+23$

   c) $xy-5x^2y$ d) $5c^2x^3-15cx-10c$

   e) $-8x^2y-12xy+4x$ f) $2x(x+y)+3(x+y)$

   g) $5x(x-3y)-4(x-3y)$ h) $2x(x-y)-7(y-x)$

   i) $6x^3(3x+4)-5(3x+4)$ j) $2y(x-5y)+7(-5y+x)$

2. Factor by grouping.

   a) $x^3+3x^2+5x+15$ b) $3x^2+3x-5x-5$

   c) $6ax-6ay+x-y$ d) $xy^2-10-2y^2+5x$

   e) $xy^4-3xy-y^4+3y$ f) $24ax-8bx+9ay^2-3by^2$

3. Factor the following expression as described below.

   $$3x^2-x+9x-3$$

   a) Factor by grouping the first two terms together and the last two terms together.
   b) Factor by grouping the first and third terms together and the second and fourth terms together.
   c) Do both methods give the same final answer?

Teaching Notes:

- Consider including examples that cannot be factored.
- Remind students to check their factoring answers by multiplication.
- Some students have trouble with signs when a negative must be factored out of the second grouping, as in problem 2b).

Answers: 1a) $4(2x+7)$, b) Cannot be factored, c) $xy(1-5x)$, d) $5c(cx^3-3x-2)$, e) $4x(-2xy-3y+1)$, f) $(x+y)(2x+3)$, g) $(x-3y)(5x-4)$, h) $(x-y)(2x+7)$, i) $(3x+4)(6x^3-5)$, j) $(x-5y)(2y+7)$; 2a) $(x+3)(x^2+5)$, b) $(x+1)(3x-5)$, c) $(x-y)(6a+1)$, d) $(x-2)(y^2+5)$, e) $y(x-1)(y^3-3)$, f) $(3a-b)(8x+3y^2)$; 3a) $(3x-1)(x+3)$, b) $(x+3)(3x-1)$, c) yes

Factoring Trinomials

Learning Objectives:

1. Factor trinomials when the coefficient of the second-degree term is 1.
2. Factor trinomials when the coefficient of the second-degree term is not 1.
3. Use an alternative method for factoring trinomials.
4. Factor by substitution.

Examples:

1. Factor each trinomial.

   a) $x^2 - 3x + 2$ b) $x^2 + 6x + 8$ c) $x^2 - 6x + 8$ d) $x^2 - 10x + 9$

   e) $x^2 + x - 2$ f) $x^2 + 7x - 8$ g) $x^2 - 2x - 8$ h) $x^2 - 3x - 10$

   i) $x^2 + 12x + 35$ j) $x^2 + 2x - 48$ k) $x^4 - 11x^2 + 24$ l) $x^4 - 4x^2 - 21$

2. Factor. You may use the grouping method or the trial–and–error method.

   a) $2x^2 + 7x + 3$ b) $3x^2 - 2xy - 8y^2$ c) $5x^2 - 17x - 6$

   d) $6x^2 + 19x - 20$ e) $9x^2 + 29x + 6$ f) $8x^2y^2 + 18xy + 9$

   g) $10y^2 + 23y + 12$ h) $6z^4 + 5z^2 - 6$ i) $15z^4 - 4z^2 - 4$

3. Factor each trinomial.

   a) $3x^2 + 21x + 30$ b) $5x^2 + 20x + 15$ c) $4x^2 - 8x - 96$

   d) $10x^2 - 35x - 20$ e) $-4x^2 + 10x + 6$ f) $-45x^3 + 96x^2 - 48x$

4. Factor each trinomial.

   a) $x^4 - 5x^2 - 6$ b) $9x^6 + 6x^3 - 8$ c) $(a+4)^2 + 7(a+4) + 12$

Teaching Notes:

- Some students can factor trinomials quickly and easily using the trial–and–error method.
- Some students become very frustrated with the trial–and–error method and appreciate seeing the grouping method (ac method) because it provides a recipe that works every time (provided the trinomial is factorable).

Answers: *1a) $(x-2)(x-1)$), b) $(x+4)(x+2)$, c) $(x-4)(x-2)$, d) $(x-9)(x-1)$, e) $(x+2)(x-1)$, f) $(x+8)(x-1)$, g) $(x-4)(x+2)$, h) $(x-5)(x+2)$, i) $(x+7)(x+5)$, j) $(x+8)(x-6)$, k) $(x^2-8)(x^2-3)$, l) $(x^2-7)(x^2+3)$; 2a) $(2x+1)(x+3)$, b) $(3x+4y)(x-2y)$, c) prime, d) $(6x-5)(x+4)$, e) $(9x+2)(x+3)$, f) $(4xy+3)(2xy+3)$, g) $(5y+4)(2y+3)$, h) $(3z^2-2)(2z^2+3)$, i) $(5z^2+2)(3z^2-2)$; 3a) $3(x+5)(x+2)$, b) $5(x+3)(x+1)$, c) $4(x-6)(x+4)$, d) $5(2x+1)(x-4)$, e) $-2(2x+1)(x-3)$, f) $-3x(5x-4)(3x-4)$; 4a $(x^2-6)(x^2+1)$, b) $(3x^3+4)(3x^3-2)$, c) $(a+8)(a+7)$*

Special Factoring

Learning Objectives:

1. Factor a difference of squares.
2. Factor a perfect square trinomial.
3. Factor a difference of cubes.
4. Factor a sum of cubes.

Examples:

1. Factor each polynomial.

a) $x^2 - 4$ b) $x^2 + 49$ c) $9x^2 - 25$ d) $25x^2 - 49y^2$

e) $100 - x^2$ f) $81x^2$-1 g) $81x^2 - y^2$ h) $1 - 25x^2y^2$

2. Factor each polynomial.

a) $x^2 - 18x + 81$ b) $x^2 + 2x + 1$ c) $x^2 + 12x + 36$

d) $4x^2 + 28x + 49$ e) $49x^2 - 42xy + 9y^2$ f) $16x^2 + 72xy + 81y^2$

3. Factor each polynomial.

a) $x^3 + 8$ b) $x^3 - 8$ c) $27x^3 - 64$

d) $64x^3 + 1$ e) $64m^3 - 27n^3$ f) $27c^3 + 343$

4. Factor each polynomial.

a) $98x^4 - 32y^2$ b) $375k^3m - 192m^4$ c) $3r^2 - 24r + 48$

d) $24k^3m - 375m^4$ e) $ab^4 - 16a^3b^2$ f) $16k^3m + 72k^2m^2 + 81km^3$

Teaching Notes:

- Some students understand the difference of squares formula better if 1a) and 1c) are first done using trinomial factoring (with a $0x$ middle term).
- Encourage students to always check to see if the first and last terms of a trinomial are perfect squares. If they are, the perfect square trinomial factoring might apply.
- Some students find the sum and difference of cubes formulas confusing at first and need to see several examples.

Answers: 1a) $(x+2)(x–2)$, b) prime, c) $(3x+5)(3x–5)$, d) $(5x+7y)(5x–7y)$, e) $(10+x)(10–x)$, f) $(9x+1)(9x–1)$, g) $(9x+y)(9x–y)$, h) $(1+5xy)(1–5xy)$; 2a) $(x-9)^2$, b) $(x+1)^2$, c) $(x+6)^2$, d) $(2x+7)^2$, e) $(7x–3y)^2$, f) $(4x+9y)^2$; 3a) $(x+2)(x^2–2x+4)$, b) $(x–2)(x^2+2x+4)$, c) $(3x–4)(9x^2+12x+16)$, d) $(4x+1)(16x^2–4x+1)$, e) $(4m–3n)(16m^2+12mn+9n^2)$, f) $(3c+7)(9c^2–21c+49)$; 4a) $2(7x^2+4y)(7x^2–4y)$, b) $3m(5k–4m)(25k^2+20km+16m^2)$, c) $3(r–4)^2$, d) $3m(2k–5m)(4k^2+10km+25m^2)$, e) $ab^2(b+4a)(b–4a)$, f) $km(4k+9m)^2$

A General Approach to Factoring

Learning Objectives:

1. Factor any polynomial.

Examples:

1. Factor each polynomial.

a) $x^2+20x+100$

b) $27x^3+1$

c) $36x^2-49$

d) $12x^4y-147y^3$

e) $x^2+62x+63$

f) $x^3+6x^2-25x-150$

g) $x^4+4x^3+125x+500$

h) $16x^3y-24x^2y^2+9xy^3$

i) $x^2-15x+225$

j) x^4+3x^2-4

k) $112x^8-96x^5+160x^3$

l) $-80x^2-12x+36$

m) x^2-x-35

n) $81x^2+90xy+25y^2$

o) $4x^2+18x-10$

p) $9x^2+64$

q) $4x^4y^4-16x^2y^2$

r) $8x^3y+64y$

Teaching Notes:

- This section is a summary of factoring methods and is designed to give students extra factoring practice.
- Remind students to always try factoring out a common factor first.
- Encourage students to decide which type of factoring to try next by looking at how many terms remain.
- Remind students that while there is a *difference* of squares formula for factoring, there is no *sum* of squares formula.
- Encourage students to use whichever method they are most comfortable with for factoring trinomials of the form $ax^2 + bx + c$.
- Refer students to the ***Factoring a Polynomial*** chart in the textbook.

Answers: 1a) $(x+10)^2$, b) $(3x+1)(9x^2–3x+1)$, c) $(6x+7)(6x–7)$, d) $3y(2x^2+7y)(2x^2–7y)$, e) prime, f) $(x+6)(x+5)(x–5)$, g) $(x+4)(x+5)(x^2–5x+25)$, h) $xy(4x–3y)^2$, i) prime, j) $(x^2+4)(x+1)(x–1)$, k) $16x^3(7x^5–6x^2+10)$, l) $–4(4x+3)(5x–3)$, m) prime, n) $(9x+5y)^2$, o) $2(2x–1)(x+5)$, p) prime, q) $4x^2y^2(xy+2)(xy–2)$, r) $8y(x+2)(x^2–2x+4)$

Solving Equations by the Zero-Factor Property

Learning Objectives:

1. Learn and use the zero-factor property.
2. Solve applied problems that require the zero-factor property.
3. Solve a formula for a specified variable, where factoring is necessary.

Examples:

1. Solve each equation.

   a) $3 \cdot x = 0$ b) $2(x - 5) = 0$ c) $x(x + 6) = 0$ d) $(3x + 2)(5x - 4) = 0$

2. Solve each equation.

   a) $x^2 + 9x - 36 = 0$ b) $9x^2 - 2x = 0$

   c) $3x^2 - 21x + 30 = 0$ d) $5x^2 - 3x - 8 = 0$

   e) $x^2 - x = 6$ f) $x^2 - 64 = 63x$

   g) $x(3x + 13) = 10$ h) $(x - 3)(x + 4) = -4(x + 4)$

   i) $2 + \dfrac{x^2}{3} = 5x + 2$ j) $\dfrac{x^2 - 7x}{2} = 9$

   k) $x^3 = -8x^2 - 15x$ l) $6x^2 = -9x - x^3$

3. Solve the following.

   a) Solve $x = \dfrac{7 - y}{4 + y}$ for y.

   b) A window washer accidentally drops a bucket from the top of a 144-foot building. The height h of the bucket after t seconds is given by $h = -16t^2 + 144$. When will the bucket hit the ground?

   c) The area of a circle is 169π square meters. Find its radius.

Teaching Notes:

- Remind students to always write the equations in standard form before factoring.
- Some students try to use the zero - factor property before the equation is in standard form. For example, 2e) $x^2 - x = 6 \rightarrow x(x-1) = 6 \rightarrow x = 6,\ x - 1 = 6$ etc.
- Many students find the applied problems difficult and need to see more examples.
- Remind students to check whether their answers are reasonable for applied problems.

<u>Answers</u>: *1a) {0}, b) {5}, c) {–6, 0}, d)* $\{-2/3, 4/5\}$ *; 2a) {–12, 3}, b) {0, 2/9}, c) {2, 5}, d) {–1, 8/5}, e) {–2, 3}, f) {–1, 64}, g) {–5, 2/3}, h) {–4, –1}, i) {0, 15}, j) {–2, 9}, k) {0, –5, –3}, l) {0, –3}; 3a)* $y = \dfrac{7 - 4x}{1 + x}$, *b) 3 seconds, c) 13 m*

Rational Expressions and Functions; Multiplying and Dividing

Learning Objectives:

1. Define rational expressions.
2. Define rational functions and give their domains.
3. Write rational expressions in lowest terms.
4. Multiply rational expressions.
5. Find reciprocals of rational expressions.
6. Divide rational expressions.

1. Find all numbers that are not in the domain of each function, then give the domain in set notation.

a) $f(x) = \frac{3}{x}$ b) $f(x) = \frac{5x}{x-2}$ c) $f(x) = -\frac{x-3}{x+4}$ d) $f(x) = \frac{x^2-9}{x^2-9x+14}$

2. Write each rational expression in lowest terms.

a) $\frac{2x-12}{3x-18}$ b) $\frac{9x^2+27x^3}{7x+21x^2}$ c) $\frac{x^3+8}{x^2-3x-10}$ d) $\frac{12xy^2}{9x^2y^2(y+3x)}$

e) $\frac{x^2-3x-10}{x^2-4x-12}$ f) $\frac{-4x-4}{20x^2+28x+8}$ g) $\frac{25-x^2}{2x^2-7x-15}$ h) $\frac{x^2-xy+4x-4y}{x+4}$

3. Multiply.

a) $\frac{2p-2}{p} \cdot \frac{3p^2}{9p-9}$ b) $\frac{x^2+11x+24}{x^2+12x+27} \cdot \frac{x^2+9x}{x^2+4x-32}$

c) $\frac{x^2-4x+3}{x^2-12x+20} \cdot \frac{x^2-19x+90}{x^2-10x+9}$ d) $\frac{12x^2}{9x+45} \cdot \frac{3x^3-75x}{36x^4}$

4. Divide.

a) $\frac{6p-6}{p} \div \frac{7p-7}{9p^2}$ b) $\frac{x^2+5x+6}{x^2+10x+21} \div \frac{x^2+2x}{x^2+16x+63}$

c) $\frac{(x-11)^2}{2} \div \frac{2x-22}{4}$ d) $\frac{x^2-16x+60}{x-10} \div (x-6)$

Teaching Notes:

- Many students need a review of simplifying, multiplying, and dividing numerical fractions before attempting algebraic ones.
- The factoring methods covered in Chapter 5 are essential for success in this chapter.
- Show examples of factoring – 1 out of an expression for simplifying purposes.

Answers: *1a) 0, $\{x \mid x \neq 0\}$, b) 2, $\{x \mid x \neq 2\}$, c) –4, $\{x \mid x \neq -4\}$, d) 2, 7 $\{x \mid x \neq 2,7\}$; 2a) 2/3, b) 9x/7, c) $\frac{x^2-2x+4}{x-5}$, d) $\frac{4}{3x(y+3x)}$, e) $\frac{x-5}{x-6}$, f) $\frac{-1}{5x+2}$, g) $\frac{-(5+x)}{2x+3}$, h) x–y; 3a) 2p/3, b) $\frac{x}{x-4}$, c) $\frac{x-3}{x-2}$, d) $\frac{x-5}{9x}$; 4a) 54p/7, b) $\frac{x+9}{x}$, c) x–11, d) 1*

Adding and Subtracting Rational Expressions

Learning Objectives:

1. Add and subtract rational expressions with the same denominator.
2. Find a least common denominator.
3. Add and subtract rational expressions with different denominators.

Examples:

1. Add or subtract as indicated. Write answers in lowest terms.

a) $\dfrac{7x+4}{9x+6}+\dfrac{x-2}{9x+6}$ b) $\dfrac{m^2-11m}{m-5}+\dfrac{30}{m-5}$ c) $\dfrac{6x-16}{x-2}-\dfrac{4x-7}{x-2}$

2. Find the LCD.

a) $\dfrac{3}{4xy},\dfrac{9}{5x^2y^2}$ b) $\dfrac{3}{m^2-4m},\dfrac{9}{m^2-6m+8}$ c) $\dfrac{9x}{(x-2)(x+5)},\dfrac{13xy}{(x+5)^2}$

3. Add or subtract as indicated. Write answers in lowest terms.

a) $\dfrac{6}{x^2}+\dfrac{4}{x}$ b) $\dfrac{4}{9}-\dfrac{6}{3x}$

c) $\dfrac{x-4}{x^2+5x+6}+\dfrac{5x+6}{x^2+4x+3}$ d) $\dfrac{x}{x^2-16}-\dfrac{6}{x^2+5x+4}$

e) $\dfrac{3}{x^2-3x+2}+\dfrac{7}{x^2-1}$ f) $\dfrac{3x}{5x-2}-4$

g) $x-4+\dfrac{3}{2x+1}$

Teaching Notes:

- Most students need a review of adding, subtracting, and finding LCDs for numerical fractions before attempting algebraic ones.
- Show students how to build the LCDs of numerical fractions by using the ***Finding the Least Common Denominator*** chart listed in the textbook. Even though that method is not always necessary with numerical fractions, many students have trouble finding the algebraic LCD and understand it better after seeing this.
- Show examples of distributing a negative sign through the entire numerator.

Answers: *1a)* $\dfrac{2(4x+1)}{3(3x+2)}$, *b)* $m-6$, *c)* $\dfrac{2x-9}{x-2}$; *2a)* $20x^2y^2$, *b)* $m(m-2)(m-4)$, *c)* $(x-2)(x+5)^2$; *3a)* $\dfrac{4x+6}{x^2}$, *b)* $\dfrac{4x-18}{9x}$, *c)* $\dfrac{6x^2+13x+8}{(x+1)(x+2)(x+3)}$, *d)* $\dfrac{x^2-5x+24}{(x+1)(x+4)(x-4)}$, *e)* $\dfrac{10x-11}{(x+1)(x-1)(x-2)}$, *f)* $\dfrac{-17x+8}{5x-2}$, *g)* $\dfrac{2x^2-7x-1}{2x+1}$

Complex Fractions

Learning Objectives:

1. Simplify complex fractions by simplifying the numerator and denominator (Method 1).
2. Simplify complex fractions by multiplying by a common denominator (Method 2).
3. Compare the two methods of simplifying complex fractions.
4. Simplify rational expressions with negative exponents.

Examples:

1. Use either method to simplify each complex fraction.

a) $\dfrac{\frac{2}{7}}{\frac{4}{9}}$ b) $\dfrac{\frac{1}{5}}{\frac{2}{3}+\frac{7}{15}}$

2. Simplify each complex fraction using method 1.

a) $\dfrac{\frac{1}{x}-\frac{1}{y}}{\frac{1}{xy}}$ b) $\dfrac{\frac{5}{x}}{\frac{9}{x+2}}$ c) $\dfrac{4+\frac{2}{x}}{\frac{x}{3}+\frac{1}{6}}$

d) $\dfrac{\frac{5}{9x-1}-5}{\frac{5}{9x-1}+5}$ e) $\dfrac{\frac{4}{x+5}}{\frac{1}{x-5}-\frac{2}{x^2-25}}$ f) $\dfrac{\frac{-4}{y+2}-1}{\frac{1}{2}+\frac{4}{y+2}}$

3. Simplify each complex fraction using method 2. Use the same a) through f) examples as above.

4. Simplify each expression, using only positive exponents in your answer.

a) $\dfrac{x^{-1}-y^{-1}}{\frac{1}{x^2}-\frac{1}{y^2}}$ b) $\dfrac{a^{-1}-b^{-1}}{a-b}$ c) $\dfrac{x^{-2}-y^{-2}}{xy}$

Teaching Notes:

- Refer students to the ***Method 1*** and ***Method 2*** charts in the textbook.
- Show students that $x^{-1}+y^{-1}\neq\dfrac{1}{x+y}$ and $ay^{-1}=\dfrac{a}{y}$, not $\dfrac{1}{ay}$.

<u>Answers</u>: 1a) 9/14, b) 3/17; 2a) y–x, b) $\dfrac{5x+10}{9x}$, *c) 12/x, d)* $\dfrac{-9x+2}{9x}$, *e)* $\dfrac{4x-20}{x+3}$, *f)* $-\dfrac{2y+12}{y+10}$; *3a–f) same answers as 2a–f); 4a)* $\dfrac{xy}{x+y}$, *b)* $-\dfrac{1}{ab}$, *c)* $\dfrac{y^2-x^2}{x^3y^3}$

Equations with Rational Expressions and Graphs

Learning Objectives:

1. Determine the domain of the variable in a rational equation.
2. Solve rational equations.
3. Recognize the graph of a rational function.

Examples:

1. Give the domain of each rational equation.

 a) $\frac{2}{x}-\frac{5}{x}=6$ b) $\frac{5}{x^2-9}=\frac{3}{x+3}-\frac{2}{x-3}$

2. Solve each equation.

 a) $\frac{2}{5}y-\frac{1}{3}y=5$ b) $\frac{3y+6}{5}=1+\frac{3}{4}y$

3. Solve each equation.

 a) $\frac{15}{x}=4-\frac{1}{x}$ b) $1+\frac{1}{x}=\frac{12}{x^2}$

 c) $\frac{5-a}{a}+\frac{3}{4}=\frac{7}{a}$ d) $\frac{2}{x}=\frac{x}{5x-12}$

 e) $\frac{3}{y+5}-\frac{5}{y-5}=\frac{6}{y^2-25}$ f) $\frac{x+8}{x^2-6x+5}-\frac{8}{x^2-2x+1}=\frac{x-8}{x^2-6x+5}$

4. Solve each equation.

 a) $\frac{2}{x+3}=5+\frac{2}{x+3}$ b) $\frac{x}{2x+2}=\frac{-2x}{4x+4}+\frac{2x-3}{x+1}$

 c) $\frac{6x}{x+5}-\frac{30}{x-5}=\frac{6x^2+150}{x^2-25}$ d) $\frac{-2}{x+3}=\frac{1}{x+6}-\frac{6}{x^2+9x+18}$

5. Graph the rational function $f(x)=\frac{-1}{x+3}$, and give the equations of the vertical and horizontal asymptotes.

Teaching Notes:

- Remind students to always determine the values that make the denominator zero before solving a rational equation.
- Show students a simple example of an extraneous solution, such as:
 $x=3 \rightarrow x\cdot x=3\cdot x \rightarrow x^2=3x \rightarrow x^2-3x=0 \rightarrow x=0,\ 3$; $x=0$ is extraneous.
- Point out that we keep the common denominator in our solution when adding or subtracting and clear the denominators when solving an equation.

Answers: *(graph answers at end of mini-lectures) 1a)* $\{x|x\neq 0\}$, *b)* $\{x|x\neq -3, 3\}$; *2a) {75}, b) {4/3}; 3a) {4}, b) {–4, 3}, c) {–8}, d) {4, 6} e) {–23}, f) {–3}; 4a)* $\varnothing$, *b) {3}, c)* $\varnothing$, *d)* $\varnothing$; *5)* $x=-3$, $y=0$

Applications of Rational Expressions

Learning Objectives:

1. Find the value of an unknown variable in a formula.
2. Solve a formula for a specified variable.
3. Solve applications using proportions.
4. Solve applications about distance, rate, and time.
5. Solve applications about work rates.

Examples:

1. Find the value of the unknown variable.

   a) The area of a triangle is given by $A = \frac{bh}{2}$, where b is the length of the base and h is the length of the altitude. Find h when $A = 100$ and $b = 25$.

   b) The combined resistance R of two resistances, R_1 and R_2 in parallel is given by $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$. Find the combined resistance of two resistances of 2000 and 4000 ohms.

2. Solve for the indicated variable.

   a) $\frac{PV}{T} = \frac{pv}{t}$, for V b) $\frac{1}{a} + \frac{1}{b} = \frac{1}{c}$, for c c) $P = \frac{A}{1+rt}$, for r

   d) $A = \frac{1}{2}h(B+b)$, for B e) $F = \frac{-GMm}{r^2}$, for M f) $P = \frac{Fd}{t}$, for t

3. Solve.

   a) ***Proportion*** A recent advertisement claimed that 5 out of every 6 dentists recommend a certain brand of toothpaste. If a local city has 102 dentists, how many dentists would you expect to recommend this brand of toothpaste?

   b) ***Rate*** Alex can run 5 miles per hour on level ground on a still day. One windy day he runs 11 miles with the wind, and in the same amount of time runs 4 miles against the wind. What is the rate of the wind?

   c) ***Work*** The cold water faucet can fill a bathtub in 12 minutes. When the hot water faucet is also used, the bathtub will be full in 8 minutes. How long would it take the hot water faucet to fill the bathtub if it were the only faucet in use?

Teaching Notes:

- Encourage students to draw and label a diagram whenever possible.
- Show many examples of different types of applied problems.

Answers: *1a) 8, b)* $\frac{4000}{3}$ *ohms; 2a)* $V = \frac{pvT}{tP}$*, b)* $c = \frac{ab}{a+b}$*, c)* $r = \frac{A-P}{Pt}$*, d)* $B = \frac{2A-bh}{h}$*, e)* $M = \frac{-Fr^2}{Gm}$*, f)* $t = \frac{Fd}{P}$*; 3a) 85 dentists, b)* $2\frac{1}{3}$ *mph, c) 24 minutes*

Variation

Learning Objectives:

1. Write an equation expressing direct variation.
2. Find the constant of variation, and solve direct variation problems.
3. Solve inverse variation problems.
4. Solve joint variation problems.
5. Solve combined variation problems.

Examples:

1. Determine whether each equation represents direct, inverse, joint or combined variation.

   a) $y = kx$ b) $y = kx^2$ c) $y = \frac{k}{x}$ d) $y = \frac{k}{\sqrt{x}}$ e) $y = kwd$

2. Solve each problem.

   a) If y varies directly as x, and $y = 4$ when $x = 3$, find y when $x = 9$.

   b) The distance that an object falls when it is dropped is directly proportional to the square of the amount of time since it was dropped. An object falls 512 feet in 4 seconds. Find the distance the object falls in 5 seconds.

   c) If y varies inversely as the square root of x, and $y = 2$ when $x = 64$, find y when $x = 4$.

   d) If the voltage, V, in an electric circuit is held constant, the current, I, is inversely proportional to the resistance, R. If the current is 280 milliamperes when the resistance is 3 ohms, find the current when the resistance is 12 ohms.

   e) If y is jointly proportional to x and z, and $y = 60$ when $x = 3$ and $z = 4$, find y when $x = 2$ and $z = 9$.

   f) The power that a resistor must dissipate is jointly proportional to the square of the current flowing through the resistor and the resistance of the resistor. If a resistor with resistance 8 ohms needs to dissipate 32 watts of power when 2 amperes of current is flowing through it, find the power that it needs to dissipate when 9 amperes of current are flowing through it.

Teaching Notes:

- Some students have trouble with the concept of first solving for the constant of variation, and then using that number in the original equation and solving for a different variable.

Answers: 1a) direct, b) direct, c) inverse, d) inverse, e) joint; 2a) 12, b) 800 ft; c) 8, d) 70 milliamperes; e) 90, f) 648 watts

Radical Expressions and Graphs

Learning Objectives:

1. Find roots of numbers.
2. Find principal roots.
3. Graph functions defined by radical expressions.
4. Find *n*th roots of *n*th powers.
5. Use a calculator to find roots.

Examples:

1. Find each root that is a real number.

   a) $\sqrt{25}$ b) $-\sqrt{\dfrac{9}{64}}$ c) $-\sqrt{81}$ d) $\sqrt{36}$

   e) $\sqrt[3]{8}$ f) $\sqrt[3]{-27}$ g) $\sqrt[4]{-16}$ h) $-\sqrt[4]{81}$

   i) $-\sqrt[6]{729}$ j) $\sqrt[3]{-\dfrac{27}{125}}$ k) $\sqrt[5]{(7)^5}$ l) $\sqrt[8]{(11)^8}$

2. For the given function: find the indicated function values, find the domain, and graph the function using the obtained values. Use a calculator as necessary. Round to the nearest tenth, as needed.

   a) $f(x)=\sqrt{x}$; $f(0), f(1), f(4), f(9)$

   b) $f(x)=\sqrt{2x+3}$; $f(-1.5), f(0.5), f(3), f(11)$

3. Simplify.

   a) $\sqrt{(-3)^2}$ b) $\sqrt[3]{(-3)^3}$ c) $\sqrt[4]{(-3)^4}$

   d) $\sqrt{36x^2}$ e) $\sqrt[3]{-64x^9}$ f) $\sqrt[4]{x^4y^{28}}$

4. Use a calculator to approximate each radical to three decimal places.

   a) $\sqrt{19}$ b) $-\sqrt[3]{-500}$ c) $\sqrt[4]{8000}$

Teaching Notes:

- It may be necessary to show students how to evaluate higher order roots on their specific calculators.
- It may be necessary to review function notation with your students.

Answers: *(graph answers at end of mini–lectures) 1a) 5, b) –3/8, c) –9, d) 6, e) 2, f) –3, g) not a real number, h) –3, i) –3, j) –3/5, k) 7, l) 11; 2a) 0, 1, 2, 3, domain:* $[0,\infty)$*, b) 0, 2, 3, 5, domain:* $\left[-\frac{3}{2},\infty\right)$*; 3a) 3, b) –3, c) 3, d)* $6|x|$*, e)* $-4x^3$*, f)* $|xy^7|$*; 4a) 4.359, b) 7.937, c) 9.457*

Rational Exponents

Learning Objectives:

1. Use exponential notation for nth roots.
2. Define and use expressions of the form $a^{m/n}$.
3. Convert between radicals and rational exponents.
4. Use the rules for exponents with rational exponents.

Examples:

1. Evaluate each exponential.

   a) $64^{\frac{1}{2}}$ b) $27^{\frac{4}{3}}$ c) $(-8)^{\frac{5}{3}}$

2. Rewrite radical expressions as rational exponent expressions, and vice versa. Assume that variables represent positive real numbers.

   a) $\sqrt{x}$ b) $\sqrt[3]{a}$ c) $\sqrt[4]{y^3}$ d) $(\sqrt[6]{2x})^5$

   e) $x^{\frac{2}{3}}$ f) $4^{-\frac{2}{5}}$ g) $(x+2y)^{\frac{3}{7}}$ h) $(4ab)^{\frac{2}{7}}$

3. Simplify. Express your answer with positive exponents.

   a) x^{-2} b) $x^{-\frac{1}{2}}$ c) $\dfrac{1}{x^{-\frac{3}{4}}}$ d) $3x^4y^{-\frac{1}{3}}$

   e) $(4x^2y)^4(xy)^{\frac{1}{4}}$ f) $\left(\dfrac{2xy^{-2}}{x^3}\right)^2$ g) $\dfrac{y^{\frac{3}{4}}}{y^{\frac{1}{4}}}$ h) $(x^7)^{\frac{2}{7}}$

   i) $(x^{\frac{2}{5}})^{25}$ j) $y^{-\frac{2}{7}}\cdot y^{\frac{7}{14}}$ k) $\dfrac{x^{\frac{3}{2}}\cdot x^{-\frac{1}{2}}}{x^{-\frac{2}{3}}}$ l) $(5x^{\frac{1}{7}}y^{\frac{5}{7}})^3$

   m) $\left(\dfrac{16x^3y^{-6}}{2x^{-3}y^6}\right)^{\frac{1}{3}}$ n) $(2x^{-\frac{1}{3}}y^{\frac{1}{3}})(-5x^{\frac{1}{5}})$

Teaching Notes:

- In problem 1, encourage students to take the root first and then raise to the power.
- Point out that a negative exponent does not necessarily lead to a negative result.
- If a problem is given in exponential form then use the exponent rules to simplify instead of converting to radical form.

Answers: *1a) 8, b) 81, c) –32; 2a) $x^{1/2}$, b) $a^{1/3}$, c) $y^{3/4}$, d) $(2x)^{5/6}$, e) $\sqrt[3]{x^2}$, f) $\dfrac{1}{\sqrt[5]{4^2}}$, g) $\sqrt[7]{(x+2y)^3}$, h) $\sqrt[7]{(4ab)^2}$;*

3a) $1/x^2$, b) $1/x^{1/2}$, c) $x^{3/4}$, d) $3x^4/y^{1/3}$, e) $256x^{33/4}y^{17/4}$, f) $4/(x^4y^4)$, g) $y^{1/2}$, h) x^2, i) x^{10}, j) $y^{3/14}$, k) $x^{5/3}$, l) $125x^{3/7}y^{15/7}$, m) $2x^2/y^4$, n) $(-10y^{1/3})/x^{2/15}$

Simplifying Radicals, the Distance Formula, and Circles

Learning Objectives:

1. Use the product rule for radicals.
2. Use the quotient rule for radicals.
3. Simplify radicals.
4. Simplify products and quotients of radicals with different indexes.
5. Use the Pythagorean theorem.
6. Use the distance formula.

Examples:

1. Multiply and simplify if possible. Assume all variables represent positive real numbers.
   a) $\sqrt{11}\sqrt{7}$ b) $(3\sqrt{2})(-5\sqrt{2})$ c) $(6x\sqrt{12x})(2\sqrt{4x})$

2. Simplify. Assume that all variables are positive real numbers.
   a) $\sqrt{\dfrac{25}{36}}$ b) $\sqrt{\dfrac{12x^5}{49x}}$ c) $\sqrt{\dfrac{175x^2}{y^2}}$
   d) $\sqrt{\dfrac{27x^5y^6}{64y^4}}$ e) $\sqrt[3]{\dfrac{27x^4y^6}{8x}}$ f) $\dfrac{\sqrt[3]{9x^4y^6}}{\sqrt[3]{72x}}$
   g) $\sqrt{3}\cdot\sqrt[3]{5}$

3. Use the Pythagorean Theorem to solve the applied problems.
   a) A right triangle has sides *a*, *b*, and hypotenuse *c*. If $a=\sqrt{7}$ and $b = 8$, find *c*.
   b) A right triangle has sides *a, b*, and hypotenuse *c*. If $c = 10$ and $b = 8$, find *a*.

4. Find the distance between each pair of points. Simplify your answer.
   a) (2, 3) ; (–5,–2) b) (1, 5) ; (8, 4) c) (–3, –7) ; (–9, 2)

Teaching Notes:

- It is helpful to know perfect squares and perfect cubes less than 100.
- Refer to students to the ***Product and Quotient Rule for Radicals*** charts in the textbook.
- The distance formula is easy to understand once students see that it can be derived from the Pythagorean Theorem.

Answers: 1a) $\sqrt{77}$, b) –30, c) $48x^2\sqrt{3}$; 2a) 5/6, b) $\dfrac{2x^2\sqrt{3}}{7}$, c) $\dfrac{5x\sqrt{7}}{y}$, d) $\dfrac{3x^2y\sqrt{3x}}{8}$, e) $\dfrac{3xy^2}{2}$, f) $\dfrac{xy^2}{2}$, g) $\sqrt[6]{675}$; 3a) $\sqrt{71}$, b) 6; 4a) $\sqrt{74}$, b) $5\sqrt{2}$, c) $3\sqrt{13}$

Adding and Subtracting Radical Expressions

Learning Objective:

1. Simplify radical expressions involving addition and subtraction.

Examples:

1. Simplify. Assume that all variables represent positive real numbers.

a) $3\sqrt{5}+9\sqrt{5}$ b) $3\sqrt{2}+5\sqrt{6}-7\sqrt{6}$

c) $8\sqrt{3}+4\sqrt{2}-5\sqrt{3}$ d) $-5\sqrt{2}+3\sqrt{18}$

e) $5\sqrt{27}+4\sqrt{243}-2\sqrt{12}$ f) $\sqrt{2x}+7\sqrt{8x}-5\sqrt{50x}$

g) $\sqrt{3x^2}+3\sqrt{12x^2}+4\sqrt{12x^2}$ h) $\sqrt{300x^3}-x\sqrt{12x}$

i) $\sqrt[3]{16}+5\sqrt[3]{54}$ j) $\sqrt[3]{8x}-6\sqrt[3]{27x}$

k) $4\sqrt[3]{x^3y^7}+2xy\sqrt[3]{8y^4}$ l) $8\sqrt[4]{x^7}-3x\sqrt[4]{x^3}$

Teaching Notes:

- Most students find these exercises easy once they realize that adding and subtracting like radicals is very similar to adding and subtracting like terms.
- Sometimes we must first simplify the radical(s) and they can then be added or subtracted.

<u>Answers</u>: 1a) $12\sqrt{5}$, b) $3\sqrt{2}-2\sqrt{6}$, c) $3\sqrt{3}+4\sqrt{2}$, d) $4\sqrt{2}$, e) $47\sqrt{3}$, f) $-10\sqrt{2x}$, g) $15x\sqrt{3}$, h) $8x\sqrt{3x}$, i) $17\sqrt[3]{2}$, j) $-16\sqrt[3]{x}$, k) $8xy^2\sqrt[3]{y}$, l) $5x\sqrt[4]{x^3}$

Multiplying and Dividing Radical Expressions

Learning Objectives:

1. Multiply radical expressions.
2. Rationalize denominators with one radical term.
3. Rationalize denominators with binomials involving radicals.
4. Write radical expressions in lowest terms.

Examples:

1. Multiply, and then simplify if possible. Assume all variables represent positive real numbers.

a) $4\sqrt{7}(\sqrt{11}+\sqrt{7})$ b) $2\sqrt{7x}(\sqrt{y}+8\sqrt{3})$ c) $(\sqrt{8}+5)(\sqrt{8}-5)$

d) $(3\sqrt{7}-9)^2$ e) $(\sqrt{3x+2}-6)^2$ f) $-4\sqrt{5x}\left(2\sqrt{3}+3\sqrt{5x}\right)$

g) $\sqrt[3]{9}(\sqrt[3]{3}+\sqrt[3]{2})$ h) $\sqrt[3]{x^2}\left(4\sqrt[3]{4x}-5\sqrt[3]{x^5}\right)$ i) $5\sqrt[3]{x^2}\left(\sqrt[3]{25x}-\sqrt[3]{30x^8}\right)$

2. Simplify by rationalizing the denominator. Assume all variables represent positive real numbers.

a) $\dfrac{7}{\sqrt{2}}$ b) $\sqrt{\dfrac{36}{5}}$ c) $\dfrac{1}{\sqrt{2x}}$

d) $\dfrac{\sqrt{6y}}{\sqrt{12x}}$ e) $\dfrac{x}{\sqrt{7}-\sqrt{5}}$ f) $\dfrac{\sqrt{x}}{\sqrt{2x}+\sqrt{3}}$

g) $\dfrac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}}$ h) $\dfrac{\sqrt[3]{8x^4}}{\sqrt[3]{2x^5}}$

Teaching Notes:

- Show examples of $\left(\sqrt{x}\right)^2 = x$.
- Show how finding the product of conjugates will create a rational number.
- Remind students that $(a+b)^2 = (a+b)(a+b)$.

Answers: 1a) $4\sqrt{77}+28$, b) $2\sqrt{7xy}+16\sqrt{21x}$, c) -17, d) $144-54\sqrt{7}$, e) $3x+38-12\sqrt{3x+2}$, f) $-8\sqrt{15x}-60x$, g) $3+\sqrt[3]{18}$, h) $4x\sqrt[3]{4}-5x^2\sqrt[3]{x}$, i) $5x\sqrt[3]{25}-5x^3\sqrt[3]{30x}$; 2a) $\dfrac{7\sqrt{2}}{2}$, b) $\dfrac{6\sqrt{5}}{5}$, c) $\dfrac{\sqrt{2x}}{2x}$, d) $\dfrac{\sqrt{2xy}}{2x}$, e) $\dfrac{x(\sqrt{7}+\sqrt{5})}{2}$, f) $\dfrac{x\sqrt{2}-\sqrt{3x}}{2x-3}$, g) $\dfrac{7+2\sqrt{10}}{3}$, h) $\dfrac{\sqrt[3]{4x^2}}{x}$

Solving Equations with Radicals

Learning Objectives:

1. Solve radical equations using the power rule.
2. Solve radical equations with indexes greater than 2.
3. Use the power rule to solve a formula for a specified variable.

Examples:

1. Solve each equation.

a) $\sqrt{x} = 7$ b) $\sqrt{y+3} = 6$ c) $\sqrt{10x-9} = 9$

d) $\sqrt{8x-2} = \sqrt{x+4}$ e) $\sqrt{5x-6} = x$ f) $\sqrt{y+9} = y+3$

g) $\sqrt{12y-1} - 5y = y$ h) $\sqrt{4x^2 - x} = x$ i) $\sqrt{y^2 - 3y + 64} = y+5$

j) $\sqrt[3]{4x-4} = 1$ k) $\sqrt[3]{1-3x} = 2$ l) $\sqrt[3]{2x-5} = 3$

2. Solve each equation.

a) $\sqrt{3x+1} = 3 + \sqrt{x-4}$ b) $\sqrt{6x+4} = \sqrt{4x+1} + 3$

c) $\sqrt{x+6} + \sqrt{2-x} = 4$ d) $\sqrt{4x+3} = \sqrt{2x-2} - 2$

e) $\sqrt{4x} = \sqrt{x-5} + \sqrt{2x-2}$ f) $\sqrt{1+8\sqrt{x}} = 1 + \sqrt{x}$

g) $\sqrt[3]{x+3} = 2$ h) $\sqrt[3]{x^2} + \sqrt[3]{x} = 20$

3. When an object is dropped to the ground from a height of h meters, the time it takes for the object to reach the ground is given by the equation $t = \sqrt{\dfrac{h}{4.9}}$, where t is measured in seconds. Solve the equation for h. Use the result to determine the height from which an object was dropped if it hits the ground after falling for 2 seconds.

Teaching Notes:

- Remind students to isolate one radical before squaring both sides.
- Show how to square both sides of an equation. This includes squaring groups of terms and not individual terms.
- Show students a simple example of an extraneous solution, such as:
  $x = 3$ → square both sides → $x^2 = 9$ → $x = \pm 3$ → $x = -3$ is extraneous.

<u>Answers:</u> 1a) {49}, b) {33}, c) {9}, d) {6/7}, e) {2,3}, f) {0}, g) {1/6}, h) {0,1/3}, i) {3}, j) {5/4}, k) {–7/3}, l) {16}; 2a) {5,8}, b) {42}, c) {–2}, d) ∅ , e) {9}, f) {0,36}, g) {5}, h) {–125, 64}; 3) h=4.9t^2, 19.6 meters

Complex Numbers

Learning Objectives:

1. Simplify numbers of the form $\sqrt{-b}$, where $b>0$.
2. Recognize subsets of the complex numbers.
3. Add and subtract complex numbers.
4. Multiply complex numbers.
5. Divide complex numbers.
6. Simplify powers of *i*.

Examples:

1. Write each number as a product of a real number and *i*. Simplify all radical expressions.

   a) $\sqrt{-36}$ b) $\sqrt{-72}$ c) $4+\sqrt{-5}$ d) $\left(\sqrt{-25}\right)\left(\sqrt{-16}\right)$

2. Perform the addition or subtraction.

   a) $(-10-4i)+(6-5i)$ b) $\left(\frac{3}{2}-\frac{1}{2}i\right)+\left(-\frac{5}{2}+\frac{3}{2}i\right)$ c) $(4.8-1.7i)-(2.6-4.8i)$

3. Multiply and simplify your answers. Express factors using i notation before doing any other operations.

   a) $(4i)(5i)$ b) $(2-3i)(2+i)$

   c) $(i\sqrt{3})(i\sqrt{8})$ d) $(2+\sqrt{-2})(6+\sqrt{-3})$

4. Find each quotient.

   a) $\frac{3+2i}{2-i}$ b) $\frac{-4i}{3+5i}$ c) $\frac{7+10i}{4i}$ d) $\frac{5}{i}$

5. Find each power of *i*.

   a) i^3 b) i^4 c) i^5 d) i^6 e) i^{27}

Teaching Notes:

- Show students how the complex number system relates to the real number system.
- Be sure the concepts of $\sqrt{-1}=i$ and $i^2=-1$ are understood.
- Show examples of real numbers and pure imaginary numbers written in standard form.

Answers: *1a) 6i, b)* $6i\sqrt{2}$*, c)* $4+\sqrt{5}i$*, d) –20; 2a) –4–9i, b) –1+i, c) 2.2+3.1i; 3a) –20, b) 7–4i, c)* $-2\sqrt{6}$*, d)* $12-\sqrt{6}+2i\sqrt{3}+6i\sqrt{2}$*; 4a)* $\frac{4}{5}+\frac{7}{5}i$*, b)* $-\frac{10}{17}-\frac{6}{17}i$*, c)* $\frac{5}{2}-\frac{7}{4}i$*, d) –5i; 5a) –i, b) 1, c) i, d) –1, e) –i*

The Square Root Property and Completing the Square

Learning Objectives:

1. Review the zero factor property.
2. Learn the square root property.
3. Solve quadratic equations of the form $(ax+b)^2 = c$ by extending the square root property.
4. Solve quadratic by completing the square.
5. Solve quadratic equations with solutions that are not real numbers.

Examples:

1. Solve the equation by using the square root property. Express any complex numbers using *a+bi* notation.

   a) $x^2 = 9$ b) $x^2 = 20$ c) $x^2 - 75 = 0$

   d) $4x^2 = 16$ e) $3x^2 + 4 = 64$ f) $(x-5)^2 = 25$

   g) $(x+3)^2 = 11$ h) $(4x+1)^2 = 36$ i) $(5x-3)^2 = 48$

   j) $x^2 = -9$ k) $2x^2 + 72 = 0$ l) $4x^2 + 44 = 0$

2. Solve each equation by completing the square. Simplify your answers. Express any complex numbers using *a+bi* notation.

   a) $x^2 + 4x = -3$ b) $x^2 - 2x = 35$ c) $x^2 + 20x + 30 = 0$

   d) $x^2 + 18x + 67 = 0$ e) $x^2 - 16x = 0$ f) $6x^2 - 36x = 0$

   g) $2x^2 - 5x = 3$ h) $2x^2 + 11x = -12$ i) $2x^2 + 5x - 3 = 0$

   j) $6x^2 + 10x + 2 = 0$ k) $x^2 + 1 = -x$ l) $\dfrac{x^2}{2} + \dfrac{1}{2}x = 3$

Teaching Notes:

- Many students forget to include the +/- when using the square root property.
- Most students are confused by completing the square at first and need to see many examples of how to figure out what number must be added to complete the square.
- Explain to students that the completing the square process learned in this section will be used again later in the book. (example: finding the vertex of a parabola or the center of a circle)

Answers: *1a)* $\{-3,3\}$, *b)* $\{-2\sqrt{5}, 2\sqrt{5}\}$, *c)* $\{-5\sqrt{3}, 5\sqrt{3}\}$, *d)* $\{-2,2\}$, *e)* $\{-2\sqrt{5}, 2\sqrt{5}\}$, *f)* $\{0,10\}$, *g)* $\{-3-\sqrt{11}, -3+\sqrt{11}\}$, *h)* $\{-7/4, 5/4\}$, *i)* $\left\{\dfrac{3-4\sqrt{3}}{5}, \dfrac{3+4\sqrt{3}}{5}\right\}$, *j)* $\{-3i, 3i\}$, *k)* $\{-6i, 6i\}$, *l)* $\{-i\sqrt{11}, i\sqrt{11}\}$;

2a) $\{-3,-1\}$, *b)* $\{-5,7\}$, *c)* $\{-10-\sqrt{70}, -10+\sqrt{70}\}$, *d)* $\{-9-\sqrt{14}, -9+\sqrt{14}\}$, *e)* $\{0,16\}$, *f)* $\{0,6\}$, *g)* $\{-1/2, 3\}$, *h)* $\{-4, -3/2\}$, *i)* $\{-3, 1/2\}$, *j)* $\left\{\dfrac{-5-\sqrt{13}}{6}, \dfrac{-5+\sqrt{13}}{6}\right\}$, *k)* $\left\{\dfrac{-1-i\sqrt{3}}{2}, \dfrac{-1+i\sqrt{3}}{2}\right\}$, *l)* $\{2, -3\}$

The Quadratic Formula

Learning Objectives:

1. Derive the quadratic formula.
2. Solve quadratic equations using the quadratic formula.
3. Use the discriminant to determine the number and type of solutions.

Examples:

1. Use the quadratic formula to solve each equation. Express any complex numbers using $a+bi$ notation.

   a) $x^2+5x+6=0$ b) $x^2+4x-7=0$ c) $3x^2-9x=-2$

   d) $5x^2=-10x-3$ e) $5x^2=-8$ f) $9+3x(x-2)=8$

2. Use the discriminant to find what type of solutions (two rational, two irrational, one rational, or two nonreal complex) each equation has. Do not solve the equation.

   a) $4x^2-8x+4=0$ b) $6x^2=2x-5$

   c) $x^2+8x+7=0$ d) $10-5x^2=6x+5$

Teaching Notes:

- It is helpful to at least show the derivation of the quadratic formula.
- Students must thoroughly memorize the quadratic formla.
- Many students reduce final answers incorrectly: $\dfrac{6\pm\sqrt{5}}{8}\neq\dfrac{3\pm\sqrt{5}}{4}$.
- Some students prefer to always use the quadratic formula because it has no restrictions on when it can be used. Encourage them to also master the other methods, which are often quicker and easier to apply.

Answers: 1a) $\{-3,-2\}$, b) $\left\{-2-\sqrt{11},-2+\sqrt{11}\right\}$, c) $\left\{\dfrac{9-\sqrt{57}}{6},\dfrac{9+\sqrt{57}}{6}\right\}$, d) $\left\{\dfrac{-5-\sqrt{10}}{5},\dfrac{-5+\sqrt{10}}{5}\right\}$, e) $\left\{\dfrac{-2i\sqrt{10}}{5},\dfrac{2i\sqrt{10}}{5}\right\}$, f) $\left\{\dfrac{3-\sqrt{6}}{3},\dfrac{3+\sqrt{6}}{3}\right\}$; *2a) one rational solution, b) two nonreal complex solutions, c) two rational solutions, d) two irrational solutions*

Equations Quadratic in Form

Learning Objectives:

1. Solve an equation with fractions by writing it in quadratic form.
2. Use quadratic equations to solve applied problems.
3. Solve an equation with radicals by writing it in quadratic form.
4. Solve an equation that is quadratic in form by substitution.

Examples:

1. Solve each equation. Check your solutions. Express any complex numbers using $a+bi$ notation.

   a) $x^4 - 13x^2 + 36 = 0$ b) $x^4 - 2x^2 - 8 = 0$ c) $10x^4 = 8x^2 + 2$

   d) $\frac{1}{x+2} + \frac{1}{x} = \frac{1}{2}$ e) $(x-4)(x+3) = \frac{2x-17}{2}$

2. Solve each equation. Check your solutions. Express any complex numbers using $a+bi$ notation.

   a) $x^6 - 19x^3 - 216 = 0$ b) $x^6 - 12x^3 = 28$

   c) $x^8 = 92x^4 - 891$ d) $5x^8 + 28x^4 = 12$

   e) $x^{\frac{2}{3}} - 3x^{\frac{1}{3}} + 2 = 0$ f) $x^{\frac{2}{5}} - x^{\frac{1}{5}} - 6 = 0$

   g) $2x^{\frac{1}{2}} - 13x^{\frac{1}{4}} - 24 = 0$ h) $\sqrt{8x+3} = 4x$

3. Solve each equation. Check your solutions. Express any complex numbers using $a+bi$ notation.

   a) $(x^2 - 4x)^2 - 17(x^2 - 4x) + 60 = 0$ b) $2x - 9x^{\frac{1}{2}} - 5 = 0$

   c) $10x^{-2} + 11x^{-1} + 1 = 0$ d) $(x^2 - 2x)^2 + 2(x^2 - 2x) = 63$

4. It takes two painters 8 hours to paint a living room. If each worked alone, one of them could do the job 2 hours faster than the other. How long would it take each painter to paint the living room alone?

Teaching Notes:

- Some students prefer to solve problems such as 1a) without using substitution. Encourage them to use substitution anyway in order to master it.
- Remind students that a 6^{th}- degree equation can have up to 6 distinct solutions.
- Refer students to the ***Solving an Equation that is Quadratic in Form by Substitution*** chart in the textbook.

Answers: *1a) {–3, –2, 2, 3}, b)* $\left\{-2, 2, i\sqrt{2}, -i\sqrt{2}\right\}$, *c)* $\left\{-1, 1, -\frac{i\sqrt{5}}{5}, \frac{i\sqrt{5}}{5}\right\}$, *d)* $\left\{1-\sqrt{5}, 1+\sqrt{5}\right\}$, *e)* $\left\{\frac{2-3\sqrt{2}}{2}, \frac{2+3\sqrt{2}}{2}\right\}$; *2a)* $\left\{-2, 3\right\}$, *b)* $\left\{\sqrt[3]{14}, -\sqrt[3]{2}\right\}$, *c)* $\left\{-3, 3, -\sqrt[4]{11}, \sqrt[4]{11}\right\}$, *d)* $\left\{-\frac{\sqrt[4]{250}}{5}, \frac{\sqrt[4]{250}}{5}\right\}$, *e) {1,8}, f) {243,–32}, g) {4096}, h)* $\left\{\frac{3}{4}\right\}$; *3a) {–2,–1,6,5}, b) {25}, c) {–1,–10}, d)* $\left\{1+2\sqrt{2}, 1-2\sqrt{2}, 1+2i\sqrt{2}, 1-2i\sqrt{2}\right\}$; *4) approximately 17 and 15 hours.*

Formulas and Further Applications

Learning Objectives:

1. Solve formulas involving squares and square roots for specified variables.
2. Solve applied problems using the Pythagorean theorem.
3. Solve applied problems using area formulas.
4. Solve applied problems using quadratic functions as models.

Examples:

1. Solve for the variable specified. Assume that all other variables are non-zero. Leave $\pm$ in your answers.

   a) $M = \pi r^2hd$; for r b) $rm = t^2 - mt$; for t

   c) $S = \lambda (b^2 + B^2)w$; for b d) $T = 2\pi\sqrt{\dfrac{L}{9.8}}$; for L

2. Solve each problem.

   a) A triangle has sides *a*, *b*, and hypotenuse *c*. If $c = 8$ and $b = 4a$, find *b* and *a*.

   b) A tour bus is traveling along a path that forms a right triangle. One leg of the triangle represents a distance of 24 miles. The other leg of the triangle is 16 miles shorter than the hypotenuse. What is the length of the hypotenuse of this triangle? Of the other leg?

3. Solve each problem.

   a) A swimming pool is in the shape of a rectangle, 10 meters wide and 18 meters long. It is surrounded by a walk of uniform width whose area is 52 square meters. How wide is the walk?

   b) The number *B* (measured in thousands) of a certain type of endangered bird can be approximated by the equation $B = 1.04x^2 + 3.9x + 6.47$, where *x* is the number of years after 2010. How many birds are predicted for the year 2016?

   c) The formula $P = 0.63x^2 - 0.049x + 2$ models the approximate population *P*, in thousands, for a species of fish in a local pond, *x* years after 2012. During what year will the population reach 34,620 fish?

   d) An express train travels 166 miles between two cities. During the first 64 miles of a trip, the train traveled through mountainous terrain. The train traveled 19 miles per hour slower through mountainous terrain than through level terrain. If the total time to travel between the cities was 4 hours, find the speed of the train on level terrain.

Teaching Notes:

- In problem 1, suggest having students circle the variable being solved for and treat the remaining letters as constants.
- Encourage students to draw and label diagrams whenever possible.

Answers: *1a)* $r = \pm\dfrac{\sqrt{\pi Mhd}}{\pi hd}$, *b)* $t = \dfrac{m \pm \sqrt{m^2 + 4rm}}{2}$, *c)* $b = \pm\dfrac{\sqrt{\lambda w(S - \lambda wB^2)}}{\lambda w}$, *d)* $L = \dfrac{2.45T^2}{\pi^2}$; *2a)* $a = \dfrac{8\sqrt{17}}{17}$, $b = \dfrac{32\sqrt{17}}{17}$, b*) hypotenuse=26 miles, shorter leg=10 miles; 3a)* $-7 + \sqrt{62}$ *meters, b) 67,310 birds, c) 2019, d) 51 mph*

Graphs of Quadratic Functions

Learning Objectives:

1. Graph a quadratic function.
2. Graph parabolas with horizontal and vertical shifts.
3. Use the coefficient of x^2 to predict the shape and direction in which a parabola opens.
4. Find a quadratic function to model data.

Examples:

1. Graph each of the following quadratic functions.

   a) $f(x) = x^2$ b) $f(x) = (x-3)^2$

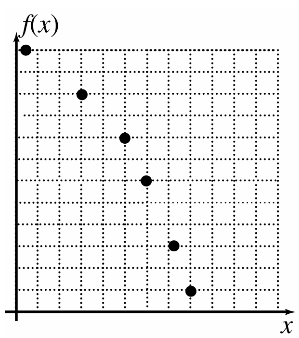
   c) $f(x) = -x^2 + 4$ d) $f(x) = (x+2)^2 - 3$

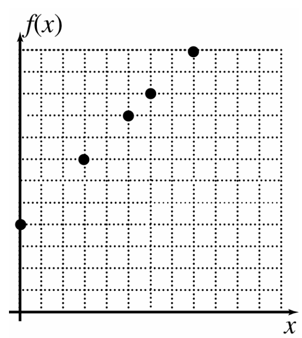
2. Identify the vertex, axis, domain, and range of each parabolas in exercise 1.

3. For each quadratic function, tell whether the graph opens up or down and whether the graph is wider, narrower, or the same shape as the graph of $f(x) = x^2$.

   a) $f(x) = \frac{1}{3}x^2$ b) $f(x) = 3 - 5x^2$ c) $f(x) = -\left(x - \frac{1}{5}\right)^2 - 2$

4. Tell whether a linear or a quadratic function would be a more appropriate model. If linear, tell whether the slope is positive or negative. If quadratic, tell whether the coefficient of x^2 should be positive or negative.

   a)
   

   b)
   

5. Find the equation of the parabola that goes through the points $(-1,-2), (0,-1)$, and $(2,7)$.

Teaching Notes:

- Most students are comfortable using the vertex formula, but some wonder at first why the calculated x-coordinate must be put back into the quadratic equation.
- Most students prefer graphing by vertex and intercepts rather than by finding points using substitution.
- Consider showing students how to use technology to model real world data.

<u>Answers</u>: (graph answers at end of mini-lectures) 2a) vertex: $(0,0)$, axis: $x = 0$, domain: $(-\infty,\infty)$, range: $[0,\infty)$, b) vertex: $(3,0)$, axis: $x = 3$, domain: $[-\infty,\infty)$, range: $[0,\infty)$, c) vertex: $(0,4)$, axis: $x = 0$, domain: $(-\infty,\infty)$, range: $(-\infty,4]$, d) vertex: $(-2,-3)$, axis: $x = -2$, domain: $(-\infty,\infty)$, range: $[-3,\infty)$; 3a) up, wider, b) down, narrower, c) down, same shape; 4a) quadratic model, negative, b) linear model, slope is positive; 5) $f(x) = x^2 + 2x - 1$

More about Parabolas and Their Applications

Learning Objectives:

1. Find the vertex of a vertical parabola.
2. Graph a quadratic function.
3. Use the discriminant to find the number of x-intercepts of a parabola with a vertical axis.
4. Use quadratic functions to solve problems involving maximum or minimum value.
5. Graph horizontal parabolas with horizontal axes.

Examples:

1. Find the vertex of each parabola. For each equation, tell whether the graph opens up, down, to the left, or to the right, and whether it is wider, narrower, or the same shape as the graph of $y = x^2$.

   a) $y = -x^2 + 2x + 1$ b) $y = x^2 + 2x + 1$

   c) $y = 2x^2 + 4x - 3$ d) $x = -\frac{1}{2}y^2 - 2y - 6$

2. Graph each parabola in exercise 1. Give the vertex, axis, domain, and range.

3. Use the discriminant to determine the number of x-intercepts.

   a) $f(x) = x^2 - 4x + 4$ b) $f(x) = x^2 - 6x + 13$

   c) $f(x) = x^2 - 9x$ d) $f(x) = \frac{1}{2}x^2 - 2x + 2$

4. *Maximum Profit* The daily profit in dollars of a ski and snowboard shop is described by the function, $P(x) = -2x^2 + 400x - 1824$, where x is the number of snowboards sold in one day. How many snowboards should be sold per day in order to maximize profit?

5. The difference of two numbers is 66. Find the numbers if their product is to be a minimum, and also find this product.

Teaching Notes:

- Show the derivation of the axis formula.
- Encourage students to identify the axis when graphing and to plot a couple of points to the right and to the left of the axis.
- Refer students to the ***Graphing a Quadratic Function*** and ***Graph of a Horizontal Parabola*** charts in the textbook.

Answers: (graph answers at end of mini-lectures) 1a) vertex: $(1,2)$*, down, same shape, b) vertex:* $(-1,0)$*, up, same shape, c) vertex:* $(-1,-5)$*, up, narrower, d) vertex:* $(-4,-2)$*, left, wider; 2a) vertex:* $(1,2)$*, axis:* $x=1$*, domain:* $(-\infty,\infty)$*, range:* $(-\infty,2]$*, b) vertex:* $(-1,0)$*, axis:* $x=-1$*, domain:* $(-\infty,\infty)$*, range:* $[0,\infty)$*, c) vertex:* $(-1,-5)$*, axis:* $x=-1$*, domain:* $(-\infty,\infty)$*, range:* $[-5,\infty)$*, d) vertex:* $(-4,-2)$*, axis:* $y=-2$*, domain:* $(-\infty,-4]$*, range:* $(-\infty,\infty)$*; 3a) one, b) none, c) two, d) one; 4) 100; 5) –33 and 33, product is –1089*

Polynomial and Rational Inequalities

Learning Objectives:

1. Solve quadratic inequalities.
2. Solve polynomial inequalities of degree 3 or greater.
3. Solve rational inequalities.

Examples:

1. The graph of the quadratic function $f(x) = -x^2 - 2x + 3$ is given. Use the graph to find the solution set of each of the following:

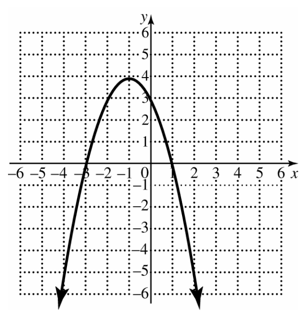
   a) $-x^2 - 2x + 3 = 0$

   b) $-x^2 - 2x + 3 > 0$

   c) $-x^2 - 2x + 3 < 0$

   d) $-x^2 - 2x + 3 \geq 0$

   e) $-x^2 - 2x + 3 \leq 0$



2. Graph the solution set of 1b and 1e of exercise 1.

3. Solve each inequality and graph the solution set.

   a) $(x-5)(x+2)(3x-1) > 0$ b) $\dfrac{2x-1}{x-4} < 0$

4. Solve.

   a) $x^2 - 9x + 18 \geq 0$ b) $(x+1)(x-3)(x-6) > 0$

   c) $\dfrac{4}{y-3} \leq 0$ d) $\dfrac{x-7}{x-2} > 0$

Teaching Notes:

- Many students understand the concepts of this section better if they are shown a graph of the quadratic function in exercise 1 and can see where the parabola is above or below the x-axis.
- Some students are confused by how to pick test points. Remind them that they can pick any convenient number except for the numbers that define regions.

Answers: *(graph answers at end of mini-lectures)* *1a)* $\{-3,1\}$, *b)* $(-3,1)$, *c)* $(-\infty,-3)\cup(1,\infty)$, *d)* $[-3,1]$, *e)* $(-\infty,-3]\cup[1,\infty)$; *3a)* $\left(-2,\frac{1}{3}\right)\cup(5,\infty)$, *b)* $\left(\frac{1}{2},4\right)$; *4a)* $(-\infty,3]\cup[6,\infty)$, *b)* $(-1,3)\cup(6,\infty)$, *c)* $(-\infty,3)$, *d)* $(-\infty,2)\cup(7,\infty)$

Inverse Functions

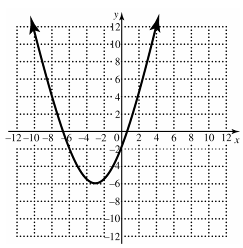
Learning Objectives:

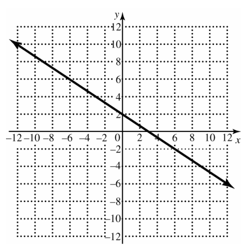
1. Decide whether a function is one–to–one and, if it is, find its inverse.
2. Use the horizontal line test to determine whether a function is one–to–one.
3. Find the equation of the inverse of a function.
4. Graph f^{-1} given the graph of f.

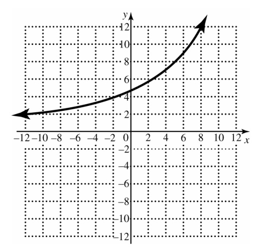
Examples:

1. Indicate whether each function is one–to–one.

   a) $B = \{(6, 2), (9, 9), (1, 4), (-1, 5)\}$ b) $C = \{(8, -1), (9, -1), (11, 7), (12, 2)\}$

   c) 

   d) 

   e) 

2. Find the inverse of each function.

   a) $A = \{(1, 2), (-1, 3), (-3, 4)\}$ b) $f(x) = 2x + 3$ c) $f(x) = \dfrac{4x-5}{3}$

   d) $f(x) = \sqrt[3]{x+9}$ e) $f(x) = \dfrac{2}{x}$ f) $f(x) = \dfrac{6}{7-x}$

3. Find the inverse of each function. Graph the function and its inverse on one coordinate plane. Graph the line $y = x$ as a dashed line.

   a) $R = \{(-9, 6), (-6, 9), (3, 4)\}$ b) $f(x) = 3x + 5$ c) $f(x) = \dfrac{3}{4}x - 2$

Teaching Notes:

- Tell students early on that f^{-1} means "the inverse function of f". It does not mean $\dfrac{1}{f}$.
- Most students understand the concept of an inverse function better if they are told that an inverse function f^{-1} "undoes" whatever the function f did to x, i.e. $f^{-1}\left[f(x)\right] = x$.
- Tell students to always check that their graphs of f and f^{-1} are symmetric about $y = x$.

<u>Answers:</u> (graph answers at end of mini–lectures) 1a) one–to–one, b) not one–to–one, c) not one–to–one, d) one–to–one, e) one–to–one; 2a) A^{-1}={(2,1), (3,–1), (4,–3)}, b) $f^{-1}(x) = \dfrac{x-3}{2}$, c) $f^{-1}(x) = \dfrac{3x+5}{4}$, d) $f^{-1}(x) = x^3 - 9$, e) $f^{-1}(x) = \dfrac{2}{x}$, f) $f^{-1}(x) = \dfrac{7x-6}{x}$; 3a) R^{-1}={(6,–9), (9,–6), (4,3)}, b) $f^{-1}(x) = \dfrac{x-5}{3}$, c) $f^{-1}(x) = \dfrac{4x+8}{3}$

Exponential Functions

Learning Objectives:

1. Use a calculator to find approximations of exponentials.
2. Define and graph an exponential functions.
3. Solve exponential equations of the form $a^x = a^k$ for x.
4. Use exponential functions in applications involving growth or decay.

Examples:

1. Use a calculator to find an approximation for each exponential expression to three decimal places.

   a) $3^{2.4}$ b) $4^{-2.5}$ c) $3^{2/3}$

2. Graph each exponential function.

   a) $f(x) = 2^x$ b) $f(x) = 2^{-x}$ c) $f(x) = 2^{x+2}$

   d) $f(x) = 2^{x-3}$ e) $f(x) = 2^x + 3$ f) $f(x) = 2^x - 2$

   g) $f(x) = 2^{-x} + 3$

3. Solve each equation.

   a) $3^x = 9$ b) $3^x = 27$ c) $3^x = 1$

   d) $3^{-x} = \frac{1}{9}$ e) $2^x = \frac{1}{16}$ f) $4^{x+2} = 64$

   g) $5^{3x-3} = 125$ h) $3^{3x-1} = 9$ i) $\left(\frac{1}{4}\right)^{x+1} = 2$

4. Application problem.

   Tony and Linda are investing \$25,000 at an annual rate of 2.8% compounded annually. How much money will they have after 3 years? Use the future value formula $A = P(1+r)^t$. Round to the nearest cent.

Teaching Notes:

- Most students understand the graphs better if the first few are done by plotting points instead of using shifting ideas.
- Encourage students to use scientific calculators in this section. Show the $y^x, 10^x, \log x, e^x, \ln x$ keys on the calculator.
- Some students find exponential equations confusing at first and need to see a step-by-step process for solving them.

<u>Answers</u>: *(graph answers at end of mini–lectures) 1a) 13.967, b) 0.031, c) 2.080; 3a) {2}, b) {3}, c) {0}, d) {2}, e) {–4}, f) {1}, g) {2}, h) {1}; i)* $-3/2$ *, 4) \$27,159.35*

Logarithmic Functions

Learning Objectives:

1. Define a logarithm.
2. Convert between exponential and logarithmic forms and evaluate logarithms.
3. Solve logarithmic equations of the form $\log_a b = k$ for *a, b,* or *k*.
4. Use the definition of logarithm to simplify logarithmic expressions.
5. Define and graph logarithmic functions.
6. Use logarithmic functions in applications involving growth or decay.

Examples:

1. Write in logarithmic form.

   a) $25 = 5^2$ b) $1000 = 10^3$ c) $\frac{1}{512} = 8^{-3}$ d) $y = 2^7$

2. Use a calculator to approximate each logarithm to four decimal places.

   a) $\log_3 8$ b) $\log_{1/3} 11$ c) $\log_3 0.5$

3. Write in exponential form.

   a) $2 = \log_4 16$ b) $3 = \log_2 8$ c) $\frac{1}{2} = \log_{25} 5$ d) $-\frac{2}{3} = \log_7 x$

4. Solve.

   a) $\log_2 x = 5$ b) $\log_{10} x = -2$ c) $\log_4 16 = y$

   d) $\log_6\left(\frac{1}{36}\right) = y$ e) $\log_a 27 = 3$ f) $\log_w 3 = \frac{1}{2}$

5. Evaluate.

   a) $\log_{10}(0.01)$ b) $\log_{25} 1$ c) $\log_7 343$ d) $\log_5 \frac{1}{125}$

6. Graph.

   a) $\log_2 x = y$ b) $\log_{10} x = y$ c) $\log_{\frac{1}{2}} x = y$

7. Sales (in thousands of units) of a new product are approximated by the function defined by $P(t) = 90 + 30\log(2t)$ where t is the number of years after the product is introduced. What were the sales after 3 years?

Teaching Notes:

- Use the definition of a logarithm $y = a^x \Leftrightarrow \log_a y = x$ early and often with students.
- Tell students early on that a logarithm is an exponent.
- Use a graph of a log function to clarify the characteristics of a logarithmic function.

Answers: *(graph answers at end of mini–lectures) 1a)* $\log_5 25 = 2$, *b)* $\log_{10} 1000 = 3$, *c)* $\log_8 \frac{1}{512} = -3$, *d)* $\log_2 y = 7$ *; 2a) 1.8928, b) -2.1827, c) -0.6309; 3a)* $4^2=16$*, b)* $2^3=8$*, c)* $25^{1/2}=5$*, d)* $7^{-2/3}=x$*; 4a) {32}, b) {1/100}, c) {2}, d) {–2}, e) {3}, f) {9}; 5a) –2, b) 0, c) 3, d) –3; 7) 113.3 thousand*

Properties of Logarithms

Learning Objectives:

1. Use the product rule for logarithms.
2. Use the quotient rule for logarithms.
3. Use the power rule for logarithms.
4. Use properties to write alternative forms of logarithmic expressions.

Examples:

1. Use the properties of logarithms to express each expression as a sum or difference of logarithms. Assume that all variables are defined in such a way that the variable expressions are positive, and bases are positive numbers not equal to 1.

   a) $\log_2 AB$ b) $\log_7(5\cdot 11)$ c) $\log_b 13g$

   d) $\log_6\left(\frac{3}{7}\right)$ e) $\log_b\left(\frac{J}{12}\right)$ f) $\log_a\left(\frac{P}{Q}\right)$

   g) $\log_2 b^9$ h) $\log_a B^{-3}$ i) $\log_7 \sqrt{n}$

   j) $\log_6 xy^2$ k) $\log_4\left(\frac{5P}{Q}\right)$ l) $\log_9\left(\frac{2x^3y}{\sqrt{z}}\right)$

   m) $\log_a \sqrt[3]{\frac{y}{z^4}}$

2. Use properties of logarithms to write each expression as a single logarithm. Assume that all variables are defined in such a way that the variable expressions are positive, and bases are positive numbers not equal to 1.

   a) $\log_3 12 + \log_3 x + \log_3 2$ b) $6\log_4 x - \log_4 9$ c) $3\log_a 2 + 6\log_a y - \frac{1}{3}\log_a z$

3. Use the properties of logarithms to simplify each expression.

   a) $\log_5 5$ b) $\log_{10} 1$

   c) $4\log_6 6 + 3\log_6 1$ d) $\frac{1}{2}\log_4 4 - 9\log_4 1$

Teaching Notes:

- Refer students to the five properties and definitions in the textbook.
- Show student that $\log_a x - \log_a y \neq \frac{\log_a x}{\log_a y}$.
- Show the difference between $\log_a x^r$ and $(\log_a x)^r$

Answers: *1a)* $\log_2 A + \log_2 B$, *b)* $\log_7 5 + \log_7 11$, *c)* $\log_b 13 + \log_b g$, *d)* $\log_6 3 - \log_6 7$, *e)* $\log_b J - \log_b 12$, *f)* $\log_a P - \log_a Q$, *g)* $9\log_2 b$, *h)* $-3\log_a B$, *i)* $\frac{1}{2}\log_7 n$, *j)* $\log_6 x + 2\log_6 y$, *k)* $\log_4 5 + \log_4 P - \log_4 Q$, *l)* $\log_9 2 + 3\log_9 x + \log_9 y - \frac{1}{2}\log_9 z$, *m)* $\frac{1}{3}\log_a y - \frac{4}{3}\log_a z$; *2a)* $\log_3 24x$, *b)* $\log_4\left(\frac{x^6}{9}\right)$, *c)* $\log_a\left(\frac{2^3y^6}{\sqrt[3]{z}}\right)$; *3a) 1, b) 0, c) 4, d) 1/2*

Common and Natural Logarithms

Learning Objectives:

1. Evaluate common logarithms using a calculator.
2. Use common logarithms in applications.
3. Evaluate natural logarithms using a calculator.
4. Use natural logarithms in applications.
5. Use the change-of-base rule.

Examples:

1. Find each logarithm. Give approximations to the nearest hundredth.

   a) $\log 10$ b) $\log 23.1$ c) $\log 45,600$ d) $\log 0.369$

   e) $\ln e$ f) $\ln 9.82$ g) $\ln 132,000$ h) $\ln 0.015$

2. The average number of boats on a lake can be approximated by the function $N = 40\ln t + 50$, where t is the high temperature for the day, in degrees Fahrenheit. Approximate the number of boats on the lake if the high temperature is 90º F.

3. The function $P = 90 - 30\log n$ models the percent P of the recruits remaining in a rigorous military training program, where n is the number of days into the program. What percent of the recruits are still in the program after 7 days? Round your answer to the nearest whole percent.

4. Use the change of base formula to find each logarithm to the nearest hundredth.

   a) $\log_6 8$ b) $\log_{1/3} 12$ c) $\log_5 0.5$ d) $\log_3 6$ e) $\log_5 9.2$

   f) $\log_{15} 0.13$ g) $\log_7 232$

Teaching Notes:

- Most students need help with calculator keystrokes for this section.
- Tell students that an antilogarithm is just another name for an exponential.
- Show the derivation of the change – of – base formula.

Answers: 1a) 1, b) 1.36, c) 4.66, d) –0.43, e) 1, f) 2.28, g) 11.79, h) –4.20; 2) 230 boats; 3) 65%; 4a) 1.16, b) –2.26, c) –0.431 d) 1.63, e) 1.38, f) – 0.75, g) 2.80

Exponential and Logarithmic Equations; Further Applications

Learning Objectives:

1. Solve equations involving variables in the exponents.
2. Solve equations involving logarithms.
3. Solve applications of compound interest.
4. Solve applications involving base *e* exponential growth and decay.

Examples:

1. Solve each equation. Give solutions to three decimal places.

   a) $3^x = 9$ b) $2^x = 9$ c) $3^{x+8} = 7$ d) $6^{4x-5} = 18$

2. Solve each equation. Give the exact solution.

   a) $\log(5+x) - \log(x-5) = \log 3$ b) $\log_7(6x-1) + \log_7 x = 1$

   c) $\log_5 x + \log_5(x-1) = \log_5 20$ d) $\log_2(x+5) - 3 = \log_2 x$

   e) $\ln 8 - \ln x = \ln(x-2)$ f) $\ln(3+3x) = 2\ln(x+1)$

3. Solve each equation. Give solutions to the nearest thousandth.

   a) $7^{2x-1} = 90$ b) $4^x = 5^{x+1}$ c) $34 = e^{x-2}$ d) $75 = e^{3x+1}$

4. a) The size of the elk population at a national park increases at the rate of 3.3% per year. If the size of the current population is 124, find how many elk there should be in 6 years. Use $A = A_0 e^{0.033t}$, where A_0 is the current population, and *t* is time, in years. Round to the nearest whole number.

   b) When the principal *P* earns an annual interest rate *r* compounded yearly, the amount *A* after *t* years is $A = P(1+r)^t$. How long will it take \$24000 to grow to \$48000 at 2% compounded annually? Round to the nearest whole year.

Teaching Notes:

- Remind students that it is not possible to take the logarithm of a non-positive number. Therefore some of the solutions in objective 1 must be discarded.
- Refer students to the 3-step process for solving logarithmic equations in the textbook.
- Most students can now understand the change of base formula as follows:

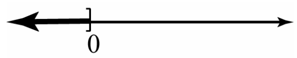
$$\log_b x = y \quad\to\quad b^y = x \quad\to\quad \log_a b^y = \log_a x \quad\to\quad y\log_a b = \log_a x \quad\to\quad y = \frac{\log_a x}{\log_a b}$$

<u>Answers:</u> 1a) {2}, b){3.170}, c){–6.229}, d){1.653}; 2a) {10}, b) {7/6}, c) {5}, d) {5/7}, e) {4}, f) {2};3a) {1.656}, b) { –7.213}, c) {5.526}, d) {1.106}; 4a) 151 elk, b) 35 years

Mini-Lecture 1.5

1a)

b)



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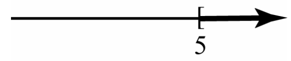


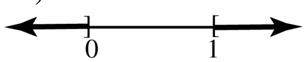
Mini-Lecture 1.6

2a)

b)

c)

3a)



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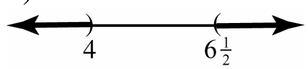

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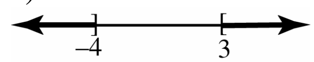

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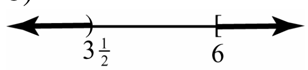
b)



c)

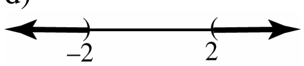
Mini-Lecture 1.7

2a)

b)

c)

d)



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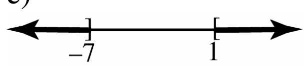

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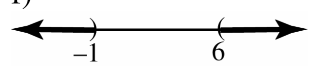

Mini-Lecture 2.1

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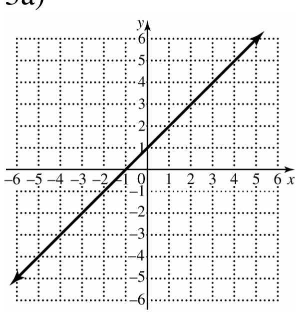

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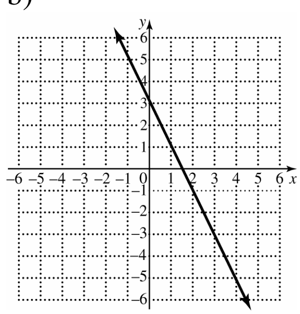

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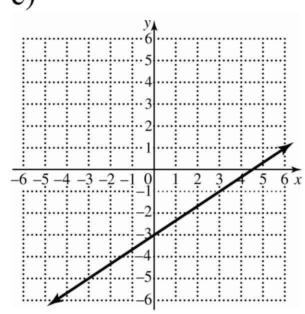

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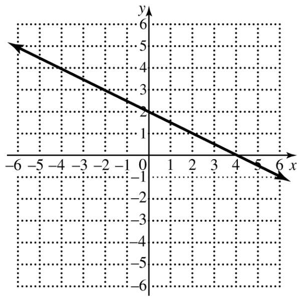

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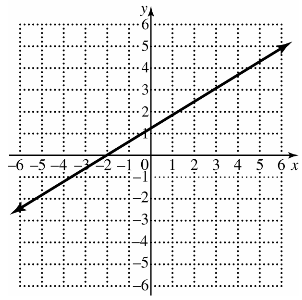

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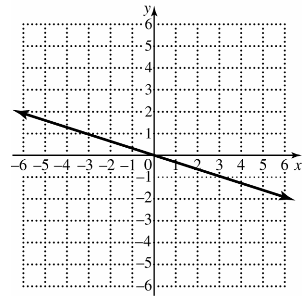
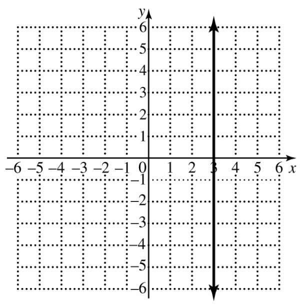


5a)



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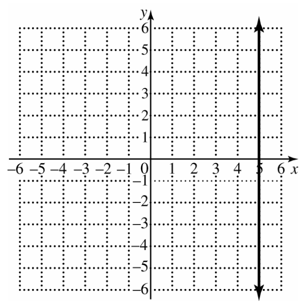

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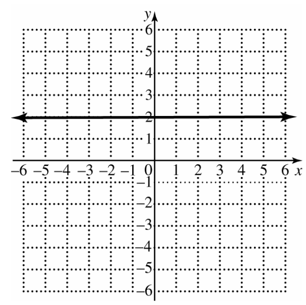
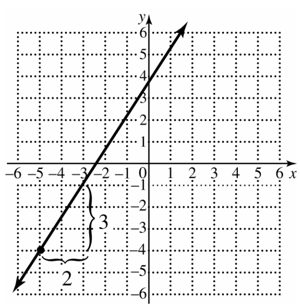


Mini-Lecture 2.2

4a)



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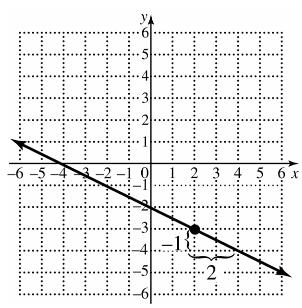
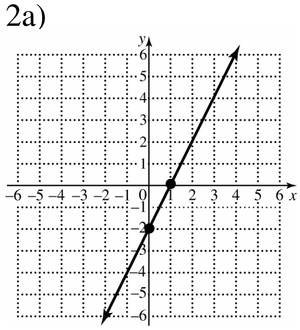


Mini-Lecture 2.3

2a)



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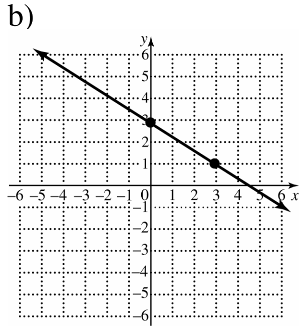
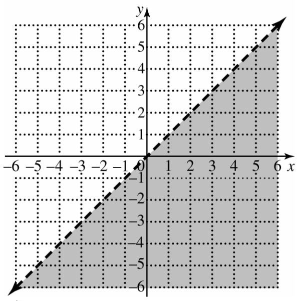


Mini-Lecture 2.4

2a)



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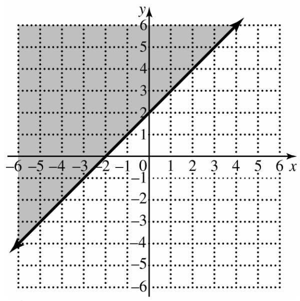

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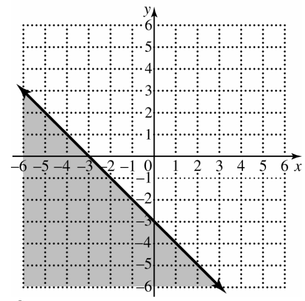

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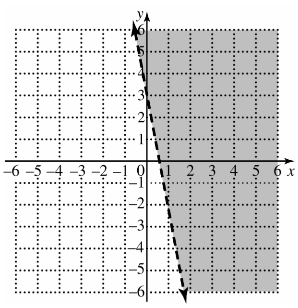

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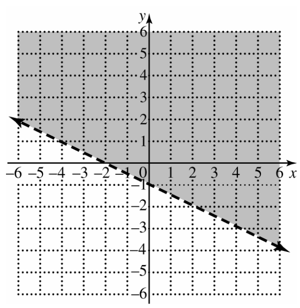

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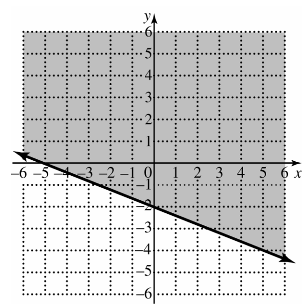

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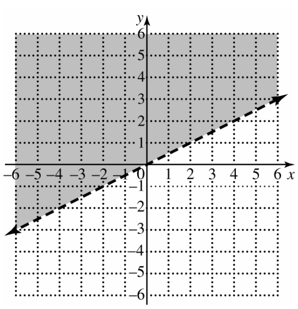

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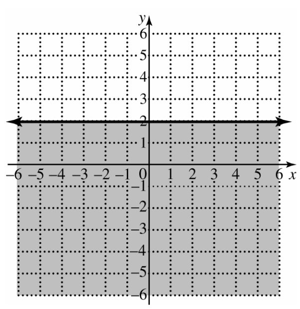
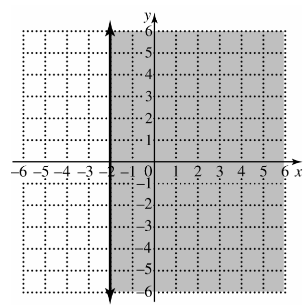


i)



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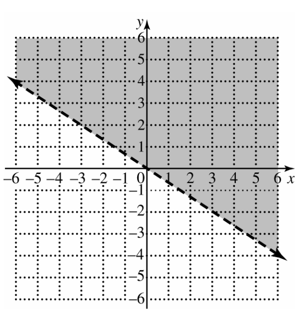

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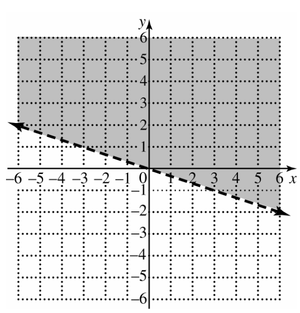
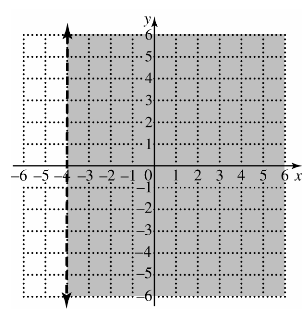


l)



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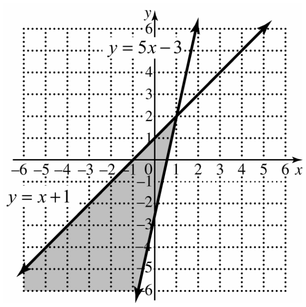

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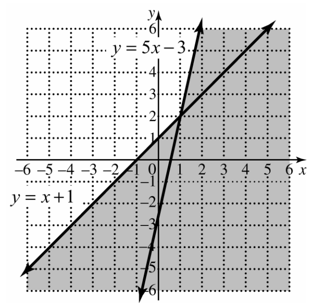
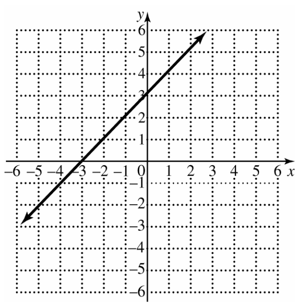


Mini-Lecture 2.6

2a)



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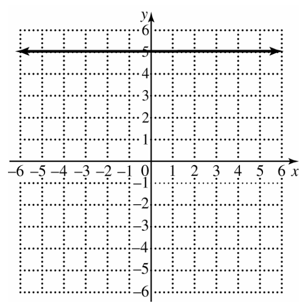

Mini-Lecture 3.1

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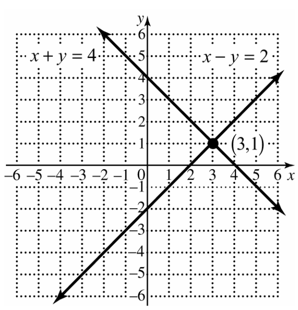

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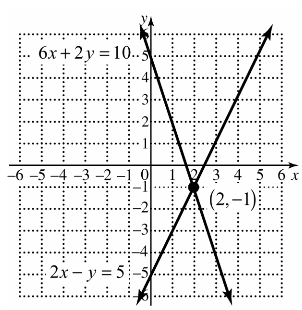

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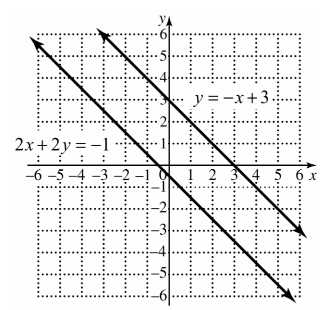
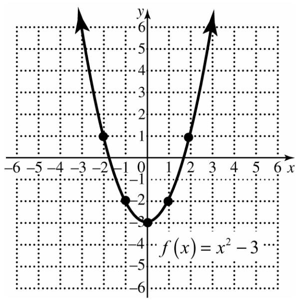


Mini-Lecture 4.3

5a)



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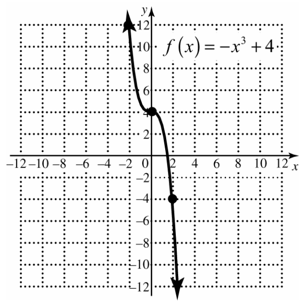
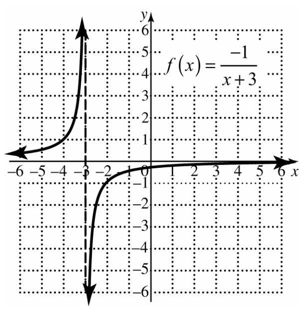


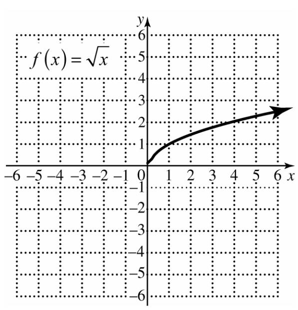
Mini-Lecture 6.4

5)



Mini-Lecture 7.1

2a)



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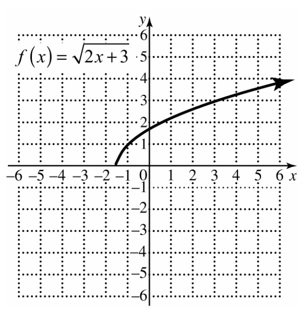
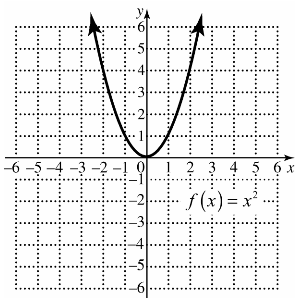


Mini-Lecture 8.5

1a)



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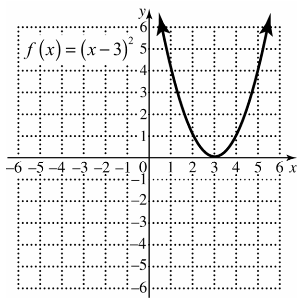

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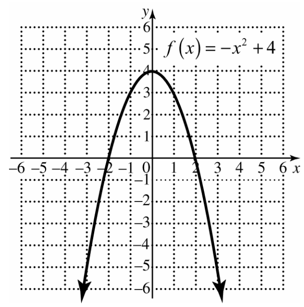
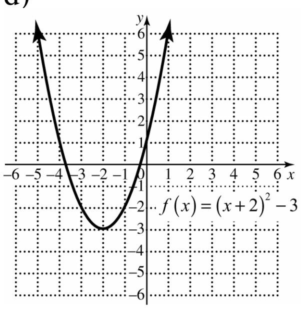
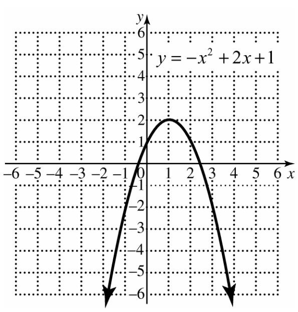


d)

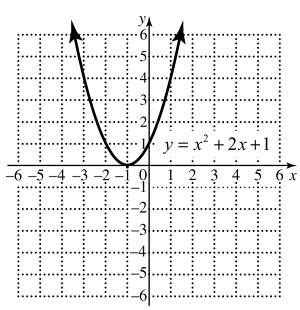


Mini-Lecture 8.6

2a)



b)



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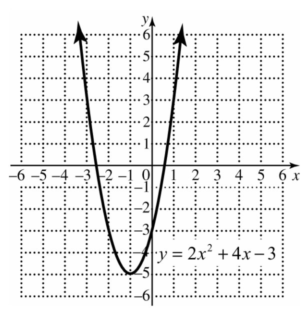

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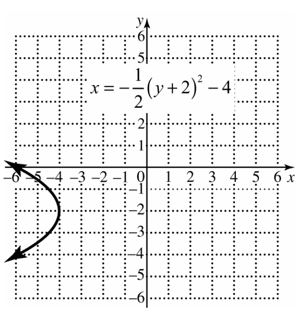
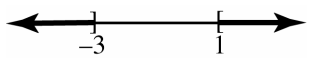


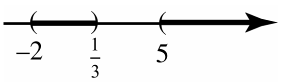
Mini-Lecture 8.7

2b)

e)



3a)



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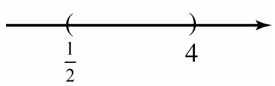
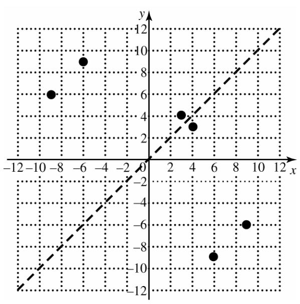


Mini-Lecture 9.1

3a)



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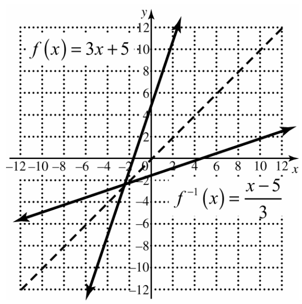

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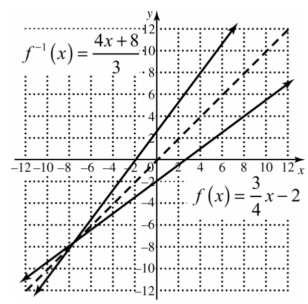
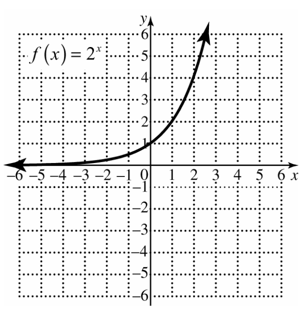


Mini-Lecture 9.2

2a)



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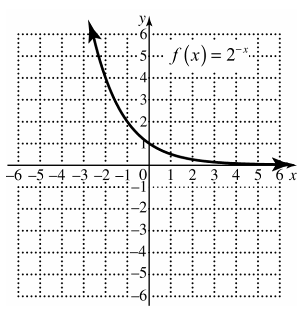

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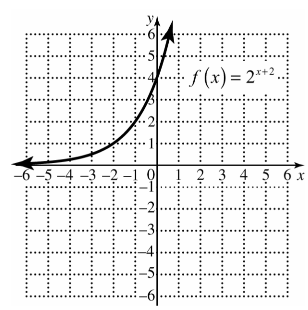

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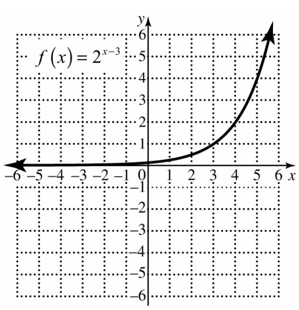

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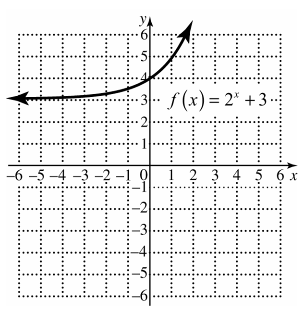
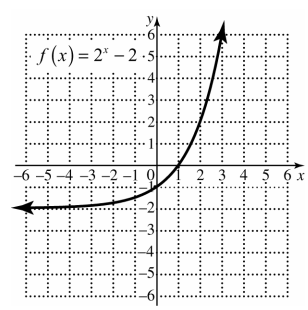
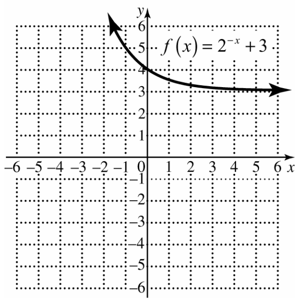


f)

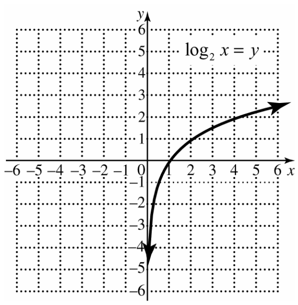


g)

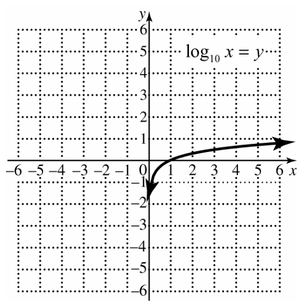


Mini-Lecture 9.3

6a)



b)



c)