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Use the limit process to find the area of the region between the graph of  $y = 4 - x^2$ , the x-axis, and the vertical lines x = -2 and x = 2.

a. Sketch the region.

b. We will find the area of the region by dividing it into upright rectangles. First, we determine the width of each rectangle by dividing the interval [-2, 2] into n subintervals of equal width. What is the width of each subinterval?

c. Determine the endpoints of the first three subintervals by filling in the following:

$$a = x_0 = -2 + 0() < -2 + 1() < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2 + () < -2$$

d. Determine the endpoints of the last two subintervals by filling in the following:

$$\dots < -2 + (n - 2)() < -2 + () < + () = x_n = b$$

e. Use the right endpoint of each subinterval to determine the height of each rectangle. What is the height of a representative rectangle?

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f. Write an expression for the area of the representative rectangle by choosing an appropriate c<sub>i</sub>.

g. How many such rectangles are in the region? \_\_\_\_\_

h. Write an expression for the sum of all the rectangles in the region in terms of n. Use Theorem 4.2 to write the appropriate summation formulas.

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i. Find the sum of 100 rectangles.

j. If the number of rectangles approaches infinity, find this sum. This is the area of the region.