

Give a geometric description of the set of points whose coordinates satisfy the given conditions.

1) $y^2 + z^2 = 2, x = 5$

2) $2 \leq y \leq 3, 2 \leq z \leq 3$

3) $x^2 + y^2 \leq 9, z = -10$

4) $x^2 + y^2 + z^2 > 16$

Describe the given set of points with a single equation or with a pair of equations.

5) The plane perpendicular to the y -axis and passing through the point $(4, -9, -4)$

6) The circle of radius 4 centered at the point $(7, -1, 1)$ and lying in a plane perpendicular to the x -axis

Write one or more inequalities that describe the set of points.

7) The interior of the sphere $x^2 + y^2 + z^2 = 36$

8) The half-space consisting of the points on and behind the yz -plane

9) The closed region bounded by the spheres of radius 5 and 7, both centered at the origin, and the planes $x = 4$ and $x = 6$

10) The exterior of the sphere of radius 2 centered at the point $(-3, 4, 4)$

Find the distance between points P_1 and P_2 .

11) $P_1(2, 6, 6)$ and $P_2(4, 10, 10)$

Find the center and radius of the sphere.

12) $x^2 + (y + 8)^2 + (z - 4)^2 = 49$

Find an equation for the sphere with the given center and radius.

13) Center $(0, -4, -9)$, radius = 3

Solve the problem.

14) Find a formula for the distance from the point $P(x, y, z)$ to the yz plane.

15) Find the perimeter of the triangle with vertices $A(6, 1, 3)$, $B(2, -2, 6)$, and $C(3, 5, 7)$.

16) Show that the point $P(-1, 6, -6)$ is equidistant from the points $A(-2, 4, -5)$ and $B(0, 8, -7)$.

Find the indicated vector.

17) Let $\mathbf{u} = \langle -3, 4 \rangle$, $\mathbf{v} = \langle -3, 6 \rangle$. Find $\mathbf{u} - \mathbf{v}$.

Find the component form of the specified vector.

18) The vector \overrightarrow{PQ} , where $P = (-4, 5)$ and $Q = (10, 5)$

Express the vector in the form $\mathbf{v} = v_1\mathbf{i} + v_2\mathbf{j} + v_3\mathbf{k}$.

19) $\overrightarrow{P_1P_2}$ if P_1 is the point $(6, -4, 4)$ and P_2 is the point $(8, -7, 0)$

Express the vector as a product of its length and direction.

20) $8\mathbf{j} - 6\mathbf{k}$

Calculate the direction of $\overrightarrow{P_1P_2}$ and the midpoint of line segment P_1P_2 .

21) $P_1(-5, 2, 7)$ and $P_2(-1, 6, 9)$

Solve the problem.

22) Let $\mathbf{u} = 2\mathbf{i} + 4\mathbf{j}$, $\mathbf{v} = 2\mathbf{i} + 3\mathbf{j}$, and $\mathbf{w} = \mathbf{i} - \mathbf{j}$. Write $\mathbf{u} = \mathbf{u}_1 + \mathbf{u}_2$ where \mathbf{u}_1 is parallel to \mathbf{v} and \mathbf{u}_2 is parallel to \mathbf{w} .

23) A force of magnitude 19 pounds pulling on a suitcase makes an angle of 60° with the ground. Express the force in terms of its \mathbf{i} and \mathbf{j} components.

24) An airplane is flying in the direction 80° west of north at 709 km/hr. Find the component form of the velocity of the airplane, assuming that the positive x -axis represents due east and the positive y -axis represents due north.

25) For the triangle with vertices located at $A(5, 5, 4)$, $B(2, 2, 4)$, and $C(1, 1, 1)$, find a vector from vertex C to the midpoint of side AB .

Find $\mathbf{v} \cdot \mathbf{u}$.

26) $\mathbf{v} = 2\mathbf{i} + 6\mathbf{j}$ and $\mathbf{u} = 8\mathbf{i} + 9\mathbf{j}$

Find the angle between \mathbf{u} and \mathbf{v} in radians.

27) $\mathbf{u} = 9\mathbf{i} + 10\mathbf{j} + 5\mathbf{k}$, $\mathbf{v} = 5\mathbf{i} + 3\mathbf{j} + 3\mathbf{k}$

Find the vector $\text{proj}_{\mathbf{v}} \mathbf{u}$.

28) $\mathbf{v} = \mathbf{i} + \mathbf{j} + \mathbf{k}$, $\mathbf{u} = 12\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}$

Find an equation for the line that passes through the given point and satisfies the given conditions.

29) $P = (8, 7)$; perpendicular to $\mathbf{v} = 3\mathbf{i} + 5\mathbf{j}$

30) $P = (8, 4)$; parallel to $\mathbf{v} = 4\mathbf{i} + 6\mathbf{j}$

Find the acute angle between the lines.

31) $3x - y = 2$ and $2x + y = 15$

Find the length and direction (when defined) of $\mathbf{u} \times \mathbf{v}$.

32) $\mathbf{u} = 3\mathbf{i} + 7\mathbf{j}$, $\mathbf{v} = \mathbf{i} - \mathbf{j}$

33) $\mathbf{u} = -3\mathbf{i} - 2\mathbf{j} + 5\mathbf{k}$, $\mathbf{v} = 6\mathbf{i} + 4\mathbf{j} - 10\mathbf{k}$

Sketch the coordinate axes and then include the vectors \mathbf{A} , \mathbf{B} , and $\mathbf{A} \times \mathbf{B}$ as vectors starting at the origin.

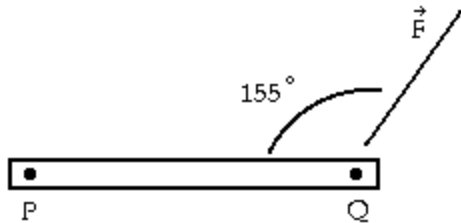
34) $\mathbf{u} = \mathbf{i} - \mathbf{j}$, $\mathbf{v} = \mathbf{k}$

Solve the problem.

35) Find the area of the triangle determined by the points $P(1, 1, 1)$, $Q(9, -10, 10)$, and $R(6, -4, -7)$.

36) Find a unit vector perpendicular to plane PQR determined by the points $P(2, 1, 1)$, $Q(1, 0, 0)$ and $R(2, 2, 2)$.

37) Find the magnitude of the torque in foot-pounds at point P for the following lever:



$|\vec{PQ}| = 9$ in. and $|\mathbf{F}| = 5$ lb

Determine whether the following is always true or not always true. Given reasons for your answers.

38) $|\mathbf{u}| = \sqrt{\mathbf{u} \cdot \mathbf{u}}$

39) $\mathbf{u} \times \mathbf{0} = \mathbf{0}$

40) $\mathbf{u} \times (\mathbf{v} + \mathbf{w}) = \mathbf{u} \times \mathbf{v} + \mathbf{u} \times \mathbf{w}$

41) $(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{w} = \mathbf{u} \cdot (\mathbf{w} \times \mathbf{v})$

42) $(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{v} = 0$

43) $(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{v} = \mathbf{u} \cdot (\mathbf{u} \times \mathbf{v})$

44) $c(\mathbf{u} \cdot \mathbf{v}) = \mathbf{cu} \cdot \mathbf{cv}$ (any number c)

45) $c(\mathbf{u} \times \mathbf{v}) = \mathbf{cu} \times \mathbf{cv}$ (any number c)

Find parametric equations for the line described below.

46) The line through the point $P(-4, -4, -2)$ parallel to the vector $-7\mathbf{i} + 7\mathbf{j} - 7\mathbf{k}$

47) The line through the point $P(-7, -4, 3)$ and perpendicular to the plane $-4x + 4y + 7z = 3$

Write the equation for the plane.

48) The plane through the point $P(4, 7, 4)$ and parallel to the plane $7x + 5y + 4z = 75$.

49) The plane through the points $P(5, -3, -1)$, $Q(-3, -6, 35)$ and $R(-1, 8, -27)$.

50) The plane through the point $P(-5, 6, -2)$ and perpendicular to the line $x = 7 + 2t, y = 2 + 3t, z = 5 + 7t$

51) The plane through the point $A(10, 10, 5)$ perpendicular to the vector from the origin to A .

Calculate the requested distance.

52) The distance from the point $S(3, 4, -1)$ to the line $x = -9 + 4t, y = 9 + 12t, z = 2 + 3t$

Use a calculator to find the acute angle between the planes to the nearest thousandth of a radian.

53) $8x + 8y + 6z = -6$ and $2x + 3y + 8z = 1$

Find the intersection.

54) $-8x - 4y - 4z = 8, -5x - 5y - 7z = 8$

Match the equation with the surface it defines.

55) $\frac{y^2}{100} + \frac{z^2}{25} = 1$

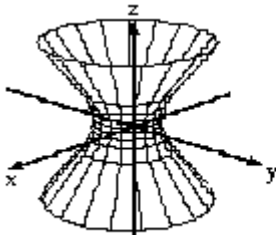


Figure 1

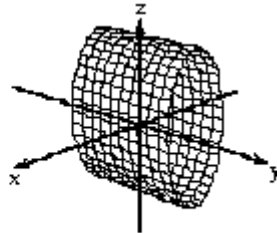


Figure 2

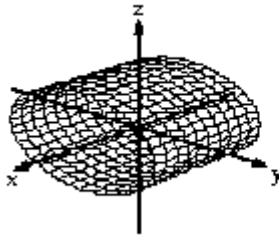


Figure 3

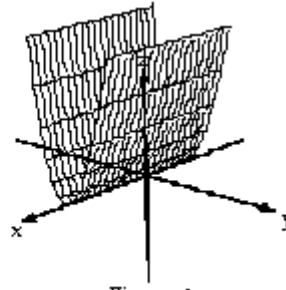


Figure 4

$$56) \frac{x^2}{9} + \frac{y^2}{36} + \frac{z^2}{9} = 1$$

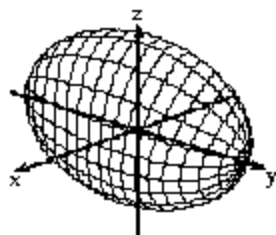


Figure 1

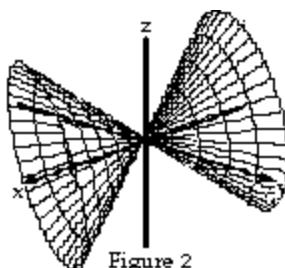


Figure 2

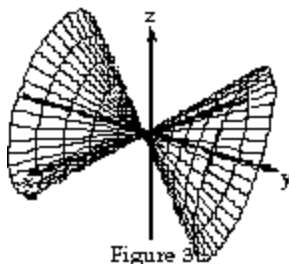


Figure 3

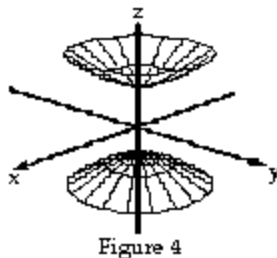


Figure 4

$$57) \frac{x^2}{16} + \frac{y^2}{16} = \frac{z^2}{4}$$

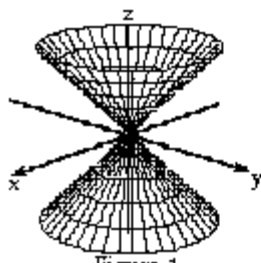


Figure 1

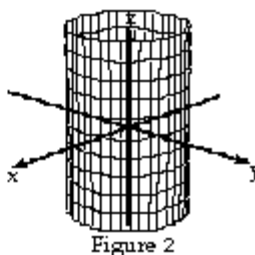


Figure 2

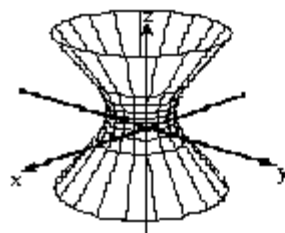


Figure 3



Figure 4

$$58) -\frac{x^2}{64} + \frac{y^2}{16} + \frac{z^2}{16} = 1$$

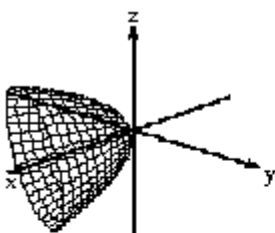


Figure 1

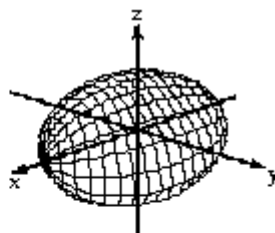


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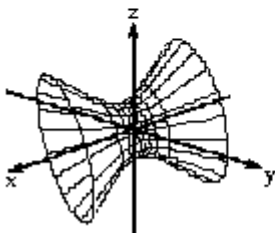


Figure 3

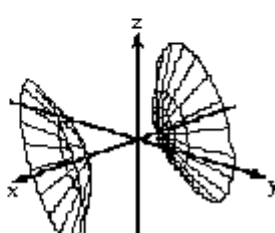


Figure 4

$$59) \frac{z^2}{36} - \frac{x^2}{100} - \frac{y^2}{100} = 1$$

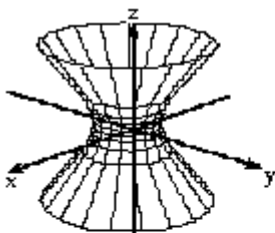


Figure 1

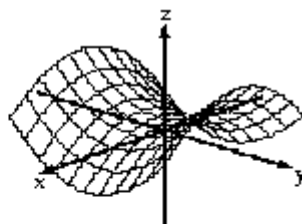


Figure 2

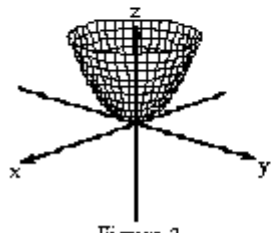


Figure 3

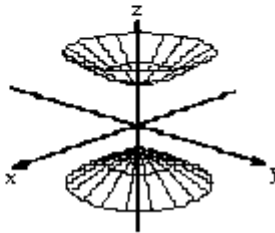


Figure 4

$$60) \frac{y^2}{4} - \frac{x^2}{4} = \frac{z}{8}$$

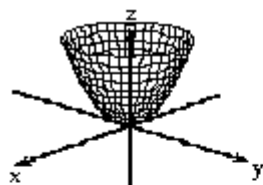


Figure 1

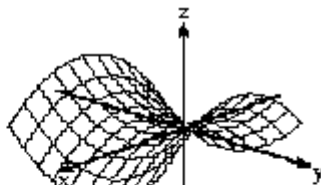


Figure 2

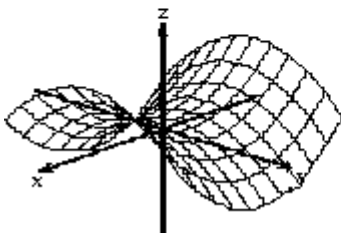


Figure 3

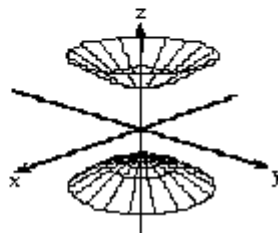


Figure 4

Identify the type of surface represented by the given equation.

$$61) y^2 + z^2 = 5$$

$$62) \frac{x^2}{9} + \frac{y^2}{7} = 2$$

$$63) \frac{x^2}{3} + \frac{y^2}{9} + \frac{z^2}{8} = 1$$

$$64) \frac{x^2}{10} + \frac{z^2}{10} = \frac{y}{9}$$

$$65) \frac{x^2}{2} + \frac{y^2}{8} = \frac{z^2}{4}$$

$$66) \frac{x^2}{10} - \frac{y^2}{3} - \frac{z^2}{9} = 1$$

$$67) \frac{z^2}{2} - \frac{x^2}{10} = \frac{y}{8}$$

Answer Key

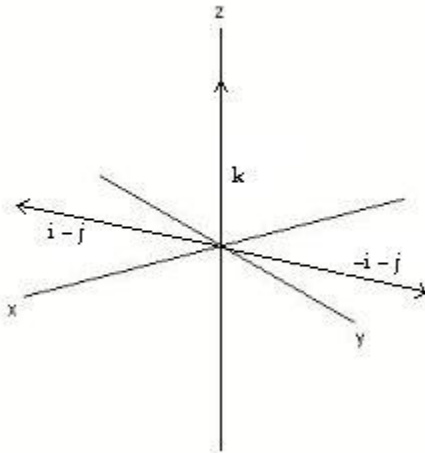
Testname: MA2415X1REV

- 1) The circle $y^2 + z^2 = 2$ in the plane $x = 5$
- 2) The infinitely long square prism parallel to the x -axis
- 3) All points on or within the circle $x^2 + y^2 = 9$ and in the plane $z = -10$
- 4) All points outside the sphere of radius 4
- 5) $y = -9$
- 6) $(y + 1)^2 + (z - 1)^2 = 16$ and $x = 7$
- 7) $x^2 + y^2 + z^2 < 36$
- 8) $x \leq 0$
- 9) $25 \leq x^2 + y^2 + z^2 \leq 49$ and $4 \leq x \leq 6$
- 10) $(x + 3)^2 + (y - 4)^2 + (z - 4)^2 > 4$
- 11) 6
- 12) $C(0, -8, 4)$, $a = 7$
- 13) $x^2 + y^2 + z^2 + 8y + 18z = -88$
- 14) x
- 15) $\sqrt{34} + \sqrt{51} + \sqrt{41}$
- 16) The distance between P and A is $\sqrt{6}$; the distance between P and B is $\sqrt{6}$
- 17) $\langle 0, -2 \rangle$
- 18) $\langle 14, 0 \rangle$
- 19) $\mathbf{v} = 2\mathbf{i} - 3\mathbf{j} - 4\mathbf{k}$
- 20) $10\left(\frac{4}{5}\mathbf{j} - \frac{3}{5}\mathbf{k}\right)$
- 21) $\frac{2}{3}\mathbf{i} + \frac{2}{3}\mathbf{j} + \frac{1}{3}\mathbf{k}; (-3, 4, 8)$
- 22) $\mathbf{u}_1 = 1.200 \mathbf{v}$
 $\mathbf{u}_2 = -0.4000\mathbf{w}$
- 23) $9.500\mathbf{i} + 16.45 \mathbf{j}$
- 24) $\langle -698.2, 123.1 \rangle$
- 25) $\frac{5}{2}\mathbf{i} + \frac{5}{2}\mathbf{j} + 3\mathbf{k}$
- 26) 70
- 27) 0.30
- 28) $\frac{19}{3}\mathbf{i} + \frac{19}{3}\mathbf{j} + \frac{19}{3}\mathbf{k}$
- 29) $3x + 5y = 59$
- 30) $6x - 4y = 32$
- 31) 45°
- 32) 10; $-\mathbf{k}$
- 33) 0; no direction

Answer Key

Testname: MA2415X1REV

34)



35) $\frac{\sqrt{29,795}}{2}$

36) $\frac{1}{\sqrt{2}}(\mathbf{j} - \mathbf{k})$

37) 1.58 ft-lb

38) Always true by definition

39) Always true by definition of $\mathbf{0}$

40) Always true by distributive property

41) Not always true; $(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{w} = \mathbf{u} \cdot (\mathbf{v} \times \mathbf{w})$, but $\mathbf{v} \times \mathbf{w} = -(\mathbf{w} \times \mathbf{v})$ from which it follows that the original equation false if $\mathbf{w} \times \mathbf{v} \neq \mathbf{0}$.

42) Always true because $\mathbf{u} \times \mathbf{v}$ and \mathbf{v} are orthogonal

43) Not always true; The statement is false if $\mathbf{u} \neq \mathbf{v}$.

44) Not always true; The statement if false if $c \neq 0,1$.

45) Not always true; The statement if false if $c \neq 0,1$.

46) $x = -7t - 4, y = 7t - 4, z = -7t - 2$

47) $x = -4t - 7, y = 4t - 4, z = 7t + 3$

48) $7x + 5y + 4z = 79$

49) $3x + 4y + z = 2$

50) $2x + 3y + 7z = -6$

51) $10x + 10y + 5z = 225$

52) $\frac{\sqrt{29,641}}{13}$

53) 0.671 rad

54) $x = 8t - \frac{2}{5}, y = -36t - \frac{6}{5}, z = 20t$

55) Figure 3

56) Figure 1

57) Figure 1

58) Figure 3

59) Figure 4

60) Figure 3

61) Cylinder

62) Elliptical cylinder

Answer Key

Testname: MA2415X1REV

- 63) Ellipsoid
- 64) Elliptical paraboloid
- 65) Elliptical cone
- 66) Hyperboloid of two sheets
- 67) Hyperbolic paraboloid