If r(t) is the position vector of a particle in the plane at time t, find the indicated vector.

1) Find the acceleration vector.
$\mathbf{r}(t) = (8 \cos t)\mathbf{i} + (5 \sin t)\mathbf{j}$

The position vector of a particle is r(t). Find the requested vector.

2) The velocity at t = 0 for $\mathbf{r}(t) = \cos(2t)\mathbf{i} + 8\ln(t - 6)\mathbf{j} - \frac{t^3}{10}\mathbf{k}$

3) The acceleration at $t = \frac{\pi}{8}$ for $\mathbf{r}(t) = (3t - 5t^3)\mathbf{i} + 4\tan(2t)\mathbf{j} + e^{3t}\mathbf{k}$

The vector r(t) is the position vector of a particle at time t. Find the angle between the velocity and the acceleration vectors at time t = 0.

4) $\mathbf{r}(t) = \sin^{-1}(9t)\mathbf{i} + \ln(7t^2 + 1)\mathbf{j} + \sqrt{6t^2 + 1}\mathbf{k}$

For the smooth curve r(t), find the parametric equations for the line that is tangent to r at the given parameter value $t = t_0$.

5) $\mathbf{r}(t) = (3 \sin t)\mathbf{i} - (5 \cos 2t)\mathbf{j} + e^{-10t}\mathbf{k}$; $t_o = 0$

Provide an appropriate response.

6) The following equations each describe the motion of a particle. For which path is the particle's speed constant?
(1) $\mathbf{r}(t) = t^3\mathbf{i} + t^5\mathbf{j}$
(2) $\mathbf{r}(t) = \cos(7t)\mathbf{i} + \sin(7t)\mathbf{j}$
(3) $\mathbf{r}(t) = t\mathbf{i} + t\mathbf{j}$
(4) $\mathbf{r}(t) = \cos(3t^2)\mathbf{i} + \sin(3t^2)\mathbf{j}$

7) Prove that if vectors **v** and **u** are differentiable functions of t, then $\frac{d}{dt}(\mathbf{u} + \mathbf{v}) = \frac{d\mathbf{u}}{dt} + \frac{d\mathbf{v}}{dt}$.

Evaluate the integral.

8) $\int_0^{\pi/4} [(10\sec^2 t)\mathbf{i} - (3 + \sin t)\mathbf{j} - (2\sec t \tan t)\mathbf{k})]$

9) $\int_0^1 \left[\frac{1}{\sqrt{2t - t^2}}\mathbf{i} + te^{9t}\mathbf{j} - \frac{27t^2}{(2 + 9t^3)^2}\mathbf{k}\right] dt$

Solve the initial value problem.

10) Differential Equation: $\frac{d\mathbf{r}}{dt} = -9t\mathbf{i} + 7t\mathbf{j} + 5t\mathbf{k}$
Initial Condition: $\mathbf{r}(0) = -4\mathbf{i} + 4\mathbf{k}$

Solve the problem. Unless stated otherwise, assume that the projectile flight is ideal, that the launch angle is measured from the horizontal, and that the projectile is launched from the origin over a horizontal surface

11) An ideal projectile is launched from level ground at a launch angle of 26° and an initial speed of 48 m/sec. How far away from the launch point does the projectile hit the ground?

12) A projectile is fired with an initial speed of 552 m/sec at an angle of 45°. What is the greatest height reached by the projectile? Round your answer to the nearest tenth.

Provide an appropriate response.

13) Prove that $\int_a^b k\mathbf{r}(t)\,dt = k\int_a^b \mathbf{r}(t)\,dt$ for any scalar constant k.

14) Prove that $\int_a^b (\mathbf{r}_1(t) + \mathbf{r}_2(t))\,dt = \int_a^b \mathbf{r}_1(t)\,dt + \int_a^b \mathbf{r}_2(t)\,dt$

Calculate the arc length of the indicated portion of the curve r(t).

15) $\mathbf{r}(t) = 11t^4\mathbf{i} + 2t^4\mathbf{j} + 10t^4\mathbf{k}$, $1 \le t \le 3$

Find the unit tangent vector of the given curve.

16) $\mathbf{r}(t) = (7t \cos t - 7 \sin t)\mathbf{j} + (7t \sin t + 7 \cos t)\mathbf{k}$

Find the arc length parameter along the curve from the point where t = 0 by evaluating $s = \int_0^t |\mathbf{v}(\tau)|\, d\tau$.

17) $\mathbf{r}(t) = (e^t\cos t)\mathbf{i} + (e^t\sin t)\mathbf{j} + 3e^t\mathbf{k}$

Find the length of the indicated portion of the curve.

18) $\mathbf{r}(t) = (4\cos t)\mathbf{i} + (4\sin t)\mathbf{j} + 3t\mathbf{k}$, $0 \le t \le \pi/2$

Find the principal unit normal vector N for the curve r(t).

19) $\mathbf{r}(t) = (t^2 + 4)\mathbf{j} + (2t - 7)\mathbf{k}$

Find the curvature of the curve r(t).

20) $\mathbf{r}(t) = (7 + 9\cos 4t)\,\mathbf{i} - (9 + 9\sin 4t)\mathbf{j} + 10\mathbf{k}$

Find the curvature of the space curve.

21) $\mathbf{r}(t) = (t + 7)\mathbf{i} + 8\mathbf{j} + (\ln(\sec t) + 5)\mathbf{k}$

22) $\mathbf{r}(t) = -9\mathbf{i} + (5 + 2t)\mathbf{j} + (t^2 + 4)\mathbf{k}$

23) $\mathbf{r}(t) = t\mathbf{i} + (\sinh t)\mathbf{j} + (\cosh t)\mathbf{k}$

For the curve r(t), write the acceleration in the form $a_T\mathbf{T} + a_N\mathbf{N}$.

24) $\mathbf{r}(t) = (t + 5)\mathbf{i} + (\ln(\sec t) - 8)\mathbf{j} + 2\mathbf{k}$, $-\pi/2 < t < \pi/2$

25) $\mathbf{r}(t) = (t^2 - 6)\mathbf{i} + (2t - 9)\mathbf{j} + 3\mathbf{k}$

26) $\mathbf{r}(t) = (2t\sin t + 2\cos t)\mathbf{i} + (2t\cos t - 2\sin t)\mathbf{j} + 2\mathbf{k}$

1) $\mathbf{a} = (-8 \cos t)\mathbf{i} + (-5 \sin t)\mathbf{j}$

2) $\mathbf{v}(0) = -\frac{4}{3}\mathbf{j}$

3) $\mathbf{a}\left(\frac{\pi}{8}\right) = -\frac{15}{4}\pi\mathbf{i} + 64\mathbf{j} + 9e^{3/8\pi}\mathbf{k}$

4) $\frac{\pi}{2}$

5) $x = 3t,\ y = -5,\ z = 1 - 10t$

6) Path (2) and Path (3)

7) $\frac{d}{dt}(\mathbf{u} + \mathbf{v}) = \frac{d}{dt}\left(u_x\mathbf{i} + u_y\mathbf{j} + v_x\mathbf{i} + v_y\mathbf{j}\right) = \frac{d}{dt}\left((u_x + v_x)\mathbf{i} + (u_y + v_y)\mathbf{j}\right)$

$= \frac{d}{dt}(u_x + v_x)\mathbf{i} + \frac{d}{dt}(u_y + v_y)\mathbf{j} = \frac{d}{dt}(u_x\mathbf{i} + u_y\mathbf{j}) + \frac{d}{dt}(v_x\mathbf{i} + v_y\mathbf{j}) = \frac{d\mathbf{u}}{dt} + \frac{d\mathbf{v}}{dt}$

8) $10\mathbf{i} + \left(\frac{2\sqrt{2} - 3\pi - 4}{4}\right)\mathbf{j} + 2(1 - \sqrt{2})\mathbf{k}$

9) $\frac{\pi}{2}\mathbf{i} + \frac{8e^9 + 1}{81}\mathbf{j} - \frac{9}{22}\mathbf{k}$

10) $\mathbf{r}(t) = \frac{-9t^2 - 8}{2}\mathbf{i} + \frac{7}{2}t^2\mathbf{j} + \frac{5t^2 + 8}{2}\mathbf{k}$

11) ≈ 185 m

12) 7765.1 m

13) $\int_a^b k\mathbf{r}(t)\,dt = \int_a^b k(x(t)\mathbf{i} + y(t)\mathbf{j})\,dt = \int_a^b kx(t)\mathbf{i} + ky(t)\mathbf{j}\,dt$

$= k\int_a^b x(t)\mathbf{i}\,dt + k\int_a^b y(t)\mathbf{j}\,dt = k\left[\int_a^b x(t)\mathbf{i}\,dt + \int_a^b y(t)\mathbf{j}\,dt\right]$

$= k\left[\int_a^b (x(t)\mathbf{i} + y(t)\mathbf{j})\,dt\right] = k\int_a^b \mathbf{r}(t)\,dt$

14) $\int_a^b (\mathbf{r}_1(t) + \mathbf{r}_2(t))\,dt = \int_a^b \left[(x_1(t)\mathbf{i} + y_1(t)\mathbf{j}) + (x_2(t)\mathbf{i} + y_2(t)\mathbf{j})\right]dt$

$= \int_a^b \left[x_1(t)\mathbf{i} + y_1(t)\mathbf{j}\right]dt + \int_a^b \left[x_2(t)\mathbf{i} + y_2(t)\mathbf{j}\right]dt$

$= \int_a^b \mathbf{r}_1(t)\,dt + \int_a^b \mathbf{r}_2(t)\,dt$

15) 1200

16) $\mathbf{T} = (-\sin t)\mathbf{j} + (\cos t)\mathbf{k}$

17) $\sqrt{11}e^t - \sqrt{11}$

18) $\frac{5}{2}\pi$

19) $\mathbf{N} = \frac{1}{\sqrt{t^2 + 1}}\mathbf{j} - \frac{t}{\sqrt{t^2 + 1}}\mathbf{k}$

20) $\kappa = \frac{1}{9}$

21) $\kappa = \cos t$

22) $\kappa = \frac{1}{2(t^2 + 1)^{3/2}}$

23) $\kappa = \frac{\text{sech}^2\, t}{2}$

24) $\mathbf{a} = (\sec t \tan t)\mathbf{T} + (\sec t)\mathbf{N}$

25) $\mathbf{a} = \frac{2t}{\sqrt{t^2 + 1}}\mathbf{T} + \frac{2}{\sqrt{t^2 + 1}}\mathbf{N}$

26) $\mathbf{a} = 2\mathbf{T} + 2t\mathbf{N}$