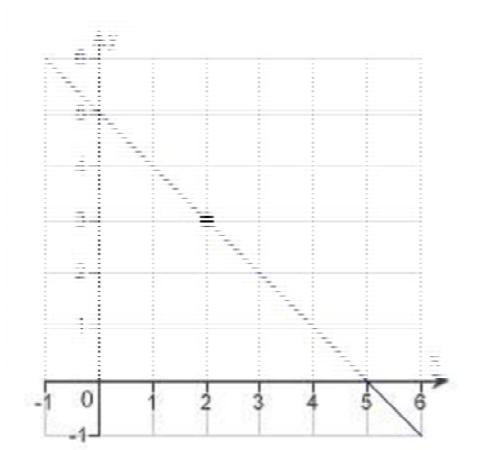


Math 2413 Final exam**Multiple Choice**

Identify the choice that best completes the statement or answers the question.

1. Determine the following limit. (Hint: Use the graph to calculate the limit.)

$$\lim_{x \rightarrow 2} (5 - x)$$



- a. 7
b. 2
c. 5
d. 3
e. does not exist
2. Find the value of c guaranteed by the Intermediate Value Theorem.

$$f(x) = \frac{x^2 + 29x}{x - 1}, \left[\frac{11}{2}, 22 \right], f(c) = 44$$

- a. 11
b. 3
c. 4
d. 12
e. 13

3. Find the derivative of the following function using the limiting process.

$$f(x) = \frac{1}{x^3}$$

- a. $f'(x) = -\frac{4}{x^4}$
b. $f'(x) = -\frac{3}{x^2}$
c. $f'(x) = -\frac{3}{x^4}$
d. $f'(x) = \frac{3}{x^4}$
e. $f'(x) = \frac{3}{x^2}$
4. Find the following limit (if it exists). Write a simpler function that agrees with the given function at all but one point.

$$\lim_{x \rightarrow -9} \frac{x^2 + 5x - 36}{x + 9}$$

- a. -13
b. 5
c. 13
d. -5
e. does not exist
5. Find the limit.

$$\lim_{x \rightarrow 4} \frac{\sqrt{x+5}}{x-1}$$

- a. 3
b. -1
c. -3
d. 1
e. 9

6. Find the x -values (if any) at which $f(x) = \frac{|x-2|}{x-2}$ is not continuous.
- $f(x)$ is not continuous at $x = 2$ and the discontinuity is removable.
 - $f(x)$ is not continuous at $x = 2$ and the discontinuity is nonremovable.
 - $f(x)$ is not continuous at $x = 0$, -2 and $x = 0$ is a removable discontinuity.
 - $f(x)$ is not continuous at $x = 0$ and the discontinuity is removable.
 - $f(x)$ is continuous for all real x .
7. Consider the function $f(x) = 7x - x^2$ and the point $P(1,6)$ on the graph of f . Estimate the slope of the tangent line of f at $P(1,6)$.
- 5
 - 6
 - 9
 - 7
 - 8
8. Find the slope m of the line tangent to the graph of the function $g(x) = 7 - x^2$ at the point $(5, -18)$.
- $m = -14$
 - $m = 5$
 - $m = -18$
 - $m = -10$
 - $m = 7$
9. Find the value of c guaranteed by the Intermediate Value Theorem.
- $$f(x) = x^2 - 4x + 4, [4, 8], f(c) = 9$$
- 1
 - 6
 - 0
 - 7
 - 5
10. Find the vertical asymptotes (if any) of the function $f(x) = \tan(11x)$.
- $x = \frac{2k+1}{11} \pi$ ($k = 0, \pm 1, \pm 2, \dots$)
 - $x = \frac{2k+1}{22} \pi$ ($k = 0, \pm 1, \pm 2, \dots$)
 - $x = \frac{2k}{11} \pi$ ($k = 0, \pm 1, \pm 2, \dots$)
 - $x = \frac{k}{11} \pi$ ($k = 0, \pm 1, \pm 2, \dots$)
 - no vertical asymptotes
11. Find the point of inflection of the graph of the function $f(x) = \sin \frac{x}{9}$ on the interval $[0, 18\pi]$.
- $(\pi, 0)$
 - $(9\pi, 1)$
 - $(0, 0)$
 - $(10\pi, 0)$
 - $(9\pi, 0)$
12. A ring has a inner circumference of 11 centimeters. What is the radius of the ring? Round your answer to four decimal places.
- 12.2599 centimeters
 - 1.7507 centimeters
 - 0.8754 centimeter
 - 1.8712 centimeters
 - 3.5014 centimeters

13. Find the derivative of the function.

$$y = \cos(3x^4 + 8)$$

- a. $y' = 12 \sin(3x^4 + 8)$
- b. $y' = -3 \sin(3x^4 + 8)$
- c. $y' = -12x^3 \sin(3x^4 + 8)$
- d. $y' = 12x^4 \cos(3x^4 + 8)$
- e. $y' = -12 \sin(3x^4 + 8)$

14. Find the slope
- m
- of the line tangent to the graph of the function
- $f(x) = 5 - 3x$
- at the point
- $(-1, 8)$
- .

- a. $m = -5$
- b. $m = 5$
- c. $m = 3$
- d. $m = -3$
- e. $m = -8$

15. Find the slope of the graph of the function at the given value.

$$f(x) = 2x^2 + 3x - \frac{6}{x^2} \text{ when } x = -5$$

- a. $f'(-5) = \frac{2137}{5}$
- b. $f'(-5) = \frac{2137}{125}$
- c. $f'(-5) = \frac{2137}{25}$
- d. $f'(-5) = -\frac{2137}{125}$
- e. $f'(-5) = \frac{2137}{25}$

16. Find the slope of the graph of the function at the given value.

$$f(x) = 5x^3 - 9x^2 \text{ at } x = 3$$

- a. $f'(3) = 189$
- b. $f'(3) = 351$
- c. $f'(3) = 297$
- d. $f'(3) = 81$
- e. $f'(3) = 21$

17. Use the alternative form of the derivative to find the derivative of the function
- $f(x) = \frac{7}{x^4}$
- at
- $x = 2$
- .

- a. $f'(2) = -\frac{35}{64}$
- b. $f'(2) = -\frac{7}{8}$
- c. $f'(2) = -\frac{7}{2}$
- d. $f'(2) = \frac{7}{16}$
- e. $f'(2) = \frac{7}{32}$

18. Differentiate the function
- $f(x) = \ln(6x^2 + 5x + 13)$
- .

- a. $\frac{1}{6x^2 + 5x + 13}$
- b. $\frac{12x}{6x^2 + 5x + 13}$
- c. $\frac{12x - 5}{6x^2 + 5x + 13}$
- d. $\frac{12x + 5}{6x^2 + 5x + 13}$
- e. $\frac{12x + 5}{(6x^2 + 5x + 13)^2}$

19. Find the derivative of the function

$$y = \ln\left(x\sqrt{x^2 + 11}\right).$$

- a. $\frac{1}{x} + \frac{x}{x^2 + 11}$
 b. $\frac{x}{x + \sqrt{x^2 + 11}}$
 c. $\frac{x}{\sqrt{x^2 + 11}} \ln(x) + \frac{1}{x} \ln\left(\sqrt{x^2 + 11}\right)$
 d. $\frac{1}{x\sqrt{x^2 + 11}}$
 e. $\frac{1}{x} + \frac{1}{\sqrt{x^2 + 11}}$

20. Find the points of inflection and discuss the concavity of the function
- $f(x) = x\sqrt{x + 14}$
- .

- a. no inflection points; concave up on $(-14, \infty)$
 b. inflection point at $x = 0$; concave up on $(-14, 0)$; concave down on $(0, \infty)$
 c. no inflection points; concave down on $(-14, \infty)$
 d. inflection point at $x = 14$; concave up on $(-14, \infty)$
 e. inflection point at $x = 14$; concave down on $(-14, \infty)$

21. The radius
- r
- of a sphere is increasing at a rate of 2 inches per minute. Find the rate of change of the volume when
- $r = 8$
- inches.

- a. $\frac{dV}{dt} = 256\pi \text{ in}^3 / \text{min}$
 b. $\frac{dV}{dt} = 512\pi \text{ in}^3 / \text{min}$
 c. $\frac{dV}{dt} = \frac{1}{256\pi} \text{ in}^3 / \text{min}$
 d. $\frac{dV}{dt} = \frac{1}{512\pi} \text{ in}^3 / \text{min}$
 e. $\frac{dV}{dt} = 640\pi \text{ in}^3 / \text{min}$

22. Find
- $f^{-1}(x)$
- if
- $f(x) = 6\sqrt[3]{8x - 7}$
- .

- a. $f^{-1}(x) = \frac{1}{6} \left(\left(\frac{x}{6} \right)^3 + 7 \right)$
 b. $f^{-1}(x) = \frac{1}{8} \left(\left(\frac{x}{6} \right)^3 + 7 \right)$
 c. $f^{-1}(x) = \frac{1}{8} \left(\left(\frac{x}{6} \right)^3 - 7 \right)$
 d. $f^{-1}(x) = \frac{1}{6} (8x - 7)^3$
 e. $f^{-1}(x)$ does not exist

23. Find the relative extremum of
- $f(x) = -7x^2 + 42x + 9$
- by applying the First Derivative Test.

- a. relative maximum: $(3, 72)$
 b. relative minimum: $(4, 65)$
 c. relative minimum: $(2, 65)$
 d. relative minimum: $(-4, -271)$
 e. relative maximum: $(-3, -180)$

24. Find all relative extrema of the function $f(x) = x^2 + 8x + 17$. Use the Second Derivative Test where applicable.

- a. relative min: $f(0) = 17$; no relative max
- b. relative max: $f(-4) = 1$; no relative min
- c. no relative min; no relative max
- d. relative max: $f(-4) = 1$; relative min: $f(0) = 17$
- e. relative max: $f(-4) = 1$; relative min: $f(0) = 17$

25. Write the following expression as a logarithm of a single quantity.

$$6 \ln x - 15 \ln(x^2 + 2)$$

- a. $\ln(6x - 15(x^2 + 2))$
- b. $\ln(x^6 - (x^2 + 2)^{15})$
- c. $\ln\left(\frac{x^6}{(x^2 + 2)^{15}}\right)$
- d. $\ln(x^6(x^2 + 2)^{15})$
- e. $\ln\left(\frac{x^6}{15(x^2 + 2)}\right)$

26. Find $\int \tan 7\theta \, d\theta$.

- a. $\int \tan 7\theta \, d\theta = \frac{1}{7} \ln|\cos 7\theta| + C$
- b. $\int \tan 7\theta \, d\theta = -\frac{1}{7} \ln|\sec 7\theta| + C$
- c. $\int \tan 7\theta \, d\theta = -7 \ln|\cos 7\theta| + C$
- d. $\int \tan 7\theta \, d\theta = -\frac{1}{7} \ln|\cos 7\theta| + C$
- e. $\int \tan 7\theta \, d\theta = -7 \ln|\sec 7\theta| + C$

27. Find $f^{-1}(x)$ if $f(x) = \sqrt{10 - x^2}$, $0 \leq x \leq \sqrt{10}$.

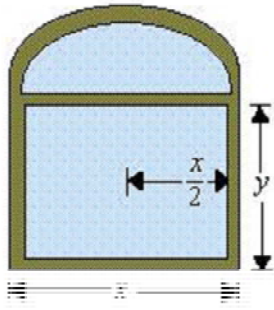
- a. $f^{-1}(x) = x + \sqrt{10}$, $0 \leq x \leq \sqrt{10}$
- b. $f^{-1}(x) = \sqrt{x^2 - 10}$, $0 \leq x \leq \sqrt{10}$
- c. $f^{-1}(x) = \sqrt{10 - x^2}$, $0 \leq x \leq \sqrt{10}$
- d. $f^{-1}(x) = (10 - x^2)^2$, $0 \leq x \leq \sqrt{10}$
- e. $f^{-1}(x) = \frac{1}{\sqrt{10 - x^2}}$, $0 \leq x \leq \sqrt{10}$

28. Evaluate the definite integral.

$$\int_1^7 \frac{1 - \cos(7\theta)}{7\theta - \sin(7\theta)} \, d\theta$$

- a. $\frac{1}{7} \ln \left| \frac{1 - \cos(49)}{7 - \sin(7)} \right|$
- b. $\frac{1}{7} \ln \left| \frac{1 - \sin(49)}{7 - \sin(7)} \right|$
- c. $\frac{1}{7} \ln \left| \frac{49 - \sin(49)}{7 - \sin(7)} \right|$
- d. $\frac{1}{7} \ln \left| \frac{7 - \cos(49)}{7 - \sin(7)} \right|$
- e. $\frac{1}{49} \ln \left| \frac{49 - \sin(49)}{7 - \sin(7)} \right|$

29. A Norman window is constructed by adjoining a semicircle to the top of an ordinary rectangular window (see figure). Find the dimensions of a Norman window of maximum area if the total perimeter is 36 feet.



- a. $x = \frac{72}{4 + \pi}$ feet; $y = \frac{36}{4 + \pi}$ feet
- b. $x = \frac{36}{4 + \pi}$ feet; $y = \frac{108}{4 + \pi}$ feet
- c. $x = \frac{108}{2 + \pi}$ feet; $y = \frac{36}{2 + \pi}$ feet
- d. $x = \frac{36}{4 + \pi}$ feet; $y = \frac{72}{4 + \pi}$ feet
- e. $x = \frac{72}{2 + \pi}$ feet; $y = \frac{36}{2 + \pi}$ feet

30. Evaluate the following definite integral by the limit definition.

$$\int_3^8 -12x^2 dx$$

- a. $\frac{-1940}{3}$
 - b. -660
 - c. $\frac{4}{3}$
 - d. -1940
 - e. -1941
31. What is the domain of the function $f(x) = 15 \ln(8x)$?
- a. $(0, 1)$
 - b. $(0, \infty)$
 - c. (e, ∞)
 - d. $\left(\frac{1}{8}, \infty\right)$
 - e. $(1, e)$
32. Use the Horizontal Line Test to determine whether the following statement is true or false.

The function $f(x) = \frac{8}{13}x + 8$ is one-to-one on its entire domain and therefore has an inverse function.

- a. false
- b. true

Name: _____

ID: A

33. Find the differential dy of the function
 $y = x \sin(5x)$.

- a. $\sin(5x) - 5x \cos(5x) dx$
- b. $\sin(5x) dx$
- c. $-5x \cos(5x) dx$
- d. $5x \cos(5x) dx$
- e. $\sin(5x) + 5x \cos(5x) dx$

Math 2413 Final exam

Answer Section

MULTIPLE CHOICE

1. ANS: D PTS: 1 DIF: Easy REF: Section 1.2
OBJ: Estimate the limit of a function from its graph MSC: Skill
2. ANS: A PTS: 1 DIF: Medium REF: Section 1.4
OBJ: Identify the value of c guaranteed by the Intermediate Value Theorem
MSC: Skill
3. ANS: C PTS: 1 DIF: Medium REF: Section 2.1
OBJ: Calculate the derivative of a function by the limit process MSC: Skill
4. ANS: A PTS: 1 DIF: Medium REF: Section 1.3
OBJ: Evaluate the limit of the function and simplify it to an identical function except at the discontinuity point
MSC: Skill
5. ANS: D PTS: 1 DIF: Medium REF: Section 1.3
OBJ: Evaluate a limit using properties of limits MSC: Skill
6. ANS: B PTS: 1 DIF: Medium REF: Section 1.4
OBJ: Identify the removable discontinuities of a function MSC: Skill
7. ANS: A PTS: 1 DIF: Medium REF: Section 1.1
OBJ: Calculate the slope of secant line passing through the given points
MSC: Skill
8. ANS: D PTS: 1 DIF: Medium REF: Section 2.1
OBJ: Calculate the slope of a line tangent to the graph of a function at a specified point
MSC: Skill
9. ANS: E PTS: 1 DIF: Easy REF: Section 1.4
OBJ: Identify the value of c guaranteed by the Intermediate Value Theorem
MSC: Skill
10. ANS: B PTS: 1 DIF: Medium REF: Section 1.5
OBJ: Identify the vertical asymptotes (if any) of the graph of a function
MSC: Skill
11. ANS: E PTS: 1 DIF: Easy REF: Section 3.4
OBJ: Identify all points of inflection for a function MSC: Skill
12. ANS: B PTS: 1 DIF: Easy REF: Section 1.2
OBJ: Solve a linear equation in applications MSC: Application
13. ANS: C PTS: 1 DIF: Medium REF: Section 2.4
OBJ: Differentiate a trigonometric function using the chain rule
MSC: Skill
14. ANS: D PTS: 1 DIF: Easy REF: Section 2.1
OBJ: Calculate the slope of a line tangent to the graph of a function at a specified point
MSC: Skill
15. ANS: D PTS: 1 DIF: Medium REF: Section 2.2
OBJ: Calculate the slope of the graph of a function at a given point
MSC: Skill
16. ANS: D PTS: 1 DIF: Easy REF: Section 2.2
OBJ: Calculate the slope of the graph of a function at a given point
MSC: Skill

17. ANS: B PTS: 1 DIF: Medium REF: Section 2.1
OBJ: Calculate the derivative of a function at a specified point using the alternative form
MSC: Skill
18. ANS: D PTS: 1 DIF: Medium REF: Section 5.1
OBJ: Differentiate a logarithmic function using the chain rule MSC: Skill
19. ANS: A PTS: 1 DIF: Medium REF: Section 5.1
OBJ: Differentiate a logarithmic function using the chain rule and product rule
MSC: Skill
20. ANS: A PTS: 1 DIF: Medium REF: Section 3.4
OBJ: Identify all points of inflection for a function and discuss the concavity
MSC: Skill
21. ANS: B PTS: 1 DIF: Easy REF: Section 2.6
OBJ: Solve a related rate problem involving the volume of a sphere and its
MSC: Application
22. ANS: B PTS: 1 DIF: Medium REF: Section 5.3
OBJ: Construct the inverse of a function MSC: Skill
23. ANS: A PTS: 1 DIF: Easy REF: Section 3.3
OBJ: Identify the relative extrema of a function by applying the First Derivative Test
MSC: Skill
24. ANS: B PTS: 1 DIF: Medium REF: Section 3.4
OBJ: Identify all relative extrema for a function using the Second Derivative Test
MSC: Skill
25. ANS: C PTS: 1 DIF: Medium REF: Section 5.1
OBJ: Write a logarithmic expression as a single quantity MSC: Skill
26. ANS: D PTS: 1 DIF: Easy REF: Section 5.2
OBJ: Evaluate the indefinite integral of a function involving logarithms
MSC: Skill
27. ANS: C PTS: 1 DIF: Medium REF: Section 5.3
OBJ: Construct the inverse of a function MSC: Skill
28. ANS: C PTS: 1 DIF: Medium REF: Section 5.2
OBJ: Evaluate the definite integral of a function using logarithms
MSC: Skill
29. ANS: A PTS: 1 DIF: Medium REF: Section 3.7
OBJ: Apply calculus techniques to solve a minimum/maximum problem involving the area of a Norman window
MSC: Application
30. ANS: D PTS: 1 DIF: Easy REF: Section 4.3
OBJ: Evaluate a definite integral by the limit definition MSC: Skill
31. ANS: B PTS: 1 DIF: Easy REF: Section 5.1
OBJ: Identify the domain of a logarithmic function MSC: Skill
32. ANS: B PTS: 1 DIF: Medium REF: Section 5.3
OBJ: Recognize invertible functions MSC: Application
33. ANS: E PTS: 1 DIF: Medium REF: Section 3.9
OBJ: Calculate the differential of y for a given function MSC: Skill