Math2414 Final review

Short Answer

1. Use the shell method to find the volume of the solid generated by revolving the plane region about the line $x = 12$.

   $y = \sqrt[3]{x}$, $y = 0$, $x = 8$

2. Use implicit differentiation to find $\dfrac{dy}{dx}$.

   $7x^2 + 8\ln xy = 14$

3. Use logarithmic differentiation to find $\dfrac{dy}{dx}$.

   $y = \dfrac{5x - 3}{(3x+2)^3}$

4. Find the indefinite integral.

   $\displaystyle\int \frac{x^2 + 16x + 6}{x^3 + 24x^2 + 18x - 1}\,dx$

5. Find the derivative of the function $y = \ln\left(x\sqrt{x^2 + 7}\right)$.

6. Find the derivative of the function $f(x) = \ln\left(\dfrac{3x}{x^2 + 4}\right)$.

Name: ________________________ **ID: A**

7. Find $\int \frac{x^2 - 14x + 10}{x + 12}\, dx$.

8. Find $\int \tan 6\theta \, d\theta$.

9. Evaluate the definite integral $\int_e^{e^5} \frac{1}{x \ln(x)^8}\, dx$.

10. Find $F'(x)$ if $F(x) = \int_1^{14x^2} \frac{1}{t}\, dt$.

11. Differentiate the function $f(x) = \ln\left(\frac{e^{5x} + 1}{e^{2x} + 1}\right)$.

12. Find $f''(x)$ if $f(x) = (7 + 4x)e^{-4x}$.

13. Find the indefinite integral.

$$\int \cos 4x \, e^{\sin 4x} dx$$

14. Use logarithmic differentiation to find $\frac{dy}{dx}$.

$y = x^{8x}$

15. Find the derivative of the function $y = \arctan\left(\frac{x}{7}\right) + \frac{4x-6}{7\left(x^2+4\right)}$.

16. Find the indefinite integral.

$$\int \frac{1}{x\sqrt{64x^2-49}}\,dx$$

17. Find the indefinite integral.

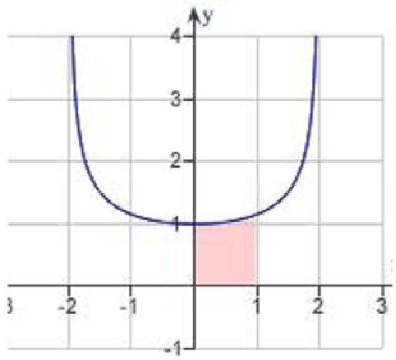
$$\int \frac{1}{36+(x-4)^2}\,dx$$

18. Find the indefinite integral.

$$\int \frac{dx}{\sqrt{-x^2-18x}}$$

19. Find the integral $\int \frac{x-15}{x^2+1}\,dx$.

20. Find the area of the shaded region for the function $y = \dfrac{2}{\sqrt{4-x^2}}$.



21. Evaluate $\sinh(\ln(8))$ and $\cosh(\ln(4))$ in that order.

22. Find the derivative of the function $y = \ln\left(\cosh^4(10x)\right)$.

23. Find the area of the region bounded by equations by integrating (i) with respect to x and (ii) with respect to y.

$y = x^2$

$y = 72 - x$

24. Find the volume of the solid generated by revolving the region bounded by the graphs of the equations $y = 2x^2$, $y = 0$, and $x = 2$ about the line $x = 2$.

25. Find the volume of the solid generated by revolving the region bounded by the graphs of the equations about the line $y = 8$.

    $y = x, y = 7, x = 0$

26. Find the volume of the solid generated by revolving the region bounded by the graphs of the equations about the x-axis.

    $y = \frac{1}{x}, y = 0, x = 8, x = 10$

27. Use partial fractions to find the integral $\int \frac{2x - 7}{x^2 - 2x - 24}\, dx$.

28. Find the fourth degree Maclaurin polynomial for the function.

    $f(x) = \frac{1}{x + 4}$

29. Find the indefinite integral.

$$\int e^{2x} \sin 7x\, dx$$

30. Find the indefinite integral.

$$\int x^3 e^{3x^2}\, dx$$

31. Find the indefinite integral.

$$\int \sin^3 5x \cos^4 5x\, dx$$

Name: ________________________ **ID: A**

32. Evaluate the limit $\lim_{x \to 0} \frac{1-\cos(3x)}{x^2}$ using L'Hopital's Rule if necessary.

33. Find the indefinite integral.

$$\int x^9 \ln x \, dx$$

34. Find the indefinite integral.

$$\int \frac{1}{\left(x^2+4\right)^{1/2}} \, dx$$

35. Find $\int \sin 2x \cos 4x dx$.

36. Use partial fractions to find $\int \frac{10x^2 + 3x + 36}{x^3 + 4x} dx$.

37. Use the Root Test to determine the convergence or divergence of the series $\sum_{n=1}^{\infty} e^{-8n}$.

38. Identify the most appropriate test to be used to determine whether the series $\sum_{n=1}^{\infty} \frac{15(-1)^{n+1}}{n}$ converges or diverges.

39. Find a geometric power series for the function $\frac{1}{3-x}$ centered at 0.

40. Determine the convergence or divergence of the series using any appropriate test from this chapter. Identify the test used.

$$\sum_{n=1}^{\infty} \frac{6n}{n+9}$$

41. Use the Root Test to determine the convergence or divergence of the series.

$$\sum_{n=1}^{\infty} \left(\frac{4n}{3n+1} \right)^n$$

42. Find the sum of the convergent series.

$$\sum_{n=1}^{\infty} \frac{7}{(n+4)(n+6)}$$

43. Find the third degree Maclaurin polynomial for the function.

$$f(x) = \frac{1}{x+4}$$

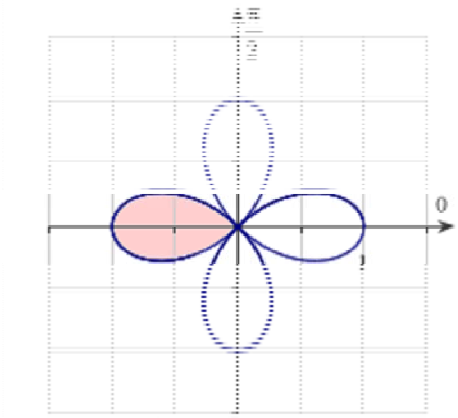
44. Determine the convergence or divergence of the sequence with the given nth term. If the sequence converges, find its limit.

$$a_n = \frac{\ln\left(\sqrt[4]{n}\right)}{7n}$$

45. Write the corresponding rectangular equation for the curve represented by the parametric equations $x = 7 + \frac{2}{t}$, $y = t - 9$ by eliminating the parameter.

Name: ________________________ **ID: A**

46. Write an integral that represents the area of the shaded region for $r = \cos 2\theta$ as shown in the figure. Do not evaluate the integral.



47. Sketch the curve represented by the parametric equations, and write the corresponding rectangular equation by eliminating the parameter.

$x = 16 + 8\cos\theta$

$y = -4 + 4\sin\theta$

48. Find the arc length of the curve on the given interval.

$x = t^2,\ y = 8t,\ 0 \le t \le 4$

49. Find the area of one petal of $r = 13\cos 3\theta$.

50. Find all points (if any) of horizontal and vertical tangency to the curve $x = t + 6,\ y = t^3 - 27t$.

Name: ________________________ **ID: A**

51. Evaluate the limit $\lim_{x \to 0} \dfrac{\arcsin(6x)}{11x}$ using L'Hopital's Rule if necessary.

52. Find $(F^{-1})'(0)$ if $F(x) = \int_4^x \sqrt{t^2 + 1}\, dt$.

53. Use the definition to find the Taylor series centered at $c = 1$ for the function $f(x) = \dfrac{1}{x}$.

54. Consider the function given by $f(x) = \sum_{n=0}^{\infty} \left(\dfrac{x}{9}\right)^n$. Find the interval of convergence for $\int f(x)\, dx$.

55. Consider the function given by $f(x) = \sum_{n=1}^{\infty} \dfrac{(-1)^{n+1}(x-6)^n}{n}$. Find the interval of convergence for $f'(x)$.

56. Find a power series for the function $\dfrac{1}{12-x}$ centered at 1.

Math2414 Final review
Answer Section

SHORT ANSWER

1. $V = \dfrac{1{,}248}{7}\pi$
2. $-\dfrac{y\left(7x^2+4\right)}{4x}$
3. $\dfrac{-30x+37}{(3x+2)^4}$
4. $\dfrac{1}{3}\ln\left|x^3+24x^2+18x-1\right|+C$
5. $\dfrac{1}{x}+\dfrac{x}{x^2+7}$
6. $f'(x)=\dfrac{1}{x}-\dfrac{2x}{x^2+4}$
7. $\displaystyle\int \frac{x^2-14x+10}{x+12}\,dx=\frac{1}{2}x^2-26x+322\ln|x+12|+C$
8. $\displaystyle\int \tan 6\theta\, d\theta=-\frac{1}{6}\ln|\cos 6\theta|+C$
9. $\dfrac{1}{8}\ln(5)$
10. $F'(x)=\dfrac{2}{x}$
11. $\dfrac{5e^{5x}}{e^{5x}+1}-\dfrac{2e^{2x}}{e^{2x}+1}$
12. $f''(x)=(80+64x)e^{-4x}$
13. $\dfrac{1}{4}e^{\sin 4x}+C$
14. $8x^{8x}(\ln x+1)$
15. $\dfrac{dy}{dx}=\dfrac{1}{7}\cdot\left(\dfrac{1}{1+(x/7)^2}+\dfrac{16+12x-4x^2}{\left(x^2+4\right)^2}\right)$
16. $\dfrac{1}{7}\operatorname{arcsec}\left(\dfrac{|8|x}{7}\right)+C$
17. $\dfrac{1}{6}\arctan\left(\dfrac{x-4}{6}\right)+C$

18. $\arcsin\left(\dfrac{x+9}{9}\right)+C$

19. $\dfrac{1}{2}\ln\left(x^2+1\right)-15\arctan(x)+C$

20. $\dfrac{\pi}{3}$

21. $\dfrac{63}{16},\dfrac{17}{8}$

22. $\dfrac{dy}{dx}=40\tanh(10x)$

23. $A=\dfrac{4913}{6}$

24. $\dfrac{16}{3}\pi$

25. $\dfrac{490}{3}\pi$

26. $\dfrac{1}{40}\pi$

27. $8\ln|x-3|+3\ln|x-8|+C$

28. $\dfrac{1}{4}-\dfrac{1}{16}x+\dfrac{1}{64}x^2-\dfrac{1}{256}x^3+\dfrac{1}{1024}x^4$

29. $\left(\dfrac{2\sin 7w-7\cos 7w}{53}\right)e^{2w}+C$

30. $\dfrac{1}{18}e^{3x^2}\left(3x^2-1\right)+C$

31. $-\dfrac{1}{175}\left(7-5\cos^2 5x\right)\cos^5 5x+C$

32. $\dfrac{27}{4}$

33. $\dfrac{x^{10}}{100}\left[10\ln(x)-1\right]+C$

34. $\dfrac{x}{8\sqrt{x^2+8}}+C$

35. $\int \sin 2x\cos 4x\,dx=\dfrac{1}{12}(3\cos 2x-\cos 6x)+C$

36. $\int \dfrac{8x^2+55x-8}{x^2(x+8)}dx=\dfrac{1}{x}+\ln\left|x^8+8x^7\right|+C$

37. converges
38. Alternating Series Test

39. $\sum_{n=0}^{\infty} \frac{x^n}{3^{n+1}}$
40. diverges; Theorem 9.9 (n^{th} Term Test for Divergence)
41. diverges
42. $\frac{77}{60}$
43. $\frac{1}{4} - \frac{1}{16}x + \frac{1}{64}x^2 - \frac{1}{256}x^3 + \frac{1}{1024}x^4$
44. The sequence converges to 0.
45. $y = \frac{2}{x-7} - 9$
46. $\frac{1}{2}\int_{3\pi/4}^{5\pi/4} (\cos 2\theta)^2 \, d\theta$
47. none of the above
48. $16\left(\sqrt{2} - \ln\sqrt{2} + \ln\left(2 + \sqrt{2}\right)\right)$
49. $\frac{169\pi}{12}$
50. horizontal tangents: $(9,-54), (3,54)$, vertical tangent: none
51. $\frac{6}{11}$
52. $F'(x) = 5$
53. $\frac{1}{x} = \sum_{n=0}^{\infty} (-1)^n (x-1)^n$
54. $[-9,9)$
55. $(5,7)$
56. $\sum_{n=0}^{\infty} \frac{(x-1)^n}{11^{n+1}}$