Math 2414 Formula Sheet

1. $\int \frac{du}{\sqrt{a^2-u^2}} = \arcsin\frac{u}{a} + C$

2. $\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a} arcsec\frac{|u|}{a} + C$

3. $\int \frac{du}{a^2+u^2} = \frac{1}{a} arctan\frac{u}{a} + C$

4. $\sinh x = \frac{e^x - e^{-x}}{2}$
5. $\cosh x = \frac{e^x + e^{-x}}{2}$

6. $V_{shell} = 2\pi \int_a^b r(x)h(x)dx$
7. $V_{disk} = \pi \int_a^b [R(x)]^2 dx$
8. $V_{washer} = \pi \int_a^b ([R(x)]^2 - [r(x)]^2)dx$
9. Arc Length: $s = \int_a^b \sqrt{1 + \left(f'(x)\right)^2}\,dx$
10. Surface Area: $S = 2\pi \int_a^b r(x)\sqrt{1 + \left(f'(x)\right)^2}\,dx$

11. $sin^2 x = \frac{1 - cos\, 2x}{2}$
12. $cos^2 x = \frac{1 + cos\, 2x}{2}$

13. $\sin A \cos B = \frac{1}{2}[sin\,(A-B) + sin(A+B)]$
14. $\sin A \sin B = \frac{1}{2}[cos(A-B) - cos(A+B)]$
15. $\cos A \cos B = \frac{1}{2}[cos(A-B) + cos(A+B)]$
16. For integrals involving $\sqrt{a^2 - u^2}$: $Let\ u = a\ sin\ \theta$
17. For integrals involving $\sqrt{a^2 + u^2}$: $Let\ u = a\ tan\ \theta$
18. For integrals involving $\sqrt{u^2 - a^2}$: $Let\ u = a\ sec\ \theta$
19. Arc Length in parametric form: $s = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$
20. Slope in parametric form: $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$
21. Area in polar form: $A = \frac{1}{2}\int_\alpha^\beta r^2 d\theta$
22. Taylor Series for a function centered at x = c:

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(c)}{n!}(x-c)^n = f(c) + f'(c)(x-c) + \frac{f''(c)(x-c)^2}{2!} + \frac{f^{(3)}(c)(x-c)^3}{3!} + \cdots$$

1. Convergence and Divergence Tests for Series

Test	When to Use	Conclusions
Divergence Test	for any series $\sum_{n=0}^{\infty} a_n$	Diverges if $\lim_{n\to\infty} \lvert a_n\rvert \neq 0$.
Integral Test	$\sum_{n=0}^{\infty} a_n$ with $a_n \geq 0$ and a_n decreasing	$\int_1^{\infty} f(x)dx$ and $\sum_{n=0}^{\infty} a_n$ both converge/diverge where $f(n) = a_n$.
Comparison Test	$\sum_{n=0}^{\infty} a_n$ and $\sum_{n=0}^{\infty} b_n$ if $0 \leq a_n \leq b_n$	$\sum_{n=0}^{\infty} b_n$ converges $\Longrightarrow \sum_{n=0}^{\infty} a_n$ converges. $\sum_{n=0}^{\infty} a_n$ diverges $\Longrightarrow \sum_{n=0}^{\infty} b_n$ diverges.
Limiting Comparison Test	$\sum_{n=0}^{\infty} a_n, (a_n > 0)$. Choose $\sum_{n=0}^{\infty} b_n, (b_n > 0)$ if $\lim_{n\to\infty} \frac{a_n}{b_n} = L$ with $0 < L < \infty$	$\sum_{n=0}^{\infty} a_n$ and $\sum_{n=0}^{\infty} b_n$ both converge/diverge
	if $\lim_{n\to\infty} \frac{a_n}{b_n} = 0$	$\sum_{n=0}^{\infty} b_n$ converges $\Longrightarrow \sum_{n=0}^{\infty} a_n$ converges.
	if $\lim_{n\to\infty} \frac{a_n}{b_n} = \infty$	$\sum_{n=0}^{\infty} b_n$ diverges $\Longrightarrow \sum_{n=0}^{\infty} a_n$ diverges.
Convergent test for alternating Series	$\sum_{n=0}^{\infty} (-1)^n a_n \quad (a_n > 0)$	converges if $\lim_{n\to\infty} a_n = 0$ and a_n is decreasing
Absolute Convergence	for any series $\sum_{n=0}^{\infty} a_n$	If $\sum_{n=0}^{\infty} \lvert a_n\rvert$ converges, then $\sum_{n=0}^{\infty} a_n$ converges, (definition of absolutely convergent series.)
Conditional Convergence	for any series $\sum_{n=0}^{\infty} a_n$	if $\sum_{n=0}^{\infty} \lvert a_n\rvert$ diverges but $\sum_{n=0}^{\infty} a_n$ converges. $\sum_{n=0}^{\infty} a_n$ conditionally converges
Ratio Test: Root Test:	For any series $\sum_{n=0}^{\infty} a_n$, Calculate $\lim_{n\to\infty} \left\lvert \frac{a_{n+1}}{a_n} \right\rvert = L$ Calculate $\lim_{n\to\infty} \sqrt[n]{\lvert a_n\rvert} = L$	there are 3 cases: if $L < 1$, then $\sum_{n=0}^{\infty} \lvert a_n\rvert$ converges ; if $L > 1$, then $\sum_{n=0}^{\infty} \lvert a_n\rvert$ diverges; if $L = 1$, no conclusion can be made.

2. Important Series to Remember

Series	How do they look	Conclusions
Geometric Series	$\sum_{n=0}^{\infty} ar^n$	Converges to $\frac{a}{1-r}$ if $\lvert r\rvert < 1$ and diverges if $\lvert r\rvert \geq 1$
p-series	$\sum_{n=1}^{\infty} \frac{1}{n^p}$	Converges if $p > 1$ and diverges if $p \leq 1$