

Test Two Review Cal 2

Short Answer

1. Set up the definite integral that gives the area of the region bounded by the graph of \( y_1 = x^2 + 2x + 1 \) and \( y_2 = 2x + 5 \).

5. Find the area of the region bounded by the graphs of the algebraic functions.

\[
\begin{align*}
  f(x) &= \sqrt[3]{x - 8} \\
  g(x) &= x - 8
\end{align*}
\]

6. Find the area of the region bounded by the graphs of the equations.

\[
\begin{align*}
  f(x) &= \frac{18x}{x^2 + 1}, \quad y = 0, \quad 0 \leq x \leq 9
\end{align*}
\]

7. Find the area of the region bounded by the graphs of the function \( f(x) = \frac{9x}{x^2 + 1}, y = 0, \quad 0 \leq x \leq 3 \).
   Round your answer to three decimal places.

8. Find the area of the region bounded by the graphs of the equations.

\[
\begin{align*}
  f(x) &= \sin(x), \quad g(x) = \cos(2x), \quad \frac{-\pi}{2} \leq x \leq \frac{\pi}{6}
\end{align*}
\]

2. Find the area of the region bounded by the equations by integrating (i) with respect to \( x \) and (ii) with respect to \( y \).

\[
\begin{align*}
  x &= 4 - y^2 \\
  x &= y - 2
\end{align*}
\]

3. Find the area of the region bounded by equations by integrating (i) with respect to \( x \) and (ii) with respect to \( y \).

\[
\begin{align*}
  y &= x^2 \\
  y &= 72 - x
\end{align*}
\]

4. Find the area of the region bounded by the graphs of the algebraic functions.

\[
\begin{align*}
  f(x) &= x^2 + 30x + 225 \\
  g(x) &= 17(x + 15)
\end{align*}
\]
9. The surface of a machine part is the region between the graphs of \( y_1 = |x| \) and \( y_2 = 0.080x^2 + k \) as shown in the figure. Find \( k \) if the parabola is tangent to the graph of \( y_1 \). Round your answer to three decimal places.

10. Find the volume of the solid generated by revolving the region bounded by the graphs of the equations \( y = 10x^2, \ y = 0, \ and \ x = 2 \) about the line \( y = 40 \).

11. Find the volume of the solid generated by revolving the region bounded by the graphs of the equations \( y = 2x^2, \ y = 0, \ and \ x = 2 \) about the line \( x = 2 \).

12. Find the volume of the solid generated by revolving the region bounded by the graphs of the equations about the line \( y = 8 \).

\[ y = x, \ y = 7, \ x = 0 \]

13. Find the volume of the solid generated by revolving the region bounded by the graphs of the equations about the line \( y = 2 \).

\[ y = \frac{1}{2}x^2, \ y = 2, \ x = 0 \]

14. Find the volume of the solid generated by revolving the region bounded by the graphs of the equations about the line \( y = 14 \).

\[ y = \sin(x), \ y = 0, \ 0 \leq x \leq \frac{\pi}{2} \]

15. Find the volume of the solid generated by revolving the region bounded by the graphs of the equations about the \( x \)-axis.

\[ y = \frac{1}{\sqrt{x + 13}}, \ y = 0, \ x = 0, \ x = 9 \]

16. Find the volume of the solid generated by revolving the region bounded by the graphs of the equations about the \( x \)-axis.

\[ y = \frac{1}{x}, \ y = 0, \ x = 8, \ x = 10 \]

17. Find the volume of the solid generated by revolving the region bounded by the graphs of the equations about the \( x \)-axis. Verify your results using the integration capabilities of a graphing utility.

\[ y = \sin(x), \ y = 0, \ x = 0, \ x = \frac{\pi}{3} \]

18. A tank on the wing of a jet aircraft is formed by revolving the region bounded by the graph of \( y = \frac{1}{15}x^2 \sqrt{2 - x} \) and the \( x \)-axis \((0 \leq x \leq 2)\) about the \( x \)-axis, where \( x \) and \( y \) are measured in meters. Find the volume of the tank. Round your answer to two decimal places.
19. Use the shell method to set up and evaluate the integral \( y = 3\sqrt{x} \) that gives the volume of the solid generated by revolving the plane region about the \( y \)-axis.

\[ y = 25 - x^2, \ y = 0 \]

20. Use the shell method to set up and evaluate an integral that gives the volume of the solid generated by revolving the plane region about the \( y \)-axis.

\[ y = 25 - x^2, \ y = 0 \]

21. Use the shell method to set up and evaluate the integral that gives the volume of the solid generated by revolving the plane region bounded by \( y = \sqrt{x} + 20, \ y = x, \ y = 0 \) about the \( x \)-axis.

22. Use the shell method to find the volume of the solid generated by revolving the plane region bounded by \( y = 4x^2, \ y = 10x - x^2, \) about the line \( x = 2 \).

23. Use the disk or shell method to find the volume of the solid generated by revolving the region bounded by the graphs of the equations about the line \( x = 22 \).

\[ y = x^3, \ y = 0, \ x = 11 \]

24. Use the disk or the shell method to find the volume of the solid generated by revolving the region bounded by the graphs of the equations \( y = 2x^3, \ y = 0, \ x = 3 \) about the \( x \)-axis.

25. Use the disk or the shell method to find the volume of the solid generated by revolving the region bounded by the graphs of the equations \( y = 2x^3, \ y = 0, \ x = 3 \) about the line \( x = 8 \).

26. Find the arc length of the graph of the function \( y = \frac{2}{3}x^\frac{3}{2} + 2 \) over the interval \([14, 16]\).

27. Find the arc length of the graph of the function \( y = \frac{x^3}{6} + \frac{1}{2x} \) over the interval \([1, 2]\).

28. Find the arc length of the graph of the function \( x = \frac{1}{3}(y^2 + 2)^{\frac{3}{2}} \) over the interval \( 0 \leq y \leq 11 \).

29. Find the arc length of the graph of the function \( x = \frac{1}{3}(y - 3)\sqrt{y} \) over the interval \( 1 \leq y \leq 144 \).

30. A barn is 75 feet long and 50 feet wide. A cross section of the roof is the inverted catenary

\[ y = 41 - 15 \left( e^{\frac{x}{30}} + e^{-\frac{x}{30}} \right) \]. Find the number of square feet of roofing on the barn. Round your answer to the nearest integer.
31. Find the area of the surface generated by revolving the curve about the $x$-axis.

$$ y = \frac{1}{7} x^3, \ 0 \leq x \leq 7. $$

32. Set up and evaluate the definite integral for the area of the surface formed by revolving the graph of

$$ y = \frac{x}{2}, \ 1 \leq x \leq 7 \text{ about the } x\text{-axis.} $$

33. Set up and evaluate the definite integral for the area of the surface formed by revolving the graph of

$$ y = 9 - x^2 \text{ about the } y\text{-axis.} $$

Round your answer to three decimal places.

34. Determine the work done by lifting a 100 pound bag of sugar 13 feet.

35. A force of 10 pounds compresses a 21-inch spring 8 inches. How much work is done in compressing the spring from a length of 15 inches to a length of 13 inches?

36. A tank with a base of 4 feet by 3 feet and a height of 4 feet is full of water. The water weighs 62.4 pounds per cubic foot. How much work is done in pumping water out over the top edge in order to empty half of the tank. Round your answer to one decimal place.

37. A cylindrical water tank 5 meters high with a radius of 2 meters is buried so that the top of the tank is 1 meter below ground level. How much work is done in pumping a full tank of water up to ground level? (The water weighs 9800 newtons per cubic meter.)

38. Two electrons repel each other with a force that varies inversely as the square of the distance between them, where $k$ is the constant of proportionality. One electron is fixed at the point $(2, 2)$. Find the work done in moving the second electron from $(-4, 2)$ to $(1, 2)$.

39. Find the center of mass of the point masses lying on the $x$-axis.

$$ m_1 = 10, \ m_2 = 1, \ m_3 = 7 $$

$$ x_1 = 3, \ x_2 = 10, \ x_3 = 4 $$
40. Find the center of mass of the point masses lying on the \(x\)-axis.

\[m_1 = 10, m_2 = 1, m_3 = 6, m_4 = 5\]
\[x_1 = 2, x_2 = -10, x_3 = -7, x_4 = -8\]

41. Find the center of mass of the point masses lying on the \(x\)-axis.

\[m_1 = 7, m_2 = 10, m_3 = 8, m_4 = 10, m_5 = 8\]
\[x_1 = -9, x_2 = -2, x_3 = -6, x_4 = 9, x_5 = 6\]

43. Find the center of mass of the given system of point masses.

\[
\begin{array}{c|c|c|c}
  m_i & 9 & 4 & 7 \\
  (x_i, y_i) & (-1, 0) & (7, -3) & (1, -9) \\
\end{array}
\]

44. Find the center of mass of the given system of point masses.

\[
\begin{array}{c|c|c|c|c|c}
  m_i & 10 & 1 & 9 & 3 \\
  (x_i, y_i) & (-8, 8) & (1, 8) & (2, -10) & (6, 6) \\
\end{array}
\]

45. Find \((\bar{x}, \bar{y})\) for the lamina of uniform density \(\rho\) bounded by the graphs of the equations \(y = \sqrt{x}, y = 0, x = 4\).

46. Find \(M_x\) for the lamina of uniform density \(\rho\) bounded by the graphs of the equations \(y = 17x^2\) and \(y = 17x^3\).

47. Find \(M_x, M_y, \) and \((\bar{x}, \bar{y})\) for the lamina of uniform density \(\rho\) bounded by the graphs of the equations \(\frac{3}{4} y = x^4, y = 0, x = 2^4\).

42. Consider a beam of length \(L = 12\) feet with a fulcrum \(x\) feet from one end as shown in the figure. Two objects weighing 36 pounds and 108 pounds are placed at opposite ends of the beam. Find \(x\) (the distance between the fulcrum and the object weighing 36 pounds) such that the system is equilibrium.

48. Find \((\bar{x}, \bar{y})\) for the lamina of uniform density \(\rho\) bounded by the graphs of the equations \(x = 9 - y^2\) and \(x = 0\).

49. Find \(M_x\) for the lamina of uniform density \(\rho\) bounded by the graphs of the equations \(x = 4y + 12\) and \(x = y^2\).

50. Set up and evaluate integrals for finding the moment about the \(y\)-axis for the region bounded by the graphs of the equations. (Assume \(\rho = 1\))

\[y = 64 - x^2, y = 0.\]
Test Two Review Cal 2
Answer Section

SHORT ANSWER

1. ANS:
\[ \int_{-2}^{2} \left(-x^2 + 4\right) \, dx \]
PTS: 1  DIF: Easy  REF: 7.1.2
OBJ: Write the definite integrals needed to calculate the area of a bounded region
MSC: Skill  NOT: Section 7.1

2. ANS:
\[ \frac{125}{6} \]
PTS: 1  DIF: Medium  REF: 7.1.17b
OBJ: Calculate the area of a region bounded by two curves
MSC: Application  NOT: Section 7.1

3. ANS:
\[ \frac{4913}{6} \]
PTS: 1  DIF: Medium  REF: 7.1.18a
OBJ: Calculate the area of a region bounded by two curves
MSC: Application  NOT: Section 7.1

4. ANS:
\[ \frac{4913}{6} \]
PTS: 1  DIF: Medium  REF: 7.1.24
OBJ: Calculate the area of a region bounded by two curves
MSC: Application  NOT: Section 7.1

5. ANS:
\[ \frac{1}{2} \]
PTS: 1  DIF: Medium  REF: 7.1.30
OBJ: Calculate the area of a region bounded by two curves
MSC: Application  NOT: Section 7.1

6. ANS:
\[ 9 \ln(82) \]
PTS: 1  DIF: Medium  REF: 7.1.44b
OBJ: Calculate the area of a region bounded by several curves
MSC: Application  NOT: Section 7.1
7. ANS: 10.362
   PTS: 1 DIF: Medium REF: 7.1.44b
   OBJ: Calculate the area of a region bounded by several curves MSC: Application
   NOT: Section 7.1

8. ANS:
   \[ A = \frac{3^{3/2}}{4} \]
   PTS: 1 DIF: Medium REF: 7.1.48
   OBJ: Calculate the area between two curves MSC: Application
   NOT: Section 7.1

9. ANS: 3.125
   PTS: 1 DIF: Medium REF: 7.1.96a
   OBJ: Calculate slopes of tangent lines in applications MSC: Application
   NOT: Section 7.1

10. ANS: \[ \frac{4,480}{3\pi} \]
    PTS: 1 DIF: Difficult REF: 7.2.12c
    OBJ: Calculate the volume using the washer method of the solid formed by revolving a region about a horizontal line MSC: Application NOT: Section 7.2

11. ANS: \[ \frac{16}{3\pi} \]
     PTS: 1 DIF: Difficult REF: 7.2.12d
     OBJ: Calculate the volume using the disk method of the solid formed by revolving a region about a vertical line MSC: Application NOT: Section 7.2

12. ANS: \[ \frac{490}{3\pi} \]
     PTS: 1 DIF: Medium REF: 7.2.15
     OBJ: Calculate the volume using the washer method of the solid formed by revolving a region about a horizontal line MSC: Application NOT: Section 7.2

13. ANS: \[ \frac{64}{15\pi} \]
     PTS: 1 DIF: Medium REF: 7.2.16
     OBJ: Calculate the volume using the disk method of the solid formed by revolving a region about a horizontal line MSC: Application NOT: Section 7.2
14. ANS: \[\pi \left(28 - \frac{\pi}{4}\right)\]

PTS: 1 DIF: Medium REF: 7.2.18
OBJ: Calculate the volume using the washer method of the solid formed by revolving a region about a horizontal line
MSC: Application NOT: Section 7.2

15. ANS: \[\ln \left(\frac{22}{13}\right)\pi\]

PTS: 1 DIF: Medium REF: 7.2.23
OBJ: Calculate the volume using the disk method of the solid formed by revolving a region about the x-axis
MSC: Application NOT: Section 7.2

16. ANS: \[\frac{1}{40}\pi\]

PTS: 1 DIF: Medium REF: 7.2.25
OBJ: Calculate the volume using the disk method of the solid formed by revolving a region about the x-axis
MSC: Application NOT: Section 7.2

17. ANS: \[\frac{1}{6}\pi^2 - \frac{\sqrt{3}}{8}\pi\]

PTS: 1 DIF: Medium REF: 7.2.33
OBJ: Calculate the volume using the disk method of the solid formed by revolving a region about the x-axis
MSC: Application NOT: Section 7.2

18. ANS: 0.03 m³

PTS: 1 DIF: Medium REF: 7.2.63
OBJ: Calculate the volume using the disk method of the solid formed by revolving a region about the x-axis
MSC: Application NOT: Section 7.2

19. ANS: \[\frac{384\pi}{5}\]

PTS: 1 DIF: Easy REF: 7.3.3
OBJ: Calculate the volume using the shell method of the solid formed by revolving a region about the y-axis
MSC: Application NOT: Section 7.3
20. ANS:
\[ V = 2\pi \int_{0}^{5} x (25 - x^2) \, dx = \frac{625}{2} \pi \]

PTS: 1  DIF: Medium  REF: 7.3.8
OBJ: Calculate the volume using the shell method of the solid formed by revolving a region about the y-axis
MSC: Application  NOT: Section 7.3

21. ANS:
\[ \frac{1,625\pi}{6} \]

PTS: 1  DIF: Medium  REF: 7.3.22
OBJ: Calculate the volume using the shell method of the solid formed by revolving a region about the x-axis
MSC: Application  NOT: Section 7.3

22. ANS:
\[ \frac{40\pi}{3} \]

PTS: 1  DIF: Medium  REF: 7.3.26
OBJ: Calculate the volume using the shell method of the solid formed by revolving a region about a vertical line
MSC: Application  NOT: Section 7.3

23. ANS:
\[ \frac{483,153}{5} \pi \]

PTS: 1  DIF: Medium  REF: 7.3.29a
OBJ: Calculate volumes of revolution by choosing an appropriate method
MSC: Application  NOT: Section 7.3

24. ANS:
\[ \frac{8,748\pi}{7} \]

PTS: 1  DIF: Medium  REF: 7.3.29a
OBJ: Calculate volumes of revolution by choosing an appropriate method
MSC: Application  NOT: Section 7.3

25. ANS:
\[ \frac{2,268\pi}{5} \]

PTS: 1  DIF: Difficult  REF: 7.3.29c
OBJ: Calculate volumes of revolution by choosing an appropriate method
MSC: Application  NOT: Section 7.3
26. ANS: 
\[ \frac{2}{3} \left( 17\sqrt{17} - 15\sqrt{15} \right) \]

PTS: 1  DIF: Medium  REF: 7.4.5  
OBJ: Calculate the arc length of a curve over a given interval  
MSC: Application  
NOT: Section 7.4

27. ANS: 
\[ \frac{17}{12} \]

PTS: 1  DIF: Medium  REF: 7.4.9  
OBJ: Calculate the arc length of a curve over a given interval  
MSC: Application  
NOT: Section 7.4

28. ANS: 
\[ \frac{1364}{3} \]

PTS: 1  DIF: Medium  REF: 7.4.15  
OBJ: Calculate the arc length of a curve over a given interval  
MSC: Application  
NOT: Section 7.4

29. ANS: 
\[ \frac{1760}{3} \]

PTS: 1  DIF: Medium  REF: 7.4.16  
OBJ: Calculate the arc length of a curve over a given interval  
MSC: Application  
NOT: Section 7.4

30. ANS: 
4200 square feet

PTS: 1  DIF: Medium  REF: 7.4.32  
OBJ: Calculate arc lengths in applications  
MSC: Application  
NOT: Section 7.4

31. ANS: 
\[ 7 \left( \frac{3}{442^2 - 1} \right) \]  
\[ \pi \]

PTS: 1  DIF: Medium  REF: 7.4.37  
OBJ: Calculate the area of a solid of revolution  
MSC: Application  
NOT: Section 7.4

32. ANS: 
\[ 12\sqrt{5} \pi \]

PTS: 1  DIF: Easy  REF: 7.4.40  
OBJ: Calculate the area of a solid of revolution  
MSC: Application  
NOT: Section 7.4
33. ANS:
   117.319

   PTS: 1   DIF: Medium   REF: 7.4.44   OBJ: Calculate the area of a solid of revolution
   MSC: Application   NOT: Section 7.4

34. ANS:
   1,300 ft-lb

   PTS: 1   DIF: Easy   REF: 7.5.1
   OBJ: Calculate the work done by a constant force   MSC: Application
   NOT: Section 7.5

35. ANS:
   17.5 ft-lb

   PTS: 1   DIF: Medium   REF: 7.5.6
   OBJ: Calculate work in problems involving springs   MSC: Application
   NOT: Section 7.5

36. ANS:
   1,497.6 ft-lb

   PTS: 1   DIF: Medium   REF: 7.5.17
   OBJ: Calculate work in problems involving pumping liquids from containers   MSC: Application
   NOT: Section 7.5

37. ANS:
   686,000\pi \text{ joules}

   PTS: 1   DIF: Medium   REF: 7.5.19
   OBJ: Calculate work in problems involving pumping liquids from containers   MSC: Application
   NOT: Section 7.5

38. ANS:
   \frac{5k}{6} \text{ units of work}

   PTS: 1   DIF: Medium   REF: 7.5.41
   OBJ: Calculate the total work done by a variable force   MSC: Application
   NOT: Section 7.5

39. ANS:
   \bar{x} = \frac{34}{9}

   PTS: 1   DIF: Easy   REF: 7.6.1
   OBJ: Calculate the center of mass in a one-dimensional system   MSC: Application
   NOT: Section 7.6
40. ANS:
\[ \bar{x} = -\frac{36}{11} \]

PTS: 1 DIF: Easy REF: 7.6.2
OBJ: Calculate the center of mass in a one-dimensional system MSC: Application
NOT: Section 7.6

41. ANS:
\[ \bar{x} = \frac{7}{43} \]

PTS: 1 DIF: Easy REF: 7.6.3
OBJ: Calculate the center of mass in a one-dimensional system MSC: Application
NOT: Section 7.6

42. ANS:
9 feet

PTS: 1 DIF: Easy REF: 7.6.7
OBJ: Calculate the center of mass in a one-dimensional system MSC: Application
NOT: Section 7.6

43. ANS:
\[ \bar{x} = \frac{13}{10}, \bar{y} = \frac{15}{4} \]

PTS: 1 DIF: Medium REF: 7.6.9
OBJ: Calculate the center of mass in a two-dimensional system MSC: Application
NOT: Section 7.6

44. ANS:
\[ \bar{x} = \frac{43}{23}, \bar{y} = \frac{16}{23} \]

PTS: 1 DIF: Medium REF: 7.6.11
OBJ: Calculate the center of mass in a two-dimensional system MSC: Application
NOT: Section 7.6

45. ANS:
\[ (\bar{x}, \bar{y}) = \left( \frac{12}{5}, \frac{3}{4} \right) \]

PTS: 1 DIF: Medium REF: 7.6.15
OBJ: Calculate the center of mass for a lamina formed by curves MSC: Application NOT: Section 7.6

46. ANS:
\[ M_x = \frac{289\rho}{35} \]

PTS: 1 DIF: Easy REF: 7.6.17
OBJ: Calculate the moments for a lamina formed by curves MSC: Application
NOT: Section 7.6
47. ANS:

\[ M_x = \frac{1,024}{5} \rho, \quad M_y = \frac{8,192}{11} \rho, \]
\[ \bar{x} = \frac{112}{11}, \quad \bar{y} = \frac{14}{5} \]

PTS: 1  DIF: Medium  REF: 7.6.21
OBJ: Calculate the moments and center of mass for a lamina formed by curves
MSC: Application  NOT: Section 7.6

48. ANS:

\[ (\bar{x}, \bar{y}) = \left( \frac{18}{5}, 0 \right) \]

PTS: 1  DIF: Difficult  REF: 7.6.23
OBJ: Calculate the center of mass for a lamina formed by curves
MSC: Application  NOT: Section 7.6

49. ANS:

\[ M_y = \frac{17,408 \rho}{15} \]

PTS: 1  DIF: Difficult  REF: 7.6.26
OBJ: Calculate the moments for a lamina formed by curves
MSC: Application  NOT: Section 7.6

50. ANS:

\[ M_y = 0 \text{ by symmetry} \]

PTS: 1  DIF: Medium  REF: 7.6.30
OBJ: Calculate the moments for a lamina formed by curves
MSC: Application  NOT: Section 7.6