Physics 2326 (91239) Summer II 2012 Chapter 18 Assignment Solutions

Problem # 18.32

The specific heat of a substance varies with temperature according to the function
\[ c = 0.20 + 0.14 T + 0.023 T^2, \]
with \( T \) in °C and \( c \) in \( \text{cal g}^{-1} \text{K}^{-1} \). Find the energy required to raise the temperature of 2.0 g of this substance from 5.0°C to 15°C.

Solution (20 points)

Data: The first thing to do is to fix the given equation so that the phrase "with \( T \) in °C and \( c \) in \( \text{cal g}^{-1} \text{K}^{-1} \)" isn't needed every time it's written to keep the units straight. We define
\[ A = 0.20 \text{ cal g}^{-1} \text{K}^{-1}, \quad B = 0.14 \text{ cal g}^{-1} \text{K}^2 \]
and
\[ C = 0.023 \text{ cal g}^{-1} \text{K}^3, \]
so the equation becomes
\[ c = A + B T + C T^2, \]
which stands on its own with correct units and no longer needs that silly phrase tagging along with it. The additional data provided are the mass of the sample, \( m = 2.0 \text{ g} \), the initial sample temperature \( T_i = 5.0 \text{ °C} \) and the final temperature \( T_f = 15.0 \text{ °C} \).

Plan: The basic fundamental equation needed here is the definition of the specific heat as \( c = \frac{Q}{m(\Delta T)}. \)
Since the given specific heat is itself a function of temperature, we need to restate this relationship to apply to infinitesimal changes in energy and temperature, so that \( m c = \frac{dQ}{dT}. \) Then the element of energy needed to raise the temperature by an element of temperature is \( dq = m c dT \)
The energy \( Q \) required to raise the temperature of the sample from the initial temperature to the final one must satisfy the equation
\[ \int_{T_i}^{T_f} dq = m \int_{T_i}^{T_f} (A + B T + C T^2) dT. \]
Integrating both sides of this equation, we have the result that
\[ Q = m\left(A(T_f - T_i) + \frac{B}{2} (T_f^2 - T_i^2) + \frac{C}{3} (T_f^3 - T_i^3)\right). \]
The inexact numerical value is
\[ Q \approx 3.8 \times 10^4 \text{ cal} \approx 1.6 \times 10^5 \text{ J}. \]

Contemplation

The first appearance of the specific heat in the textbook, Equation 18-14, expresses a constant specific heat. Specific heats for many materials are often practically constant over a considerable range of temperature. But all specific heats are somewhat temperature dependent if the temperature difference is large enough. In this problem, the temperature dependence is explicitly given, so must be taken into account in solving this question. This means that the differential form of the equation 18-14 must be used, which is the statement that
\[ c = \frac{1}{m} \frac{dQ}{dT}. \]

Given the above, how large an error would be made if the specific heat were taken to be constant at its average value over the temperature interval stated in this problem? What would be the appropriate way to calculate the average value of \( c \) in this temperature interval?

Mathematica calculations

\[
\begin{align*}
\{aa, bb, cc, ti, tf, m\} &= \{0.20, 0.14, 0.023, (5.0 + 273.15), (15.0 + 273.15), 2.0\}; \\
\{q1 = m (aa (tf - ti) + \frac{bb}{2} (tf^2 - ti^2) + \frac{cc}{3} (tf^3 - ti^3)), q2 = q1 * 4.1868\} &\approx \{37680.7, 157761.\}
\end{align*}
\]
A lab sample of gas is taken through cycle $abca$ shown in the $p$-$V$ diagram shown below. The net work done is $+1.2$ J. Along path $ab$, the change in internal energy is $+3.0$ J and the magnitude of the work done is $5.0$ J. Along path $ca$, the energy transferred to the gas as heat is $+2.5$ J. How much energy is transferred as heat along (a) path $ab$ and (b) path $bc$?

\[
\begin{align*}
\text{Solution} \quad (20 \text{ points}) \\
\text{Data:} \quad \text{The net work done around the closed path} \quad abca \quad \text{is} \quad W_{abca} = 1.2 \text{ J.} \quad \text{Along the path} \quad ab, \quad \text{the change in internal energy is} \quad \Delta E_{int, ab} = 3.0 \text{ J and the magnitude of the work done is} \quad W_{ab} = 5.0 \text{ J.} \quad \text{Along path} \quad ca, \quad \text{we have} \quad Q_{ca} = 2.5 \text{ J.} \\
\end{align*}
\]

\[
\begin{align*}
\text{Plan:} \quad & \text{Because the full path is closed, the total change in internal energy is zero. So, for the full path} \\
& \Delta E_{int, abca} = 0 = Q_{abca} - W_{abca}, \quad \text{so} \quad Q_{abca} = W_{abca}. \quad \text{That, together with the data provided for parts of the full path, allow the energies for all the other parts to be determined.} \\
& (a) \quad \text{We are to find} \quad Q_{ab}, \quad \text{and} \quad \Delta E_{int, ab} = Q_{ab} - W_{ab}, \quad \text{where both the other quantities are known. So, we have} \\
& Q_{ab} = W_{ab} + \Delta E_{int, ab}. \quad \text{The numerical value is} \quad Q_{ab} \approx 8.0 \text{ J.} \\
& (b) \quad \text{Next we are to find} \quad Q_{bc}. \quad \text{For this, we note that} \quad Q_{abca} = Q_{ab} + Q_{bc} + Q_{ca}, \quad \text{so that} \\
& Q_{bc} = Q_{abca} - Q_{ab} - Q_{ca}. \quad \text{Writing this in terms of the data, we have} \quad Q_{bc} = W_{abca} - (W_{ab} + \Delta E_{int, ab}) - Q_{ca}. \quad \text{The numerical value is} \quad Q_{bc} \approx -9.3 \text{ J.} \\
\end{align*}
\]

The mathematics needed to solve this problem are very elementary, but only make sense if the meaning of the symbols is clear, and the idea of conservation of energy is taken seriously.

\[
\begin{align*}
\text{Mathematica calculations} \\
\{wabca, wab, deleab, qca\} = \{1.2, 5.0, 3.0, 2.5\}; \\
\{(wab + deleab), (wabca - (wab + deleab) - qca)\} \\
\{8., -9.3\} \\
\end{align*}
\]

Clear[wabca, wab, deleab, qca]