Calculus 1 Test 4 Review

1. Analyze and sketch a graph of the function \( f(x) = \frac{x^2 - 2x + 9}{x} \).

2. Use differentials to approximate the value of \( \sqrt[3]{7.5} \). Round your answer to four decimal places.

3. Determine the slant asymptote of the graph of \( f(x) = \frac{x^2 + 8x + 14}{x + 6} \).

4. Find the differential \( dy \) of the function \( y = 2x^{3/7} \).

5. The measurement of the radius of the end of a log is found to be 28 inches, with a possible error of \( \frac{1}{4} \) inch. Use differentials to approximate the possible propagated error in computing the area of the end of the log.

6. Find the differential \( dy \) of the function \( y = -x^2 - 3x - 2 \).

7. The radius of a spherical balloon is measured to be 8 inches, with a possible error of 0.03 inch. Use differentials to approximate the maximum possible error in calculating the volume of the sphere. Round your answer to two decimal places.

8. Use Newton’s Method to approximate the \( x \)-value of the indicated point of intersection of the two graphs accurate to three decimal places. Continue the process until two successive approximations differ by less than 0.001. [Hint: Let \( h(x) = f(x) - g(x) \).]

\[
\begin{align*}
  f(x) &= 3x + 1 \\
  g(x) &= \sqrt{x + 5}
\end{align*}
\]

9. Find the equation of the tangent line \( T \) to the graph of \( f(x) = \frac{19}{x^2} \) at the given point \( \left( 2, \frac{19}{4} \right) \).
10. Find the point on the graph of \( f(x) = 25 - x^2 \) that is closest to the point \((1,0)\). Round all numerical values of the solution to three decimal places.

11. Complete two iterations of Newton's Method for the function \( f(x) = \cos x \) using the initial guess \( x_1 = 1.3 \). Round all numerical values in your answer to four decimal places.

12. Find two positive numbers whose product is 181 and whose sum is a minimum.

13. Determine the slant asymptote of the graph of \( f(x) = \frac{5x^2 - 9x + 5}{x - 1} \).

14. Analyze and sketch a graph of the function \( y = x\sqrt{9-x} \).

15. Approximate the positive zero(s) of the function \( f(x) = x^3 - \cos x \) to three decimal places. Use Newton's Method and continue the process until two successive approximations differ by less than 0.001.

16. Find the point on the graph of the function \( f(x) = (x + 1)^2 \) that is closest to the point \((-5,1)\). Round all numerical values in your answer to four decimal places.

17. Find the point on the graph of the function \( f(x) = \sqrt{x} \) that is closest to the point \((18,0)\).

18. Use Newton's Method to approximate the zero(s) of the function \( f(x) = x^3 + x + 1 \) accurate to three decimal places.

19. Use Newton's Method to approximate the zero(s) of the function \( f(x) = x - 2\sqrt{x+2} \) accurate to three decimal places.

20. The concentration \( C \) of a chemical in the bloodstream \( t \) hours after injection into muscle tissue is given by \( C = \frac{5t^2 + t}{50 + t^3} \). When is the concentration greatest? Round your answer to three decimal places.

21. A sector with central angle \( \Theta \) is cut from a circle of radius 10 inches, and the edges of the sector are brought together to form a cone. Find the magnitude of \( \Theta \) such that the volume of the cone is a maximum.

22. On a given day, the flow rate \( F \) (cars per hour) on a congested roadway is given by \( F = \frac{v}{17 + 0.04v^2} \), where \( v \) is the speed of the traffic in miles per hour. What speed will maximize the flow rate on the road? Round your answer to the nearest mile per hour.
Calculus 1 Test 4 Review
Answer Section

SHORT ANSWER

1. ANS:
   none of the above
   
   PTS: 1     DIF: Medium     REF: 3.6.15
   OBJ: Graph a function using extrema, intercepts, symmetry, and asymptotes
   MSC: Skill     NOT: Section 3.6

2. ANS:
   1.9583
   
   PTS: 1     DIF: Medium     REF: 3.9.44
   OBJ: Estimate the value of a radical using differentials
   MSC: Skill     NOT: Section 3.9

3. ANS:
   \( y = x + 2 \)
   
   PTS: 1     DIF: Medium     REF: 3.6.15
   OBJ: Identify the slant asymptote of the graph of a function
   MSC: Skill     NOT: Section 3.6

4. ANS:
   \( \frac{6}{7} x^{-4/7} \, dx \)
   
   PTS: 1     DIF: Medium     REF: 3.9.12
   OBJ: Calculate the differential of \( y \) for a given function
   MSC: Skill     NOT: Section 3.9

5. ANS:
   \( \pm 14\pi \) square inches
   
   PTS: 1     DIF: Easy     REF: 3.9.29
   OBJ: Estimate the propagated error using differentials
   MSC: Application     NOT: Section 3.9

6. ANS:
   \( (-2x - 3) \, dx \)
   
   PTS: 1     DIF: Medium     REF: 3.9.11
   OBJ: Calculate the differential of \( y \) for a given function
   MSC: Skill     NOT: Section 3.9
7. **ANS:**
   ±24.13 cubic inches
   
   **PTS:** 1  **DIF:** Medium  **REF:** 3.9.33a
   **OBJ:** Estimate the propagated error using differentials  **MSC:** Application
   **NOT:** Section 3.9

8. **ANS:**
   0.444
   
   **PTS:** 1  **DIF:** Medium  **REF:** 3.8.15
   **OBJ:** Estimate the intersection point of two graphs using Newton's Method
   **MSC:** Skill  **NOT:** Section 3.8

9. **ANS:**
   \[ y = -\frac{19x}{4} + \frac{57}{4} \]
   
   **PTS:** 1  **DIF:** Easy  **REF:** 3.9.2
   **OBJ:** Write an equation of a line tangent to the graph of a function at a specified point
   **MSC:** Skill  **NOT:** Section 3.9

10. **ANS:**
    \( (4.960,0.398) \)
    
    **PTS:** 1  **DIF:** Medium  **REF:** 3.8.35
    **OBJ:** Estimate an extremum involving distance between points using calculus
    **MSC:** Application  **NOT:** Section 3.8

11. **ANS:**

    | \( n \) | \( x_n \) | \( f(x_n) \) | \( f'(x_n) \) | \( \frac{f(x_n)}{f'(x_n)} \) | \( x_n - \frac{f(x_n)}{f'(x_n)} \) |
    |---|---|---|---|---|
    | 1 | 1.3 | 0.2675 | -0.9636 | -0.2776 | 1.5776 |
    | 2 | 1.5776 | -0.0068 | -1.0000 | 0.0068 | 1.5708 |

    **PTS:** 1  **DIF:** Easy  **REF:** 3.8.3
    **OBJ:** Estimate a zero of a function using two iterations of Newton's Method
    **MSC:** Skill  **NOT:** Section 3.8

12. **ANS:**
    \( \sqrt{181}, \sqrt{181} \)
    
    **PTS:** 1  **DIF:** Easy  **REF:** 3.7.4
    **OBJ:** Apply calculus techniques to solve a minimum/maximum problem involving the sum of two numbers
    **MSC:** Application  **NOT:** Section 3.7
13. **ANS:**

   \[ y = 5x - 4 \]

   PTS: 1  DIF: Medium  REF: 3.6.16
   OBJ: Identify the slant asymptote of the graph of a function  MSC: Skill
   NOT: Section 3.6

14. **ANS:**

   \[
   \begin{align*}
   f(x) & = \frac{1}{x} \\
   y & = f(x)
   \end{align*}
   \]

   PTS: 1  DIF: Medium  REF: 3.6.18
   OBJ: Graph a function using extrema, intercepts, symmetry, and asymptotes  MSC: Skill
   NOT: Section 3.6

15. **ANS:**

   0.865

   PTS: 1  DIF: Medium  REF: 3.8.14
   OBJ: Estimate a zero of a function using Newton's Method  MSC: Skill
   NOT: Section 3.8

16. **ANS:**

   \((-2.3918, 1.9370)\)

   PTS: 1  DIF: Difficult  REF: 3.7.14
   OBJ: Apply calculus techniques to solve a minimum/maximum problem involving the distance between points  MSC: Application  NOT: Section 3.7

17. **ANS:**

   \[
   \left( \frac{35}{2}, \sqrt{\frac{35}{2}} \right)
   \]

   PTS: 1  DIF: Medium  REF: 3.7.15
   OBJ: Apply calculus techniques to solve a minimum/maximum problem involving the distance between points  MSC: Application  NOT: Section 3.7
18. ANS: 
-0.682

PTS: 1  DIF: Medium  REF: 3.8.7
OBJ: Estimate a zero of a function using Newton's Method  MSC: Skill
NOT: Section 3.8

19. ANS: 
5.464

PTS: 1  DIF: Medium  REF: 3.8.10
OBJ: Estimate a zero of a function using Newton's Method  MSC: Skill
NOT: Section 3.8

20. ANS: 
\[ t \approx 4.546 \text{ hours} \]

PTS: 1  DIF: Medium  REF: 3.8.38
OBJ: Estimate an extremum involving chemical concentration of the blood using calculus and Newton's Method
MSC: Application  NOT: Section 3.8

21. ANS: 
\[
2 \left(1 - \frac{\sqrt{2}}{\sqrt[3]{3}}\right) \pi \text{ radians}
\]

PTS: 1  DIF: Medium  REF: 3.7.56
OBJ: Apply calculus techniques to solve a minimum/maximum problem involving the volume of a cone
MSC: Application  NOT: Section 3.7

22. ANS: 
21 miles per hour

PTS: 1  DIF: Medium  REF: 3.7.20
OBJ: Apply calculus techniques to solve a minimum/maximum problem involving traffic flow
MSC: Application  NOT: Section 3.7