Chapter 25

Electric Potential

Electrical Potential Energy

- When a test charge is placed in an electric field, it experiences a force
  - $$\vec{\mathbf{F}} = q_o\vec{\mathbf{E}}$$
  - The force is conservative
- If the test charge is moved in the field by some external agent, the work done by the field is the negative of the work done by the external agent
- $d\vec{\mathbf{s}}$ is an infinitesimal displacement vector that is oriented tangent to a path through space

Electric Potential Energy, cont

- The work done by the electric field is

  $$\vec{\mathbf{F}} \cdot d\vec{\mathbf{s}} = q_o \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$

- As this work is done by the field, the potential energy of the charge-field system is changed by $\Delta U = -q_o \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$
- For a finite displacement of the charge from A to B,

$$\Delta U = U_B - U_A = -q_o \int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$

Electric Potential Energy, final

- Because the force is conservative, the line integral does not depend on the path taken by the charge
- This is the change in potential energy of the system

Electric Potential

- The potential energy per unit charge, U/q_o, is the **electric potential**
  - The potential is characteristic of the field only
    - The potential energy is characteristic of the charge-field system
  - The potential is independent of the value of q_o
  - The potential has a value at every point in an electric field
- The electric potential is

$$V = \frac{U}{q_o}$$

Electric Potential, cont.

- The potential is a scalar quantity
  - Since energy is a scalar
- As a charged particle moves in an electric field, it will experience a change in potential

$$\Delta V = \frac{\Delta U}{q_o} = -\int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$

Electric Potential, final

- The ***difference* in potential is the meaningful quantity**
- We often take the value of the potential to be zero at some convenient point in the field
- **Electric potential is a scalar characteristic of an electric field**, independent of any charges that may be placed in the field

Work and Electric Potential

- Assume a charge moves in an electric field without any change in its kinetic energy
- The work **W** performed on the charge is

$$W = \Delta U = q\,\Delta V$$

Units

- 1 V = 1 J/C
  - V is a volt
  - It takes one joule of work to move a 1-coulomb charge through a potential difference of 1 volt
- In addition, 1 N/C = 1 V/m
  - This indicates we can interpret the electric field as a measure of the rate of change with position of the electric potential

Electron-Volts

- Another unit of energy that is commonly used in atomic and nuclear physics is the electron-volt

- One ***electron-volt*** is defined as the energy a charge-field system gains or loses when **a charge of magnitude *e*** (an electron or a proton) is moved through a potential difference of 1 volt

  - 1 eV = 1.60 x 10^{-19} J

Potential Difference in a Uniform Field

- The equations for electric potential can be simplified if the electric field is **uniform**:

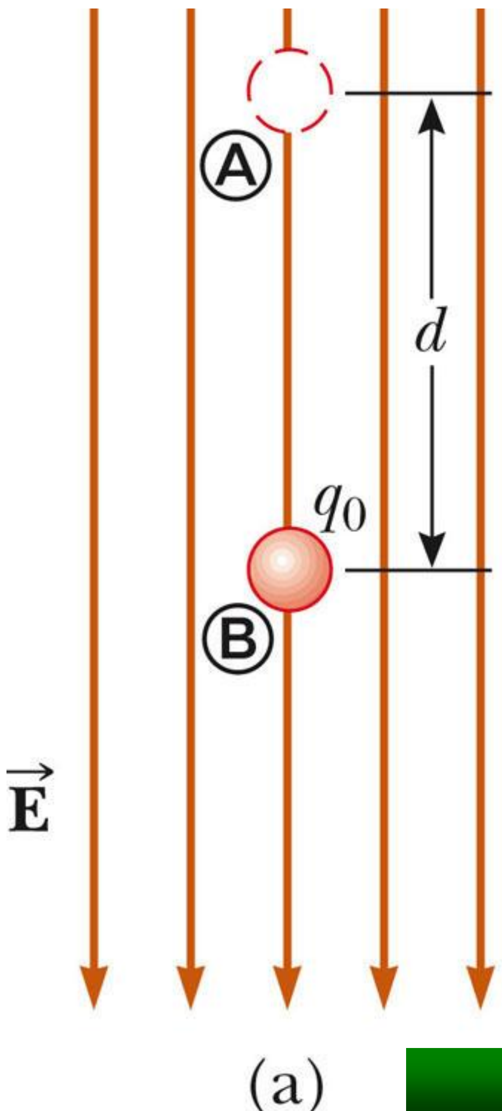
$$V_B - V_A = \Delta V = -\int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = -E\int_A^B d\mathbf{s} = -Ed$$

- The negative sign indicates that the electric potential at point *B* is lower than at point *A*

  - Electric field lines always point in the direction of decreasing electric potential

Energy and the Direction of Electric Field

- When the electric field is directed downward, point *B* is at a lower potential than point *A*
- When a positive test charge moves from *A* to *B*, the charge-field system loses potential energy
- Use the active figure to compare the motion in the electric field to the motion in a gravitational field



PLAY ACTIVE FIGURE

More About Directions

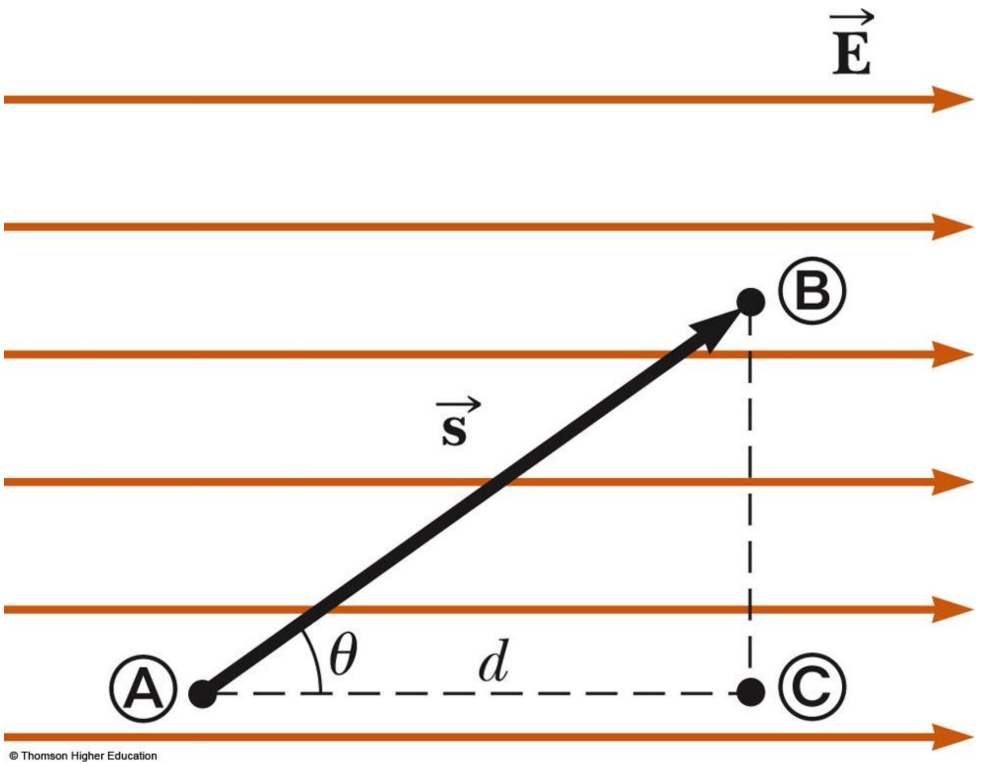
- A system consisting of a positive charge and an electric field **loses** electric potential energy when the charge moves in the direction of the field
  - An electric field does work on a positive charge when the charge moves in the direction of the electric field
- The charged particle gains kinetic energy equal to the potential energy lost by the charge-field system
  - Another example of Conservation of Energy

Directions, cont.

- If q_o is negative, then ΔU is positive
- A system consisting of a negative charge and an electric field *gains* potential energy when the charge moves in the direction of the field
  - In order for a negative charge to move in the direction of the field, an external agent must do positive work on the charge

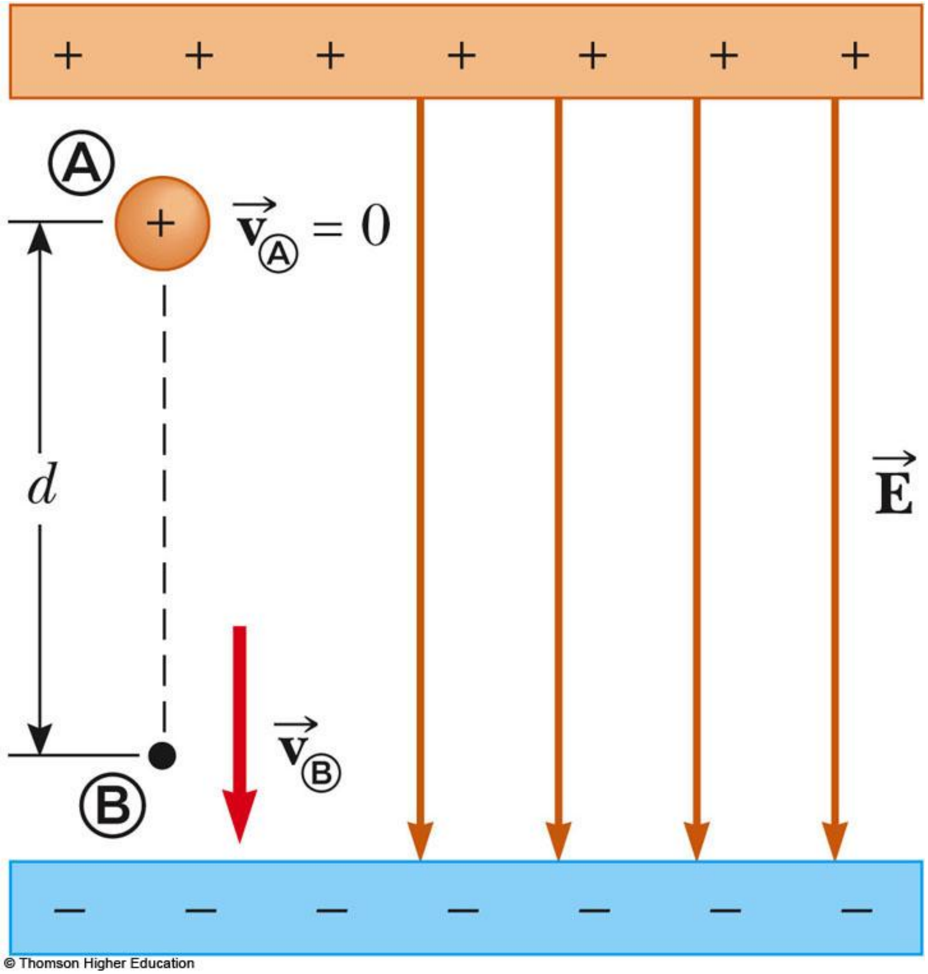
Equipotentials

- Point *B* is at a lower potential than point *A*
- Points *B* and *C* are at the same potential
  - All points in a plane perpendicular to a uniform electric field are at the same electric potential
- The name **equipotential surface** is given to any surface consisting of a continuous distribution of points having the same electric potential



Charged Particle in a Uniform Field, Example

- A positive charge is released from rest and moves in the direction of the electric field
- The change in potential is negative
- The change in potential energy is negative
- The force and acceleration are in the direction of the field
- Conservation of Energy can be used to find its speed



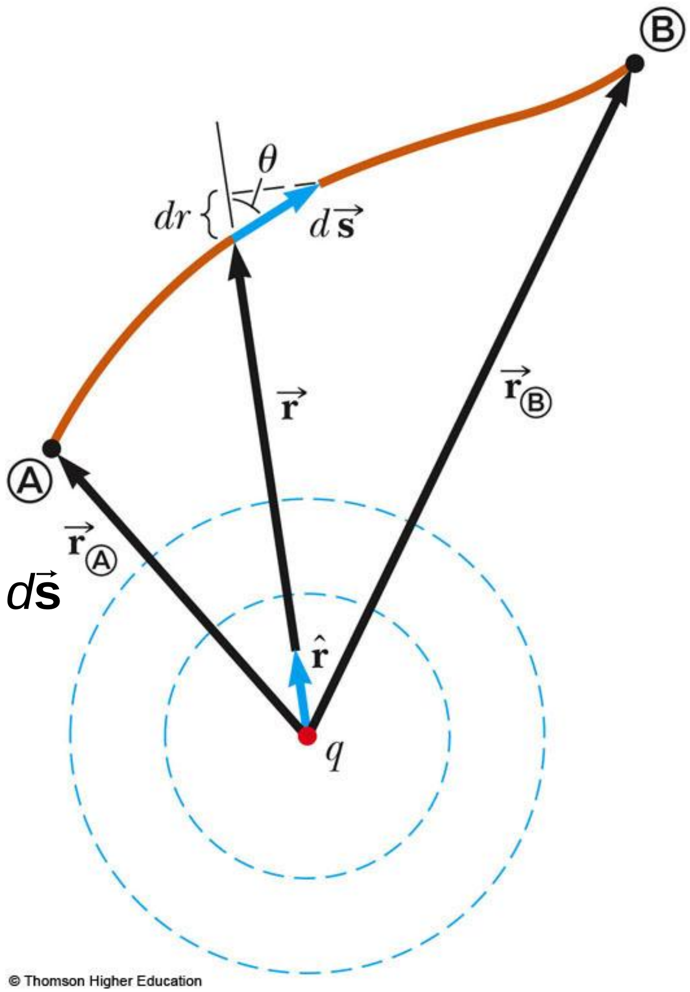
Potential and Point Charges

- A positive point charge produces a field directed radially outward
- The potential difference between points *A* and *B* will be:

$$V_B - V_A = k_e q\left[\frac{1}{r_B} - \frac{1}{r_A}\right]$$

Integrate:

$$\Delta V = \frac{\Delta U}{q_o} = -\int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$



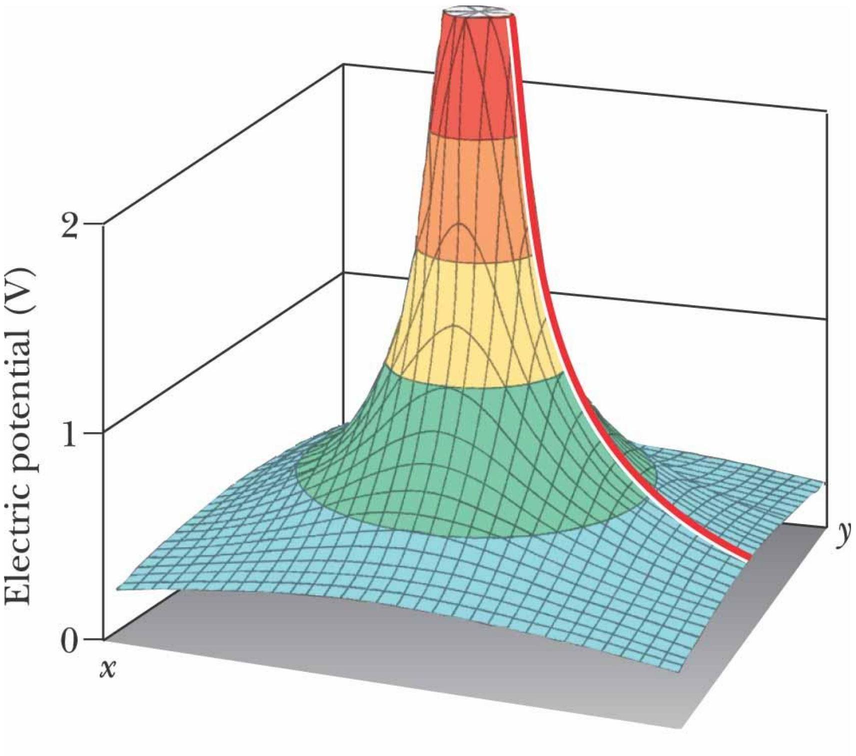
Potential and Point Charges, cont.

- The electric potential is independent of the path between points *A* and *B*
- It is customary to choose a reference potential of $V = 0$ at $r_A = \infty$
- Then the potential at some point *r* is

$$V = k_e \frac{q}{r}$$

Electric Potential of a Point Charge

- The electric potential in the plane around a single point charge is shown
- The red line shows the 1/*r* nature of the potential



Electric Potential with Multiple Charges

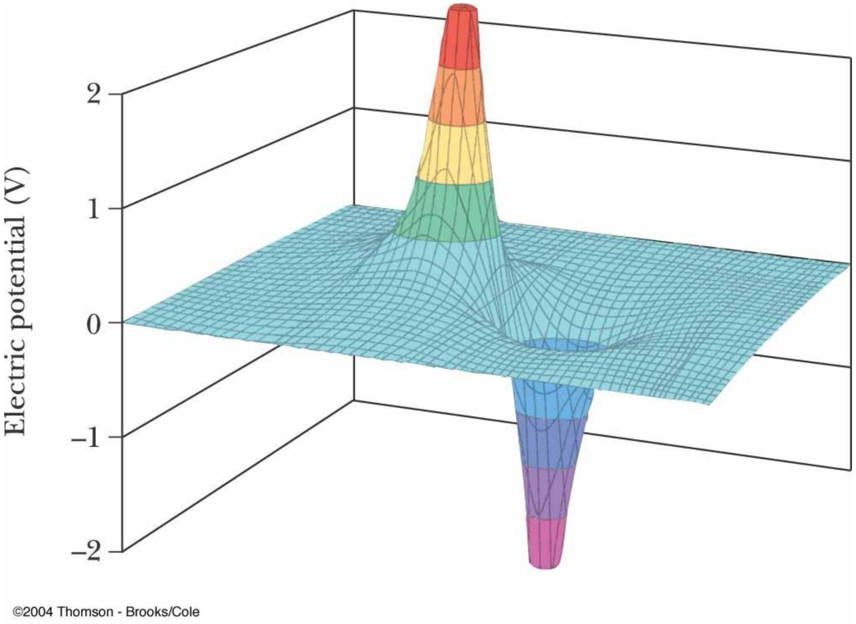
- The electric potential due to several point charges is the sum of the potentials due to each individual charge
  - This is another example of the superposition principle
  - The sum is the algebraic sum

$$V = k_e \sum_i \frac{q_i}{r_i}$$

  - $V = 0$ at $r = \infty$

Electric Potential of a Dipole

- The graph shows the potential (y-axis) of an electric dipole
- The steep slope between the charges represents the strong electric field in this region

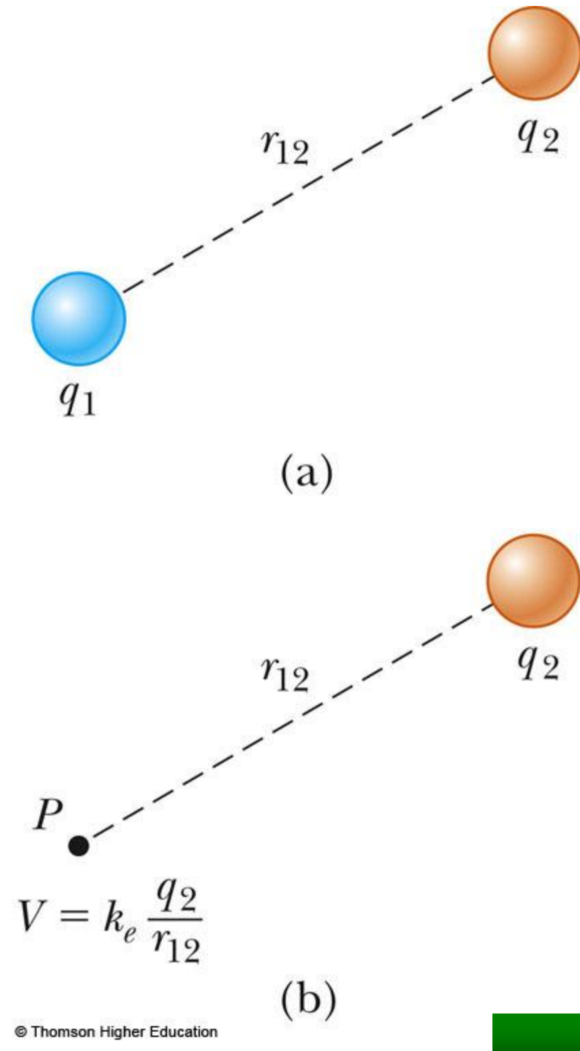


Potential Energy of Multiple Charges

- Consider two charged particles
- The potential energy of the system is

$$U = k_e \frac{q_1 q_2}{r_{12}}$$

  - Use the active figure to move the charge and see the effect on the potential energy of the system



PLAY ACTIVE FIGURE

More About *U* of Multiple Charges

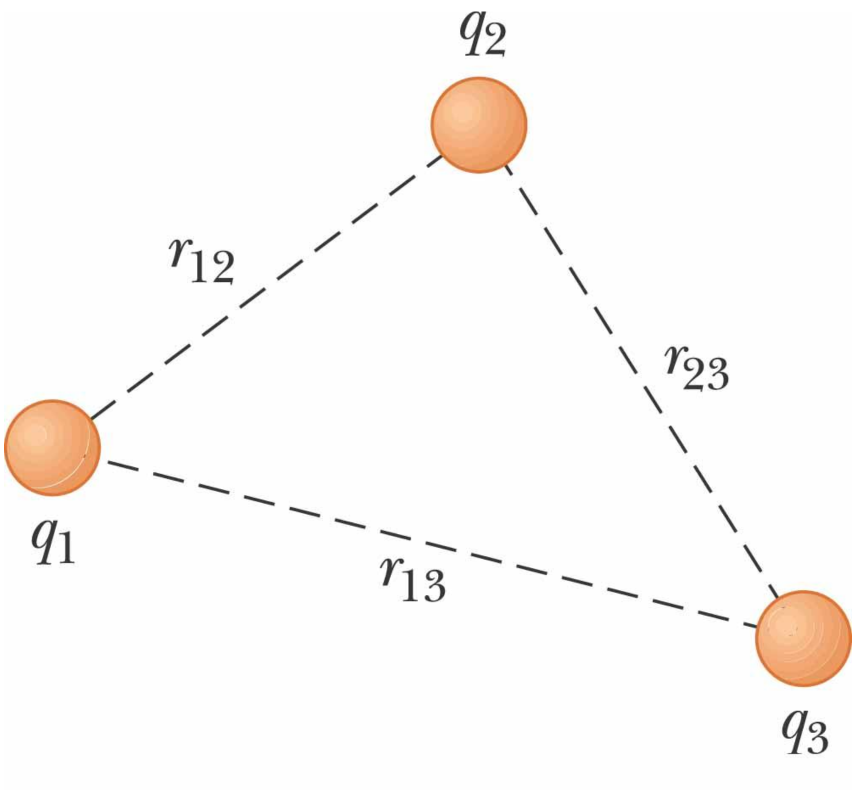
- If the two charges are the same sign, U is positive and work must be done to bring the charges together
- If the two charges have opposite signs, U is negative and work is done to keep the charges apart

U with Multiple Charges, final

- If there are more than two charges, then find *U* for each pair of charges and add them
- For three charges:

$$U = k_e\left(\frac{q_1q_2}{r_{12}} + \frac{q_1q_3}{r_{13}} + \frac{q_2q_3}{r_{23}}\right)$$

  - The result is independent of the order of the charges



Finding E From *V*

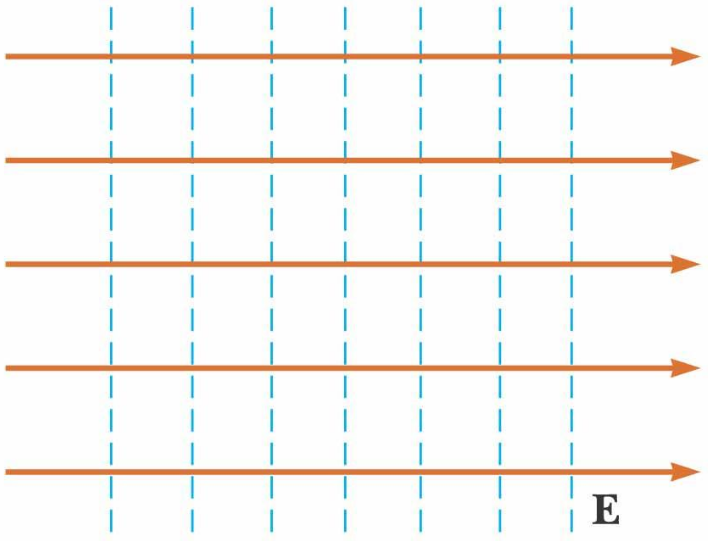
- Assume, to start, that the field has only an *x* component

$$E_x = -\frac{dV}{dx}$$

- Similar statements would apply to the *y* and *z* components
- Equipotential surfaces must always be perpendicular to the electric field lines passing through them

E and *V* for an Infinite Sheet of Charge

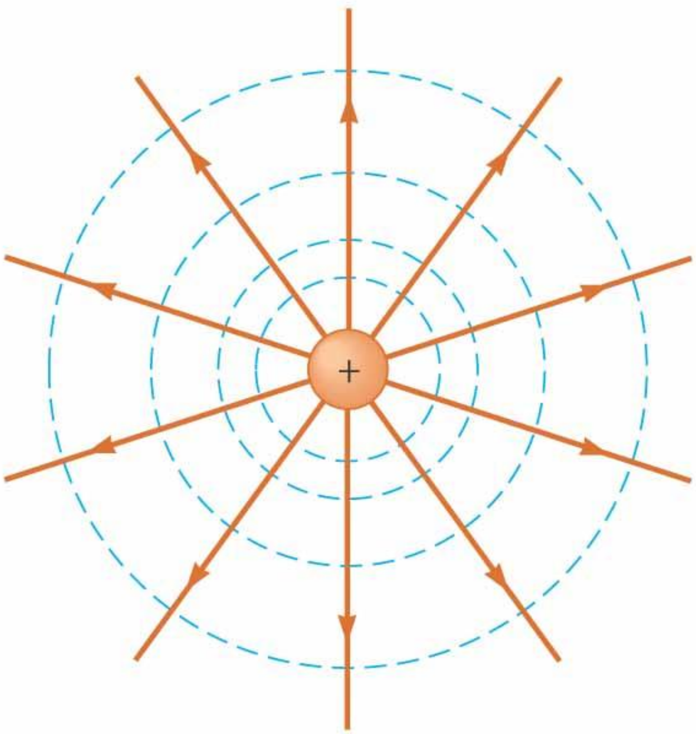
- The equipotential lines are the dashed blue lines
- The electric field lines are the brown lines
- The equipotential lines are everywhere perpendicular to the field lines



(a)

E and *V* for a Point Charge

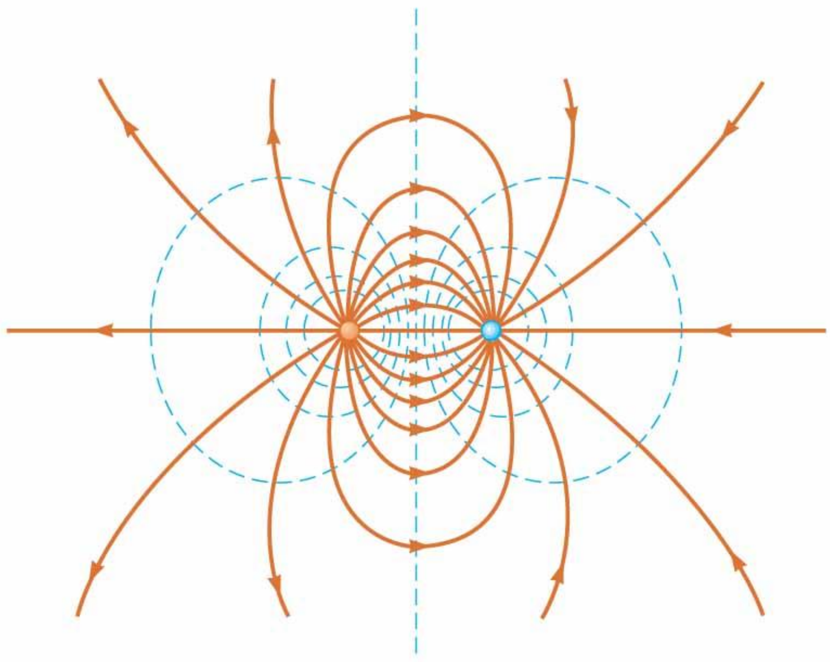
- The equipotential lines are the dashed blue lines
- The electric field lines are the brown lines
- The equipotential lines are everywhere perpendicular to the field lines



(b)

E *and V* for a Dipole

- The equipotential lines are the dashed blue lines
- The electric field lines are the brown lines
- The equipotential lines are everywhere perpendicular to the field lines


(c)

Electric Field from Potential, General

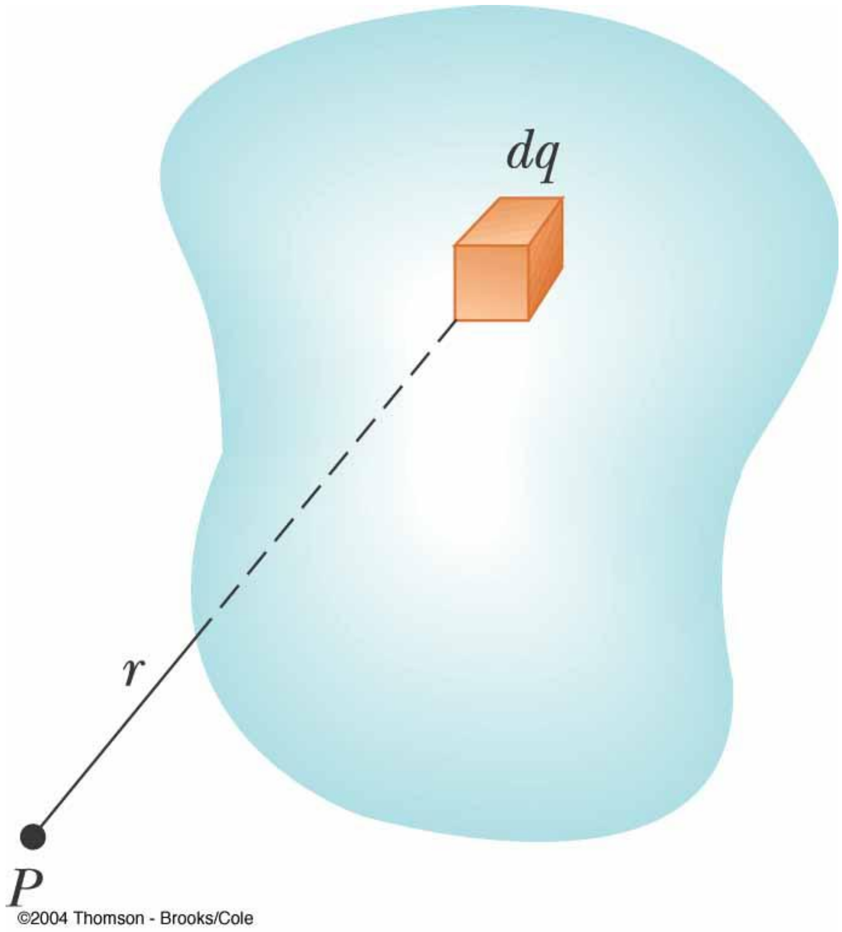
- In general, the electric potential is a function of all three dimensions
- Given $V(x, y, z)$ you can find E_x, E_y and E_z as partial derivatives

$$E_x = -\frac{\partial V}{\partial x} \quad E_y = -\frac{\partial V}{\partial y} \quad E_z = -\frac{\partial V}{\partial z}$$

Electric Potential for a Continuous Charge Distribution

- Consider a small charge element *dq*
  - Treat it as a point charge
- The potential at some point due to this charge element is

$$dV = k_e \frac{dq}{r}$$



V for a Continuous Charge Distribution, cont.

- To find the total potential, you need to integrate to include the contributions from all the elements

$$V = k_e \int \frac{dq}{r}$$

  - This value for *V* uses the reference of *V* = 0 when *P* is infinitely far away from the charge distributions

V From a Known E

- If the electric field is already known from other considerations, the potential can be calculated using the original approach

$$\Delta V = -\int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$

  - If the charge distribution has sufficient symmetry, first find the field from Gauss' Law and then find the potential difference between any two points
    - Choose V = 0 at some convenient point

Problem-Solving Strategies

- *Conceptualize*
  - Think about the individual charges or the charge distribution
  - Imagine the type of potential that would be created
  - Appeal to any symmetry in the arrangement of the charges
- *Categorize*
  - Group of individual charges or a continuous distribution?

Problem-Solving Strategies, 2

- *Analyze*
  - General
    - Scalar quantity, so no components
    - Use algebraic sum in the superposition principle
    - Only changes in electric potential are significant
    - Define V = 0 at a point infinitely far away from the charges
      - If the charge distribution extends to infinity, then choose some other arbitrary point as a reference point

Problem-Solving Strategies, 3

- *Analyze, cont*
  - If a group of individual charges is given
    - Use the superposition principle and the algebraic sum
  - If a continuous charge distribution is given
    - Use integrals for evaluating the total potential at some point
    - Each element of the charge distribution is treated as a point charge
  - If the electric field is given
    - Start with the definition of the electric potential
    - Find the field from Gauss' Law (or some other process) if needed

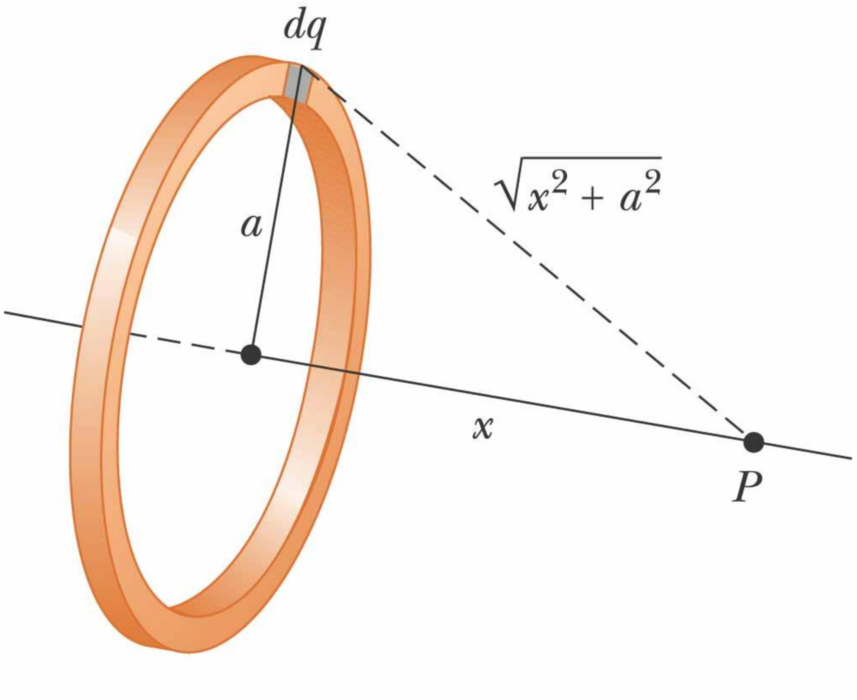
Problem-Solving Strategies, final

- *Finalize*
  - Check to see if the expression for the electric potential is consistent with your mental representation
  - Does the final expression reflect any symmetry?
  - Image varying parameters to see if the mathematical results change in a reasonable way

V for a Uniformly Charged Ring

- *P* is located on the perpendicular central axis of the uniformly charged ring
  - The ring has a radius *a* and a total charge *Q*

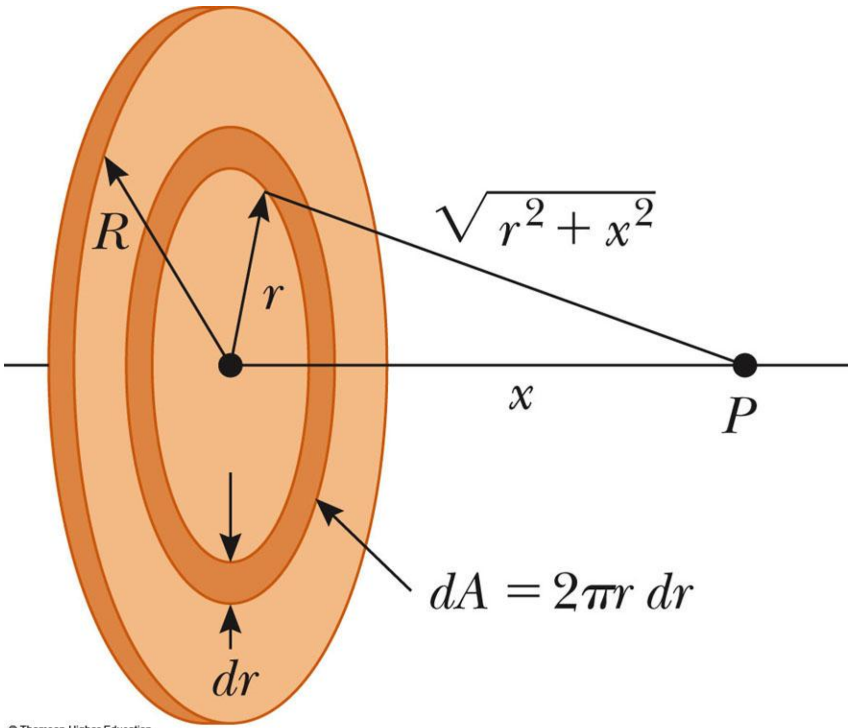
$$V = k_e \int \frac{dq}{r} = \frac{k_e Q}{\sqrt{a^2 + x^2}}$$



V for a Uniformly Charged Disk

- The ring has a radius *R* and surface charge density of *σ*
- P is along the perpendicular central axis of the disk

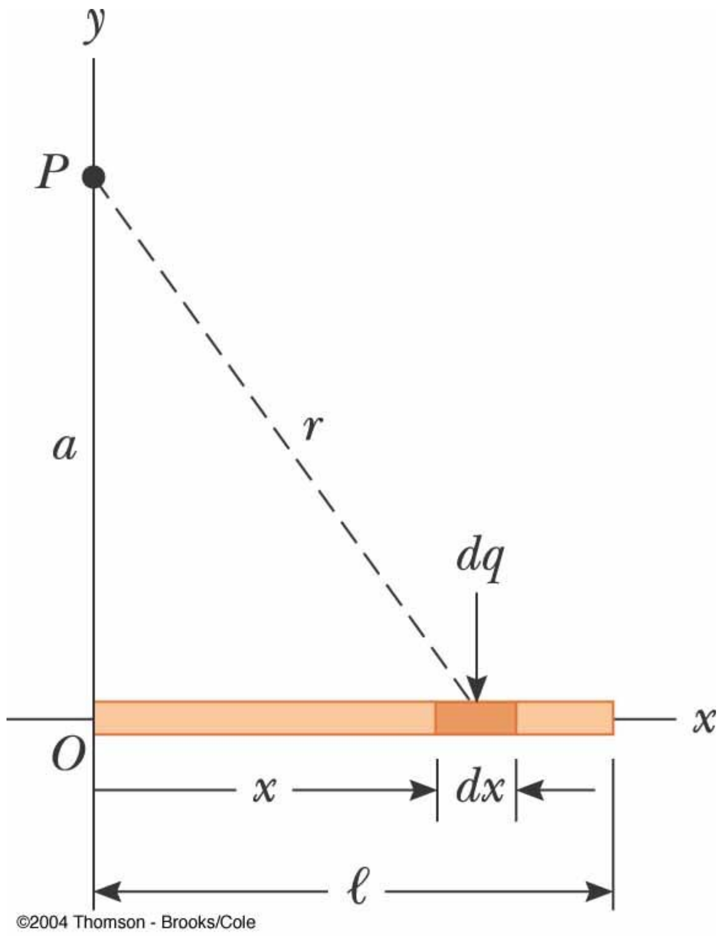
$$V = 2\pi k_e \sigma \left[\left(R^2 + x^2 \right)^{1/2} - x \right]$$



V for a Finite Line of Charge

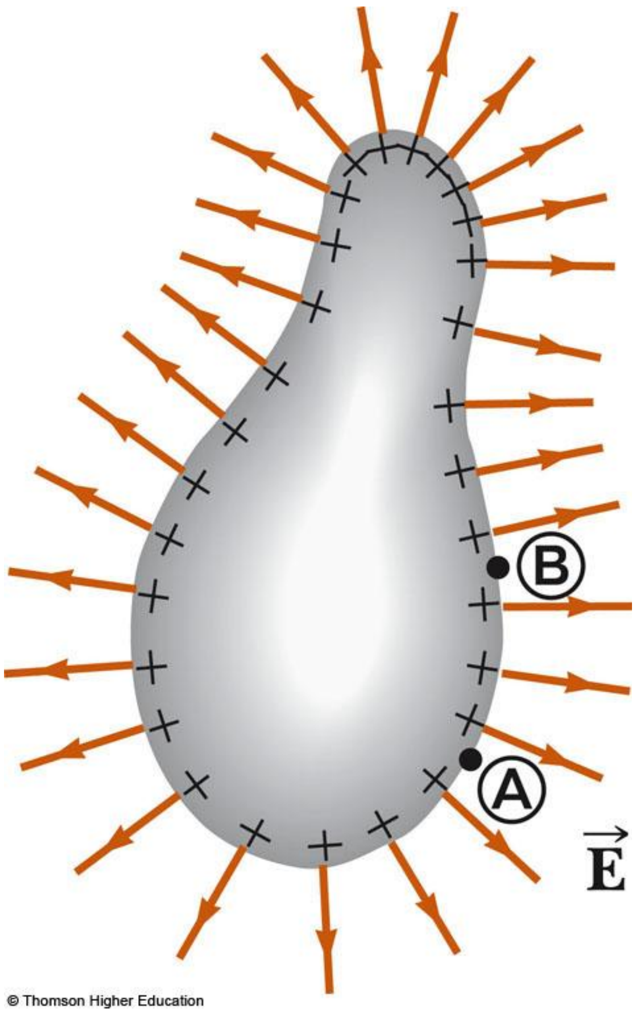
- A rod of line ℓ has a total charge of Q and a linear charge density of λ

$$V = \frac{k_e Q}{\ell} \ln\left(\frac{\ell + \sqrt{a^2 + \ell^2}}{a}\right)$$



V Due to a Charged Conductor

- Consider two points on the surface of the charged conductor as shown
- $\vec{\mathbf{E}}$ is always perpendicular to the displacement $d\vec{\mathbf{s}}$
- Therefore, $\vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = 0$
- Therefore, the potential difference between *A* and *B* is also zero

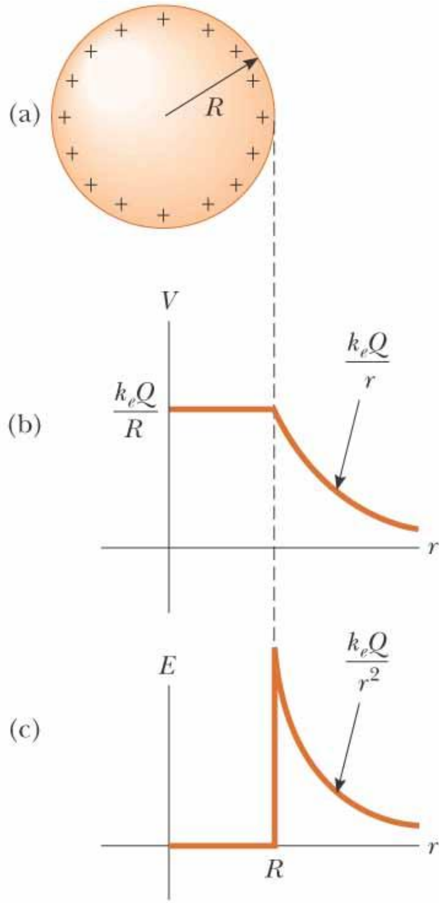


V Due to a Charged Conductor, cont.

- *V* is constant everywhere on the surface of a charged conductor in equilibrium
  - $\Delta V = 0$ between any two points on the surface
- The surface of any charged conductor in electrostatic equilibrium is an equipotential surface
- Because the electric field is zero inside the conductor, we conclude that the electric potential is constant everywhere inside the conductor and equal to the value at the surface

E **Compared to *V***

- The electric potential is a function of *r*
- The electric field is a function of r^2
- The effect of a charge on the space surrounding it:
  - The charge sets up a vector electric field which is related to the force
  - The charge sets up a scalar potential which is related to the energy

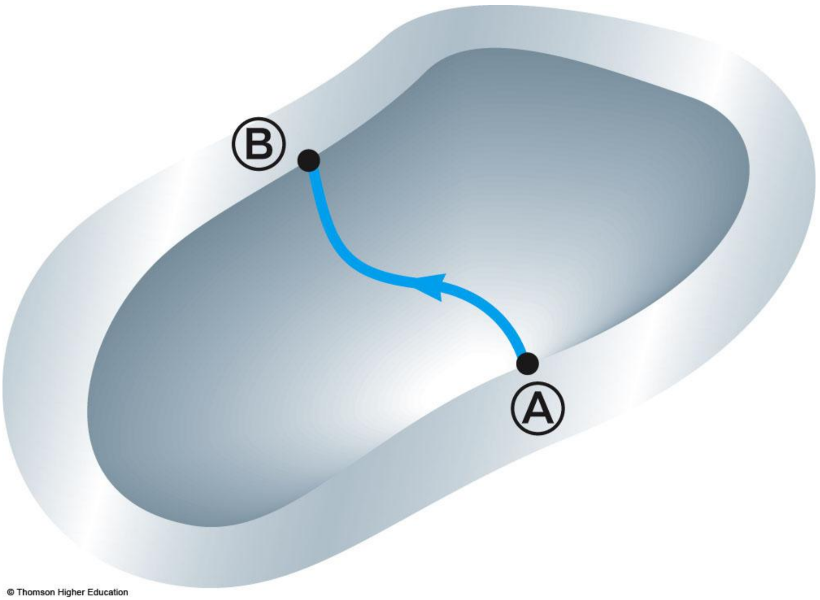


Irregularly Shaped Objects

- The charge density is high where the radius of curvature is small
  - And low where the radius of curvature is large
- The electric field is large near the convex points having small radii of curvature and reaches very high values at sharp points

Cavity in a Conductor

- Assume an irregularly shaped cavity is inside a conductor
- Assume no charges are inside the cavity
- The electric field inside the conductor must be zero



Cavity in a Conductor, cont

- The electric field inside does not depend on the charge distribution on the outside surface of the conductor
- For all paths between *A* and *B*,

$$V_B - V_A = -\int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = 0$$

- A cavity surrounded by conducting walls is a field-free region as long as no charges are inside the cavity

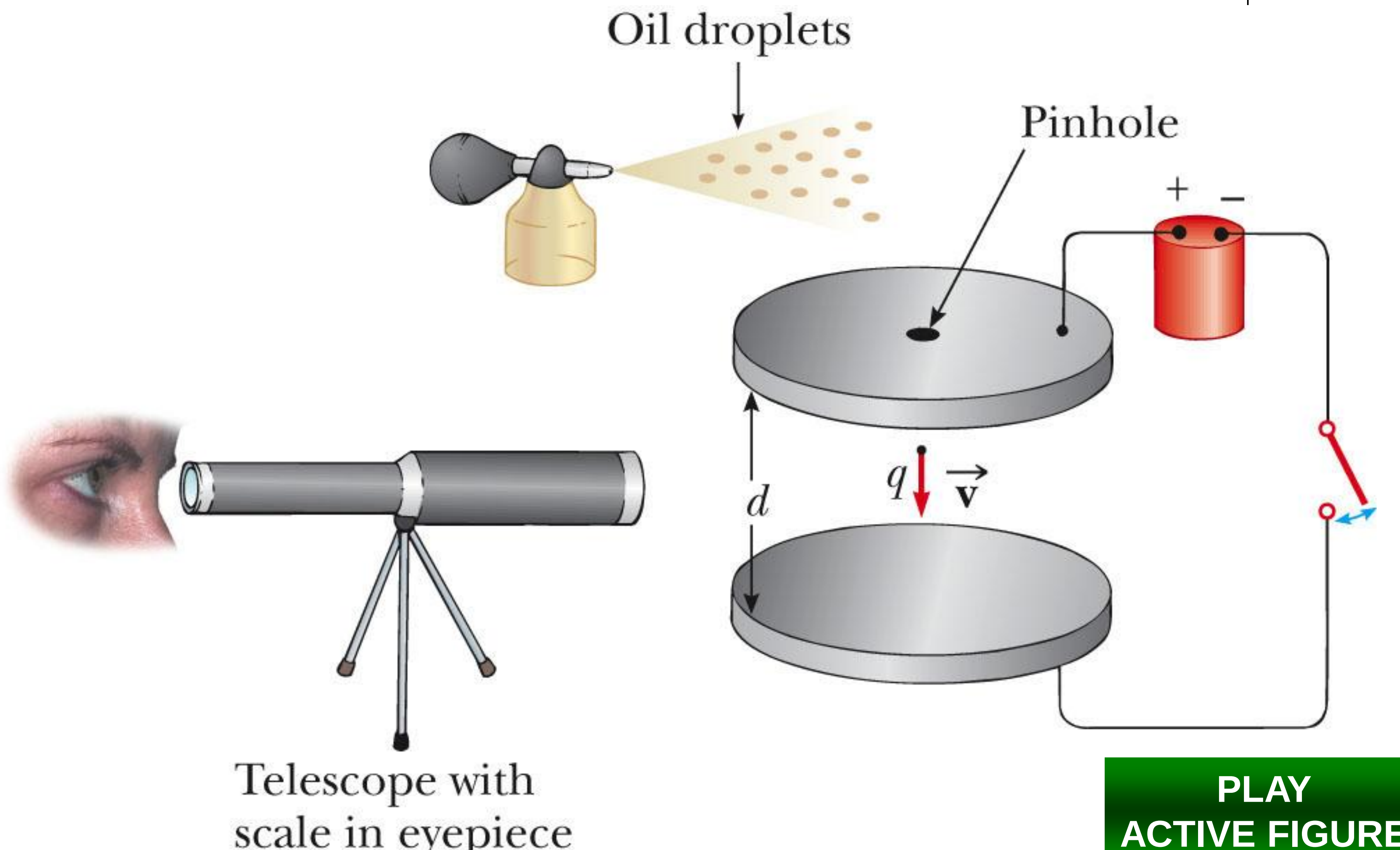
Corona Discharge

- If the electric field near a conductor is sufficiently strong, electrons resulting from random ionizations of air molecules near the conductor accelerate away from their parent molecules
- These electrons can ionize additional molecules near the conductor

Corona Discharge, cont.

- This creates more free electrons
- The **corona discharge** is the glow that results from the recombination of these free electrons with the ionized air molecules
- The ionization and corona discharge are most likely to occur near very sharp points

Millikan Oil-Drop Experiment – Experimental Set-Up



PLAY ACTIVE FIGURE

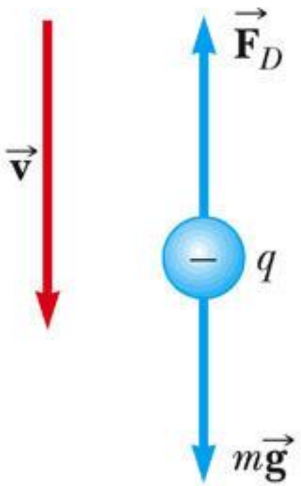
Millikan Oil-Drop Experiment

- Robert Millikan measured *e*, the magnitude of the elementary charge on the electron
- He also demonstrated the quantized nature of this charge
- Oil droplets pass through a small hole and are illuminated by a light

Oil-Drop Experiment, 2

- With no electric field between the plates, the gravitational force and the drag force (viscous) act on the electron
- The drop reaches terminal velocity with

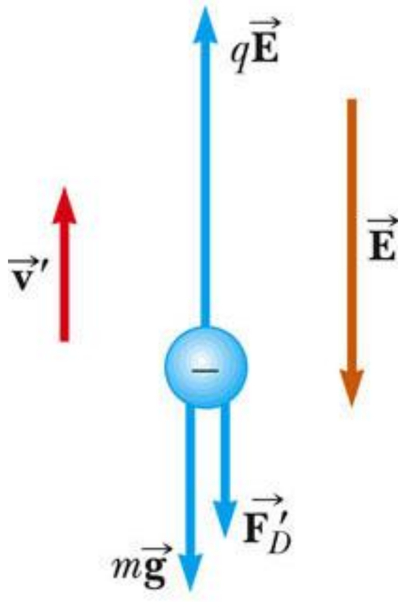
$$\vec{\mathbf{F}}_D = m\vec{\mathbf{g}}$$



(a)

Oil-Drop Experiment, 3

- When an electric field is set up between the plates
  - The upper plate has a higher potential
- The drop reaches a new terminal velocity when the electrical force equals the sum of the drag force and gravity



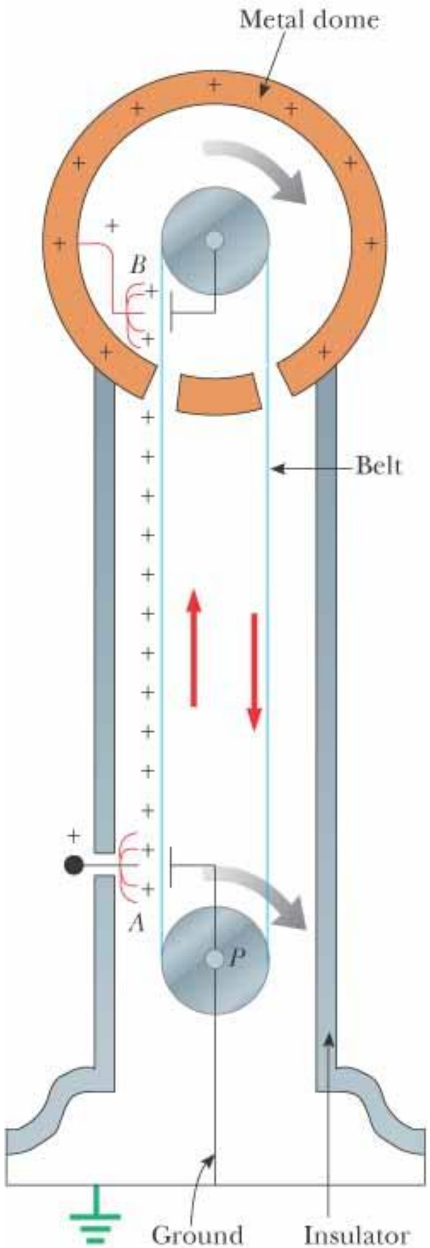
(b)

Oil-Drop Experiment, final

- The drop can be raised and allowed to fall numerous times by turning the electric field on and off
- After many experiments, Millikan determined:
  - $q = ne$ where $n = 0, -1, -2, -3, \ldots$
  - $e = 1.60 \times 10^{-19}$ C
- This yields conclusive evidence that charge is quantized
- Use the active figure to conduct a version of the experiment

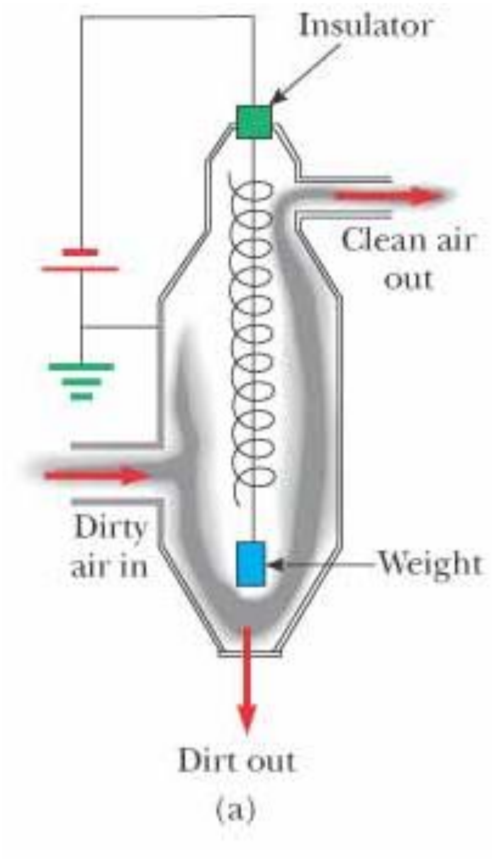
Van de Graaff Generator

- Charge is delivered continuously to a high-potential electrode by means of a moving belt of insulating material
- The high-voltage electrode is a hollow metal dome mounted on an insulated column
- Large potentials can be developed by repeated trips of the belt
- Protons accelerated through such large potentials receive enough energy to initiate nuclear reactions



Electrostatic Precipitator

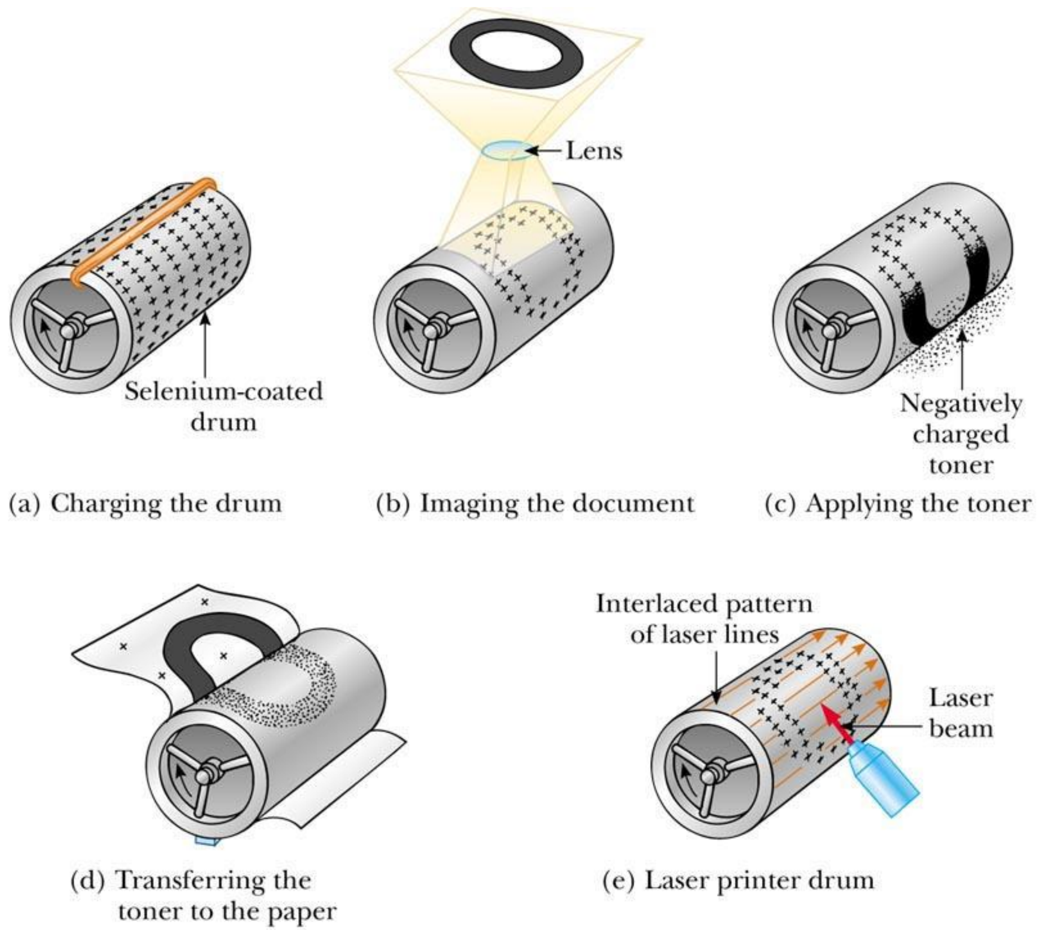
- An application of electrical discharge in gases is the electrostatic precipitator
- It removes particulate matter from combustible gases
- The air to be cleaned enters the duct and moves near the wire
- As the electrons and negative ions created by the discharge are accelerated toward the outer wall by the electric field, the dirt particles become charged
- Most of the dirt particles are negatively charged and are drawn to the walls by the electric field



Application – Xerographic Copiers

- The process of xerography is used for making photocopies
- Uses photoconductive materials
  - A photoconductive material is a poor conductor of electricity in the dark but becomes a good electric conductor when exposed to light

The Xerographic Process



Application – Laser Printer

- The steps for producing a document on a laser printer is similar to the steps in the xerographic process
- A computer-directed laser beam is used to illuminate the photoconductor instead of a lens