PHYS 2326 University Physics II –

HOMEWORK- SET #1

CHAPTERS: 23, 24, 25, 26

Ch. 23.==

1. A charge (uniform linear density = 9.0 nC/m) is distributed along the x axis from $x = 0$ to $x = 3.0$ m. Determine the magnitude of the electric field at a point on the x axis with $x = 4.0$ m.

   a. 81 N/C
   b. 74 N/C
   c. 61 N/C
   d. 88 N/C
   e. 20 N/C

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SOLUTION:
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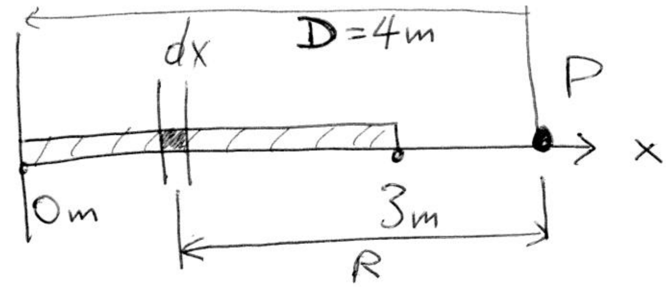
(1.)



Charge: $dq = \lambda \cdot dx$

Electric field: $d\vec{E} = k_e \cdot \dfrac{dq}{R^2}\,\hat{x}$

$$\boxed{R = D - x}$$

$$dE = k_e \frac{dq}{(D-x)^2} = k_e \cdot \frac{\lambda \cdot dx}{(D-x)^2}$$

$$E = \int dE = k_e \cdot \lambda \cdot \int_0^{3m} \frac{dx}{(D-x)^2}$$

Change variables: $\boxed{x = y + D}$ ~~at x=0m y=~~

$y = x - D$ ⟨ at $x=0$: $y = -4m$; $x=3$: $y=-1m$

$$E = k_e \cdot \lambda \cdot \int_{-4m}^{-1m} \frac{dy}{y^2} =$$

$$= k_e \cdot \lambda \cdot \left(-\frac{1}{y}\right)\Big|_{-4}^{-1} = k_e \cdot \lambda \cdot (-1)\left\{-1 + \frac{1}{4}\right\}$$

$$= k_e \cdot \lambda \cdot \left\{1 - \frac{1}{4}\right\} = k_e \cdot \lambda \cdot \frac{3}{4}$$

$$E = 60.7\,\frac{N}{C} = 61\,\frac{N}{C}.$$

2. A uniformly charged rod (length = 2.0 m, charge per unit length = 3.0 nC/m) is bent to form a semicircle. What is the magnitude of the electric field at the center of the circle?

a. 64 N/C
b. 133 N/C
c. 48 N/C
d. 85 N/C
e. 34 N/C

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SOLUTION:
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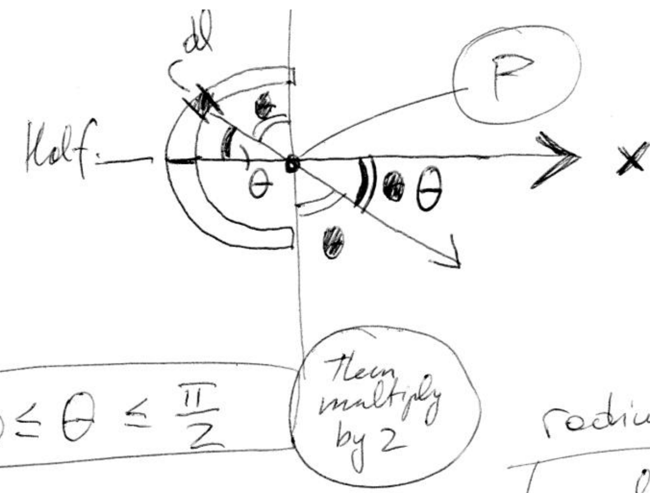
(2.)



$$0 \le \theta \le \frac{\pi}{2}$$

radius:

Semicircle: $2\pi \cdot R = 2 \cdot l \Rightarrow R = \frac{l}{\pi}$

Segment: $dl = R \cdot d\theta$

$0 \le \theta \le \frac{\pi}{2}$ $\times 2$ for x-component:

Field: $dE = k_e \cdot \frac{dq}{R^2} = k_e \cdot \frac{\lambda \cdot dl}{R^2} = k_e \cdot \frac{\lambda \cdot R\, d\theta}{R^2}$

x-component: $R = \text{constant}$

$$dE_x = dE \cdot \cos\theta$$

$$dE_x = k_e \cdot \frac{\lambda \cdot d\theta}{R} \cos\theta = \frac{k_e \cdot \lambda}{R} \cos\theta\, d\theta$$

$$E_x = 2 \cdot \int_0^{\frac{\pi}{2}} k_e \frac{\lambda \pi}{l} \cos\theta\, d\theta$$

$$E_x = 2 \cdot \frac{k_e \cdot \lambda \pi}{l} \int_0^{\frac{\pi}{2}} \cos\theta\, d\theta = 2 \frac{k_e \cdot \lambda \cdot \pi}{l} \cdot \left(\sin\theta\right)\Big|_0^{\frac{\pi}{2}}$$

$$E_x = \frac{2 \cdot 8.9876 \cdot 10^9 \frac{Nm^2}{C^2} \cdot 3 \cdot 10^{-9} \frac{C}{m} \cdot 3.14159 \cdot 1}{2\,m}$$

$$E_x = 85 \frac{N}{C}.$$

Ch. 24.===

3. A long nonconducting cylinder (radius = 6.0 mm) has a nonuniform volume charge density given by αr^2, where α = 6.2 mC/m^5 and r is the distance from the axis of the cylinder. What is the magnitude of the electric field at a point 2.0 mm from the axis?

a. 1.4 N/C
b. 1.6 N/C
c. 1.8 N/C
d. 2.0 N/C
e. 5.4 N/C

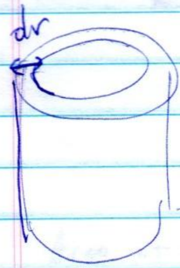
========== SOLUTION:

3.

Volume charge density

(1) $\rho = \alpha \cdot r^2$ $\left(\alpha = 6.2 \frac{mC}{m^5}\right)$

Charge: $dq = \rho \cdot dV$



$$dq = (\alpha \cdot r^2) \cdot (dV)$$

$$dV = (dr \cdot 2\pi \cdot r \cdot L)$$

$$dq_1 = \alpha \cdot r^2 \cdot (dr \cdot 2\pi \cdot r \cdot L)$$

total charge from $r = 0$ to $r = R_1 = 2\,mm$

$$q_1 = \int dq_1 = \alpha \cdot 2\pi \cdot L \int_0^{R_1} r^3\, dr$$

$$q_1 = 2\pi \cdot \alpha \cdot L \left(\frac{r^4}{4}\right)\Big|_0^{R_1}$$

$$q_1 = \frac{2\pi \cdot \alpha \cdot L \cdot R_1^4}{4} = \frac{\pi \cdot \alpha \cdot L}{2} \cdot R_1^4.$$

Gauss Law:

$$E = \frac{q_1}{\varepsilon_0 \cdot A_1}, \quad A_1 = 2\pi \cdot R_1 \cdot L$$

$$E = \frac{\pi \cdot \alpha \cdot L \cdot R_1^4}{2 \cdot \varepsilon_0 \cdot 2\pi \cdot L \cdot R_1} = \frac{\alpha \cdot R_1^3}{4 \cdot \varepsilon_0}$$

$E = 1.4 \frac{N}{C}$

$$E = \frac{6.2 \cdot 10^{-3} \frac{C}{m^5} \cdot (2 \cdot 10^{-3}\, m)^3}{4 \cdot 8.8542 \cdot 10^{-12} \frac{C^2}{N \cdot m^2}} = \frac{6.2 \cdot 8 \cdot 10^{-3} \cdot 10^{-9} \frac{m^3}{m^5} \cdot N \cdot m^2}{4 \cdot 8.8542 \cdot 10^{-12} \cdot C}$$

4. A solid nonconducting sphere (radius = 12 cm) has a charge of uniform density (30 nC/m^3) distributed throughout its volume. Determine the magnitude of the electric field 15 cm from the center of the sphere.

a. 22 N/C
b. 49 N/C
c. 31 N/C
d. 87 N/C
e. 26 N/C

============= SOLUTION: ==========

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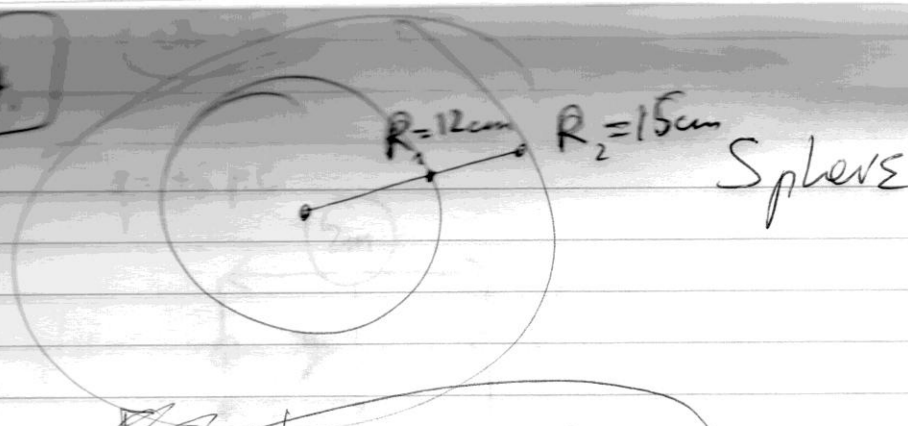


Gauss Law:

- $$E \cdot A = \frac{q}{\varepsilon_0}$$

$$E = \frac{q}{\varepsilon_0 \cdot A}$$

Area of the sphere:

$$A = 4\pi r^2 = 4\pi R^2, \quad R_2 = 15cm$$

- Total charge: $q = \sigma \cdot V'$

$$V' = \frac{4}{3}\pi r^3$$

$$V' = \frac{4}{3}\pi R_1^3$$

$$E = \frac{\frac{4}{3}\pi R_1^3 \cdot \sigma}{\varepsilon_0 (4\pi R_2^2)} = \frac{R_1^3 \cdot \sigma}{3 \varepsilon_0 R_2^2}$$

(1728)

$$E = \frac{(12 \cdot 10^{-2} m)^3 \cdot 30 \cdot 10^{-9} \frac{C}{m^3}}{3 \cdot 8.8542 \cdot 10^{-12} \frac{C^2}{N \cdot m^2} \cdot (15 \cdot 10^{-2} m)^2} = \frac{1728 \cdot 10^{-6} \cdot 10^{-9} \cdot 30 \frac{C}{m^3} \cdot m^3}{3 \cdot 8.8542 \cdot 225 \cdot 10^{-16}}$$

$$= \frac{51840 \cdot 10^{-15} \cdot 10^{1}}{5976.585 \cdot 10^{-16}} = 86.738 \frac{N}{C}$$

$$E = 86.738 \frac{N}{C} = 87 \frac{N}{C}.$$

Ch. 25===

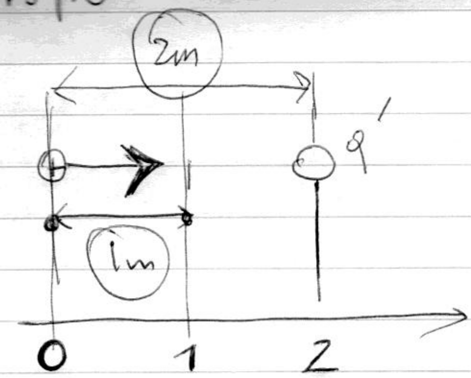
5. A particle (q = +5.0 μC) is released from rest when it is 2.0 m from a charged particle which is held at rest. After the positively charged particle has moved 1.0 m toward the fixed particle, it has a kinetic energy of 50 mJ. What is the charge on the fixed particle?

   a. –2.2 μC
   b. +6.7 μC
   c. –2.7 μC
   d. +8.0 μC
   e. –1.1 μC

================= SOLUTION =============

5.

$q = +5\ \mu C$



$E_{kin} = 50\ mJ$

$dW = F(x) \cdot dx \qquad F(x) = k_e \cdot \frac{q \cdot q'}{x^2}$

$dW = k_e \cdot \frac{q \cdot q'}{(R-x)^2} dx \qquad R = 2$

$W = \int dW = k_e \cdot q \cdot q' \int_0^1 \frac{dx}{(R-x)^2}$

$W = k_e \cdot q \cdot q' \cdot \int_{-R}^{1-R} \frac{d(y+R)}{y^2}$

$x = y + R$; $x = 0$, $y = -R$; $x = 1$, $1 = y + R$, $y = 1 - R$

$W = k_e \cdot q \cdot q' \cdot \left(-\frac{1}{y}\right) \Big|_{-R}^{1-R} \Big|_{-2}^{-1}$

$W = k_e \cdot q \cdot q' \cdot - \left(\frac{1}{+R} - \frac{1}{(1-R)}\right)$

$W = E_{kin}$

$$W = E_{kin} = k_e \cdot q \cdot q' \cdot \left[\quad \right]$$

$$= \frac{k_e \cdot q \cdot q'}{2} \quad \left[\quad \right] \left(- \left[\frac{1}{-1} - \frac{1}{-2} \right] \right)$$

Final Initial

$$= (-1) \cdot \left[\frac{1}{2} - 1 \right] = \left[1 - \frac{1}{2} \right] = \frac{1}{2}.$$

$$= \frac{1}{2}$$

$$q' = \frac{2 \cdot E_{kin}}{k_e \cdot q}$$

$$q' = \frac{2 \cdot 50 \cdot 10^{-3}\ J \cdot 1\ m}{8.9876 \cdot 10^{9} \frac{Nm^2}{C^2} \cdot 5 \cdot 10^{-6}\ C}$$

$J = N \cdot m$

$$= -2.225 \times 10^{-6}\ C, \quad = -2.225\ \mu C$$

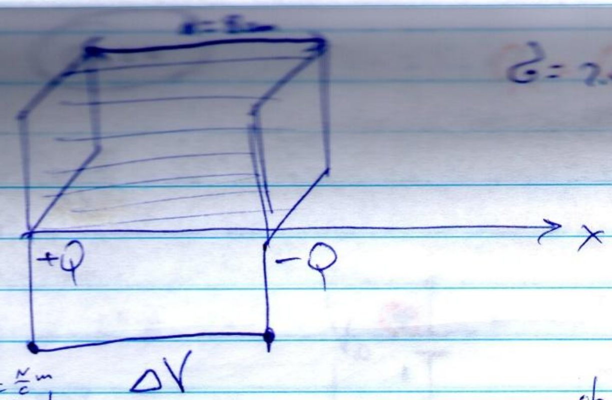
✓

6. Two large parallel conducting plates are 8.0 cm apart and carry equal but opposite charges on their facing surfaces. The magnitude of the surface charge density on either of the facing surfaces is 2.0 nC/m^2. Determine the magnitude of the electric potential difference between the plates.

a. 36 V
b. 27 V
c. 18 V
d. 45 V
e. 16 V

======SOL===========

6.



$\sigma = 2.0 \frac{nC}{m^2}$.

$E = \frac{N}{C}$

$1V = \frac{N}{C} m$

$V = E \cdot d$

$1V = E$

$\phi_E = EA = \frac{q_{in}}{\varepsilon_0}$

$q_{in} = \sigma \cdot A$

$$E = \frac{q_{in}}{\varepsilon_0 \cdot A} = \frac{\sigma \cdot A}{\varepsilon_0 \cdot A}$$

$$dV = -(\vec{E} \cdot d\vec{x})$$

$$dV = -E \cdot dx$$

$$\Delta V = -\int E \cdot dx = -E \cdot \int_0^d dx$$

$$\Delta V = -E \cdot d$$

$$\Delta V = -\frac{\sigma}{\varepsilon_0} \cdot d =$$

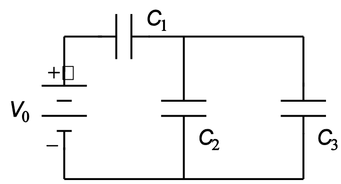
$\frac{N}{C} \cdot m = 1V$

$$|\Delta V| = \frac{\sigma}{\varepsilon_0} \cdot d = \frac{2.0 \cdot 10^{-9} \frac{C}{m^2} \cdot 8 \cdot 10^{-2} m}{8.8542 \cdot 10^{-12} \frac{C^2}{N \cdot m^2}}$$

$$= \frac{10^{-11} \cdot 10^{1}}{10^{-12}} = 18.07 \frac{N}{C} m = 18.1 \text{ Volt}$$

Ch. 26==

7. Determine the energy stored in C_2 when C_1 = 15 μF, C_2 = 10 μF, C_3 = 20 μF, and V_0 = 18 V.



a. 0.72 mJ
b. 0.32 mJ
c. 0.50 mJ
d. 0.18 mJ
e. 1.60 mJ

========= SOL ===========

Q = 26 $Q_1 = Q_2 + Q_3$

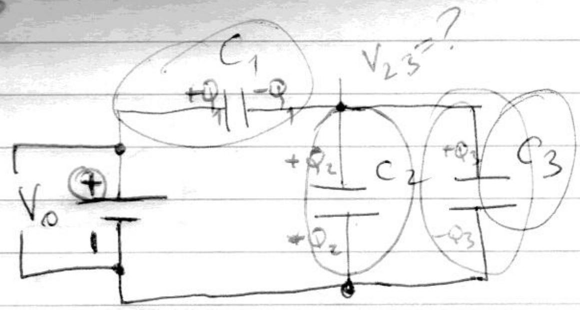
7. Energy stored in C_2 = ?

$C_1 = 15 \cdot 10^{-6} F$

$C_2 = 10 \cdot 10^{-6} F$

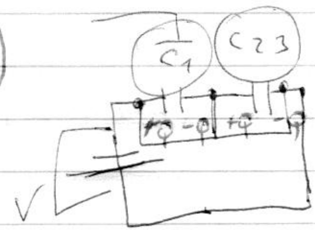
$C_3 = 20 \cdot 10^{-6} F$

$V_0 = 18 V$



Potential energy stored in the charged capacitor:

$$U = \frac{Q^2}{2C} = \frac{1}{2} Q \Delta V = \frac{1}{2} C (\Delta V)^2 \quad (1)$$



$C = \frac{Q}{V_0}$ $C_1 = \frac{Q_1}{V_1}$

need = ?

$V_0 = V_1 + V_{23}$

$Q = C \cdot V_0$ (3) $C_{23} = \frac{Q_{23}}{V_{23}}$

Parallel: C_2 & C_3: $C_{23} = C_2 + C_3$

$Q = Q_1$ (4) $Q_1 = Q_2 + Q_3$ (5)

$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_{23}}$

also $V_0 = V_1 + V_{23}$ (6)

$C = \frac{Q}{V}$

$Q = C_1 \cdot V_1$

$\frac{1}{C} = \frac{C_{23} + C_1}{C_1 \cdot C_{23}}$

$V_1 = \frac{Q}{C_1}$ $\Rightarrow V_1 = \frac{Q_1}{C_1} = \frac{Q}{C_1}$ (7)

$V_{23} = V_0 - V_1 = V_0 - \frac{Q}{C_1}$ (8)

$C = \frac{C_1 \cdot C_{23}}{C_{23} + C_1}$ (2)

$$C_{23} = C_2 + C_3 \quad (1)$$

$$C_{23} = 30 \cdot 10^{-6}\ F$$

$$C = \frac{C_1 \cdot C_{23}}{C_1 + C_{23}} \quad (2)$$

$$C = \frac{15 \cdot 10^{-6} \cdot 30 \cdot 10^{-6}\ F^2}{45 \cdot 10^{-6}\ F}$$

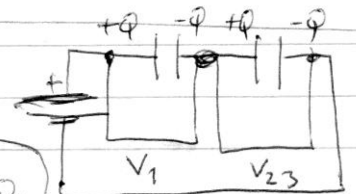
total capacitance: $\boxed{C = 10 \cdot 10^{-6}\ F} = 10^{-5}\ F$

total charge on the plates (one-side):

$$\boxed{Q = C \cdot V_0}$$

$$Q = 10^{-5}\ F \cdot 18\ V = 1.8 \cdot 10^{-4}\ C.$$

We know:



$$V_0 = V_1 + V_{23}$$

$$Q = Q_1 = Q_{23} \qquad \boxed{Q_{23} = Q_2 + Q_3}$$

$$\Rightarrow \quad C_1 = \frac{Q_1}{V_1} \Rightarrow C_1 = \frac{Q}{V_1} \Rightarrow \boxed{V_1 = \frac{Q}{C_1}}$$

$$V_1 = \frac{1.8 \cdot 10^{-4}\ C}{15 \cdot 10^{-6}\ F} = 0.12 \cdot 10^{2}\ V = 12\ V$$

then $\boxed{V_{23} = V_0 - V_1 = 18V - 12V = 6V}$

Thus, $U_2 = \frac{1}{2} C_2 \cdot V_{23}^2 = \frac{1}{2} 10^{-5} F \cdot (6V)^2 = 18 \cdot 10^{-5}\ J = 0.18 \cdot 10^{-3}\ J$

8. A 30-μF capacitor is charged to an unknown potential V_0 and then connected across an initially uncharged 10-μF capacitor. If the final potential difference across the 10-μF capacitor is 20 V, determine V_0.

   a. 13 V
   b. 27 V
   c. 20 V
   d. 29 V
   e. 60 V

========== SOL =======

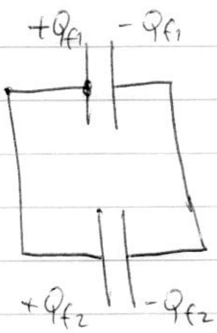
8.

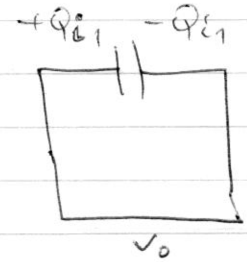
$C_1 = 30\mu F$
$C_2 = 10\mu F$
$V_f = 20V$ $\frac{1}{2}$

V_0

$Q_1 = Q_2$

$\frac{Q^2}{2C_1} + \frac{Q^2}{2C_2} = \frac{1}{2} C_1 V_0^2$





final

initial

after:

$Q_f = Q_{1f} + Q_{2f}$

$Q_f = C_1 \cdot V_f + C_2 \cdot V_f$

Charge:

$Q_{i_1} = C_1 \cdot V_0 = Q_{total}$

$Q_i = Q_{i_1} = C_1 \cdot V_0$

$Q_f = Q_i$

$C_1 \cdot V_f + C_2 \cdot V_f = C_1 \cdot V_0$

$$\Rightarrow \boxed{V_0 = \frac{C_1 + C_2}{C_1} V_f}$$

$V_0 = 20V \cdot \frac{40 \cdot 10^{-6} F}{30 \cdot 10^{-6} F} = 26.7\ V$

$= 27V.$