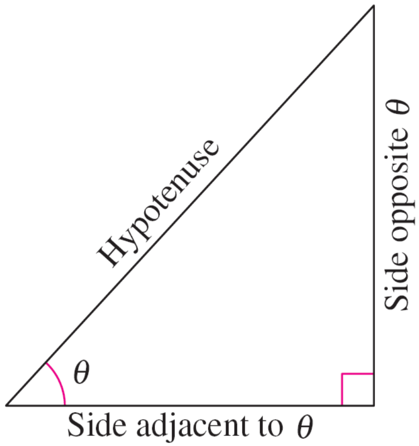
# 1.3 RIGHT TRIANGLE TRIGONOMETRY

# The Six Trigonometric Functions

Consider a right triangle, with one acute angle labeled $\theta$.



# The Six Trigonometric Functions

The three sides of the triangle are the **hypotenuse,** the **opposite side**, and the **adjacent side.**

We can form six ratios = the six trigonometric functions.

**sine  cosecant  cosine  secant  tangent  cotangent**

# The Six Trigonometric Functions

$$\sin\,\theta = \frac{\text{opp}}{\text{hyp}} \qquad \cos\,\theta = \frac{\text{adj}}{\text{hyp}} \qquad \tan\,\theta = \frac{\text{opp}}{\text{adj}}$$

$$\csc\,\theta = \frac{\text{hyp}}{\text{opp}} \qquad \sec\,\theta = \frac{\text{hyp}}{\text{adj}} \qquad \cot\,\theta = \frac{\text{adj}}{\text{opp}}$$

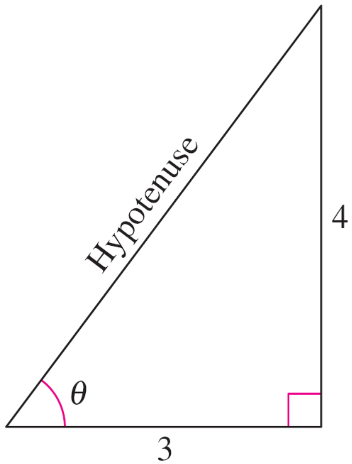
opp = the length of the side *opposite* $\theta$

adj = the length of the side *adjacent to* $\theta$

hyp = the length of the *hypotenuse*

# Example 1 – *Evaluating Trigonometric Functions*

Find the values of the six trigonometric functions of $\theta$.




By the Pythagorean Theorem,

$(\text{hyp})^2 = (\text{opp})^2 + (\text{adj})^2$, it follows that

$$\begin{aligned} \text{hyp} &= \sqrt{4^2 + 3^2} \\ &= \sqrt{25} \\ &= 5. \end{aligned}$$

# Example 1 – *Solution*

cont'd

So, the six trigonometric functions of $\theta$ are

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{4}{5} \qquad \csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{5}{4}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{3}{5} \qquad \sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{5}{3}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{4}{3} \qquad \cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{3}{4}.$$

# The Six Trigonometric Functions

## Sines, Cosines, and Tangents of Special Angles

$$\sin 30^\circ = \sin \frac{\pi}{6} = \frac{1}{2} \qquad \cos 30^\circ = \cos \frac{\pi}{6} = \frac{\sqrt{3}}{2} \qquad \tan 30^\circ = \tan \frac{\pi}{6} = \frac{\sqrt{3}}{3}$$

$$\sin 45^\circ = \sin \frac{\pi}{4} = \frac{\sqrt{2}}{2} \qquad \cos 45^\circ = \cos \frac{\pi}{4} = \frac{\sqrt{2}}{2} \qquad \tan 45^\circ = \tan \frac{\pi}{4} = 1$$

$$\sin 60^\circ = \sin \frac{\pi}{3} = \frac{\sqrt{3}}{2} \qquad \cos 60^\circ = \cos \frac{\pi}{3} = \frac{1}{2} \qquad \tan 60^\circ = \tan \frac{\pi}{3} = \sqrt{3}$$

# Trigonometric Identities

*Reciprocal Identities*

$$\sin\theta = \frac{1}{\csc\theta} \qquad \cos\theta = \frac{1}{\sec\theta} \qquad \tan\theta = \frac{1}{\cot\theta}$$

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

*Quotient Identities*

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

*Pythagorean Identities*

$$\sin^2\theta + \cos^2\theta = 1 \qquad 1 + \tan^2\theta = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

Note that $\sin^2\theta$ represents $(\sin\theta)^2$, $\cos^2\theta$ represents $(\cos\theta)^2$, and so on.

# Example 4 – *Applying Trigonometric Identities*

Let $\theta$ be an acute angle such that $\sin\theta = 0.6$. Find the values of (a) $\cos\theta$ and (b) $\tan\theta$ using trigonometric identities.

**a.** Use the Pythagorean identity $\sin^2\theta + \cos^2\theta = 1$.

$$(0.6)^2 + \cos^2\theta = 1$$

Substitute 0.6 for $\sin\theta$.

$$\cos^2\theta = 1 - (0.6)^2$$

Subtract $(0.6)^2$ from each side.

$$= 0.64$$

$$\cos\theta = \sqrt{0.64}$$

Extract the positive square root.

$$= 0.8$$

# Example 4 – *Solution*

cont'd

**b.** Knowing the sine and cosine of $\theta$, we can find the tangent of $\theta$ to be

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{0.6}{0.8} = 0.75.$$

# Evaluating Trigonometric Functions with a Calculator

To use a calculator to evaluate trigonometric functions of angles measured in degrees, first set the calculator to *degree* mode and then proceed.

For instance, you can find values of cos 28° and sec 28° as follows.

| *Function* | *Mode* | *Calculator Keystrokes* | *Display* |
|---|---|---|---|
| **a.** cos 28° | Degree | COS 28 ENTER | 0.8829476 |
| **b.** sec 28° | Degree | ( COS ( 28 ) ) $x^{-1}$ ENTER | 1.1325701 |

# Evaluating Trigonometric Functions with a Calculator

Throughout this text, angles are assumed to be measured in radians unless noted otherwise.

For example, sin 1 means the sine of 1 radian and sin 1° means the sine of 1 degree.

# Applications Involving Right Triangles

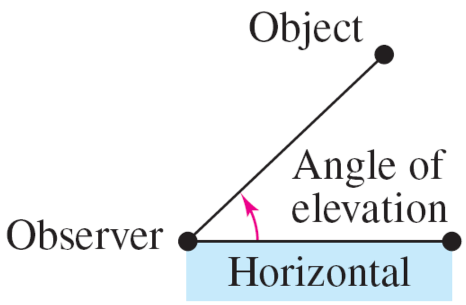
# Applications Involving Right Triangles

Many applications of trigonometry involve a process called **solving right triangles.**

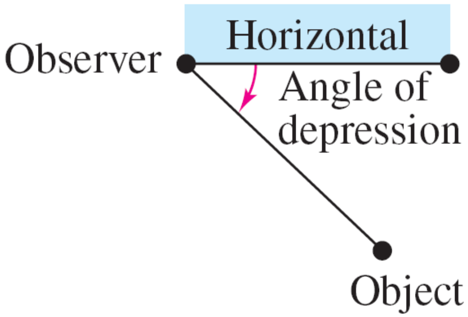
You are asked to find a missing side or angle from a right triangle.

# Applications Involving Right Triangles

The **angle of elevation** represents the angle from the horizontal upward to an object.

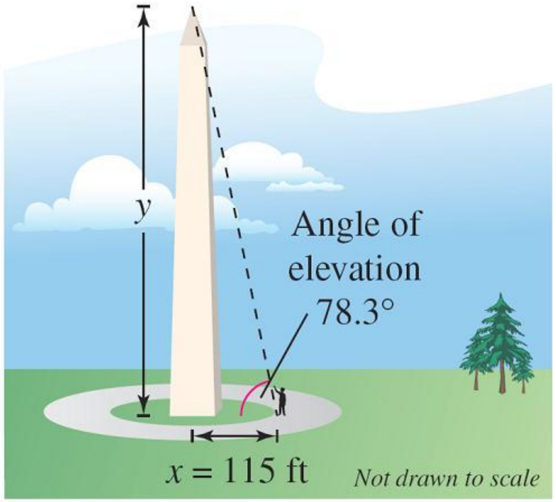



For objects that lie below the horizontal, it is common to use the term **angle of depression**



# Example 7 – *Using Trigonometry to Solve a Right Triangle*

A surveyor is standing 115 feet from the base of the Washington Monument. The surveyor measures the angle of elevation to the top of the monument as 78.3°. How tall is the Washington Monument?



# Example 7 – *Solution*

$$\tan 78.3° = \frac{\text{opp}}{\text{adj}} \qquad = \frac{y}{x}$$

where $x$ = 115 and $y$ is the height of the monument.

Solving for y, the height of the monument, we get

$$y = x \tan 78.3°$$

$$\approx 115(4.82882)$$

$$\approx 555 \text{ feet.}$$