8.4 COMPOUND INTEREST

Objective 1 – Use compound interest formulas.

Compound interest involves interest computed on the original principal as well as on any accumulated
interest.

If you deposit P dollars at rate r, in decimal form, subject to compound interest, then the amount, A, of money in the account after t years is given by $A = P(1+r)^t$.

The period of time between two interest payments is called the **compounding period**. When compound interest is paid once per year, the compounding period is one year. We say that the interest is **compounded annually**. If compound interest is paid twice per year, the compounding period is six months and we say that the interest is **compounded semiannually**. When compound interest is paid four times per year, the compounding period is three months and the interest is said to be **compounded quarterly**.

If you deposit P dollars at rate r, in decimal form, subject to compound interest paid n times per year, then the amount, A, of money in the account after t years is given by $A = P\left(1+\frac{r}{n}\right)^{nt}$.

Some banks use **continuous compounding**, where the compounding periods increase infinitely. After t years, the balance, A, in an account with principal P and annual interest rate r (in decimal form) is given by the following formulas:

1. For n compounding periods per year: $A = P\left(1+\frac{r}{n}\right)^{nt}$.

2. For continuous compounding: $A = Pe^{rt}$.

EXAMPLE

A sum of $10,000 is invested at an annual rate of 8%. Find the balance in the account after five years subject to **a.** quarterly compounding and **b.** continuous compounding.

a. $A = P\left(1+\frac{r}{n}\right)^{nt}$

$$A = 10,000\left(1+\frac{0.08}{4}\right)^{4(5)} = \$14,859.47$$

b. $A = Pe^{rt}$

$$A = 10,000e^{0.08(5)} = \$14,918.25$$

Objective 2 – Calculate present value.

Calculating Present Value: If *A* dollars are to be accumulated in *t* years in an account that pays rate *r* compounded *n* times per year, then the present value, *P*, that needs to be invested now is given by

$$P = \frac{A}{\left(1+\frac{r}{n}\right)^{nt}}.$$

EXAMPLE
How much money should be deposited today in an account that earns 7% compounded weekly so that it will accumulate to $10,000 in eight years?

A = $10,000, R = 0.07, N = 52, T =8

$$P = \frac{10{,}000}{\left(1+\frac{0.07}{52}\right)^{52\cdot 8}} \approx \frac{10{,}000}{1.750013343} \approx \$5714.25$$

Objective 3 – Understand and compute effective annual yield.

The **effective annual yield**, or the **effective rate**, is the simple interest rate that produces the same amount of money in an account at the end of one year as when the account is subject to compound interest at a stated rate.

Suppose that an investment has a nominal interest rate, *r*, in decimal form, and pays compound interest *n* times per year. The investment's effective annual yield, *Y*, in decimal form, is given by $Y = \left(1+\frac{r}{n}\right)^n - 1$. The decimal form of *Y* given by the formula should then be converted to a percent.

If you are selecting the best investment from two or more investments, the best choice is the account with the greatest effective annual yield. When borrowing money, the effective rate or effective annual yield is usually called the **annual percentage rate**.

EXAMPLE
What is the effective annual yield of an account paying 8% compounded quarterly?

$$\boldsymbol{Y = \left(1+\frac{r}{n}\right)^n - 1}$$

$$Y = \left(1+\frac{0.08}{4}\right)^4 - 1 \approx 0.0824 = 8.24\%$$

PRACTICE 8.4

Solve the following exercise using the appropriate compound interest formulas. Round answers to the nearest cent.

1. Suppose that you have $12,000 to invest. Which investment yields the greater return over three years: 7% compounded monthly or 6.85% compounded continuously?

Round your answer to the nearest cent.

2. How much money should be deposited today in an account that earns 9.5% compounded monthly so that it will accumulate to $10,000 in three years?

A passbook savings account has a rate of 6%.

3. Find the effective annual yield, rounded to the nearest tenth of a percent, if the interest is compounded monthly.