3.2: Compound Statements and Connective Notes

1. Express compound statements in symbolic form.

**Simple** statements convey one idea with no connecting words.

**Compound** statements combine two or more simple statements using connectives.

Connectives include words such as ___**and**___, _**or**_, __**if**__......... __**then**__, and if and only if.

If *p* and *q* are two simple statements, then the compound statement "*p* and *q*" is symbolized by $p \wedge q$.

The compound statement formed by connecting statements with the word *and* is called a __**conjunction**__. The symbol for *and* is $\wedge$.

Let *p* and *q* represent the following simple statements:

p: It is after 5 P.M.

q: They are working.

Write each compound statement below in symbolic form:

__$p \wedge q$__ **a.** It is after 5 P.M. and they are working.

__ $p \wedge \sim q$ ___ **b.** It is after 5 P.M. and they are not working.

Common English Expressions for *p^q*

Symbolic Statement	English Statement	Example *p*: It is after 5 P.M. *q*: They are working.
$p \wedge q$	*p* and *q*.	It is after 5 P.M. and they are working.
$p \wedge q$	*p* but *q*.	It is after 5 P.M., but they are working.
$p \wedge q$	*p* yet *q*.	It is after 5 P.M., yet they are working.
$p \wedge q$	*p* nevertheless *q*.	It is after 5 P.M.; nevertheless, they are working.

Or Statements

__**Disjunction**__ is a compound statement formed using the **inclusive or** represented by the symbol $\vee$. Thus, "*p* or *q* or both" is symbolized by $p \vee q$.

The connective *or* can mean two different things.

Consider the statement: I visited London or Paris.

This statement can mean (**exclusive or**) - I visited London or Paris but not both.
It can also mean (**inclusive or**) - I visited London or Paris or both.

Here is an example of Translating from English to Symbolic Form:

Let p and q represent the following simple statements:

p: The bill receives majority approval.

q: The bill becomes a law.

Write each compound statement below in symbolic form:

___$\boldsymbol{p \vee q}$___**a.** The bill receives majority approval or the bill becomes a law.

____$\boldsymbol{p \vee \sim q}$___**b.** The bill receives majority approval or the bill does not become a law.

If-Then Statements

The compound statement "**If** $\boldsymbol{p}$**, then** $\boldsymbol{q}$ is symbolized by $\boldsymbol{p \to q}$.

This is called a ___**conditional**___ statement.

The statement before the $\to$ is called the___**antecedent**____.

The statement after the $\to$ is called the ___**consequent**___.

Here are examples of writing if-then statements in symbolic form:

Let p and q represent the following simple statements:

p: A person is a father.

q: A person is a male.

Write each compound statement below in symbolic form:

__$\boldsymbol{p \to q}$__**a.** If a person is a father, then that person is a male.

$\boldsymbol{\sim q \to p}$**b.** If a person is not a male, then that person is not a father.

Common English Expressions for $p \rightarrow q$

Symbolic Statement	**English Statement**	**Example** ***p*: A person is a father.** ***q*: A person is a male.**
$p \rightarrow q$	If p then q.	If a person is a father, then that person is a male.
$p \rightarrow q$	q if p.	A person is a male, if that person is a father.
$p \rightarrow q$	p is sufficient for q.	Being a father is sufficient for being a male.
$p \rightarrow q$	q is necessary for p.	Being a male is necessary for being a father.
$p \rightarrow q$	p only if q.	A person is a father only if that person is a male.
$p \rightarrow q$	*Only if* q, p.	Only if a person is a male is that person a father.

**Biconditional** statements are conditional statements that are true if the statement is still true when the antecedent and consequent are reversed.

The compound statement "*p* if and only if *q*"

(abbreviated as *iff*) is symbolized by $p \leftrightarrow q$.

- If a person is a father, then that person is a male.

  true

- If a person is a male, then that person is a father.

  not necessarily true

- If a person is an unmarried male, then that person is a bachelor.

  true

- If a person is a bachelor, then that person is an unmarried male.

  also true

Common English Expressions for $p \leftrightarrow q$

Symbolic Statement	English Statement	Example *p*: A person is an unmarried male. *q*: A person is a bachelor.
$p \leftrightarrow q$	*p* if and only if *q*.	A person is an unmarried male if and only if that person is a bachelor.
$p \leftrightarrow q$	*q if and only if p.*	A person is a bachelor if and only if that person is an unmarried male.
$p \leftrightarrow q$	*If p* then *q, and if q then p.*	If a person is an unmarried male then that person is a bachelor, and if a person is a bachelor, then that person is an unmarried male.
$p \leftrightarrow q$	*p* is necessary and sufficient for *q*.	Being an unmarried male is necessary and sufficient for being a bachelor.
$p \leftrightarrow q$	*q is necessary and sufficient for p.*	Being a bachelor is necessary and sufficient for being an unmarried male.

Statements of Symbolic Logic

Name	Symbolic Form	Common English Translations
Negation	$\sim p$	Not *p*. It is not true that *p*.
Conjunction	$p \wedge q$	*p* and *q*, *p* but *q*.
Disjunction	$p \vee q$	*p* or *q*.
Conditional	$p \rightarrow q$	If *p*, then *q*, *p* is sufficient for *q*, *q* is necessary for *p*.
Biconditional	$p \leftrightarrow q$	*p* if and only if *q*, *p* is necessary and sufficient for *q*.

2. Express symbolic statements with and without parentheses in English.

Here are examples of symbolic statements in English.

Let *p* and *q* represent the following simple statements:

p: She is wealthy.

q: She is happy.

Write each of the following symbolic statements in words:

a. $\sim(p \wedge q)$

___**It is not true that she is wealthy and happy.**___

b. $\sim p \wedge q$

___**She is not wealthy and she is happy.**___

c. $\sim(p \vee q)$

___**She is neither wealthy nor happy. (Literally, it is not true that she is either wealthy or happy.**___

Let *p, q,* and *r* represent the following simple statements:

p: A student misses lecture.

q: A student studies.

r: A student fails.

Write each of these symbolic statements in words:

a. $(q \wedge \sim p) \rightarrow \sim r$

___**If a student studies and does not miss lecture, then the student does not fail.**__

Write each of these symbolic statements in words:

b. $q \wedge (\sim p \rightarrow \sim r)$

A student studies, and if the student does not miss lecture, then the student does not fail.

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3. Dominance of Connectives – Use the dominance of connectives.

If a symbolic statement appears without parentheses, statements before and after the most *dominant connective* should be grouped.

The dominance of connectives used in symbolic logic is defined in the following order.

Statement	Most Dominant Connective Highlighted in Red	Statements Meaning Clarified with Grouping Symbols	Type of Statement
$p \rightarrow q \wedge \sim r$	$p \rightarrow q \wedge \sim r$	$p \rightarrow (q \wedge \sim r)$	Conditional
$p \wedge q \rightarrow \sim r$	$p \wedge q \rightarrow \sim r$	$(p \wedge q) \rightarrow \sim r$	Conditional
$p \leftrightarrow q \rightarrow r$	$p \leftrightarrow q \rightarrow r$	$p \leftrightarrow (q \rightarrow r)$	Biconditional
$p \rightarrow q \leftrightarrow r$	$p \rightarrow q \leftrightarrow r$	$(p \rightarrow q) \leftrightarrow r$	Biconditional
$p \wedge \sim q \rightarrow r \vee s$	$p \wedge \sim q \rightarrow r \vee s$	$(p \wedge \sim q) \rightarrow (r \vee s)$	Conditional
$p \wedge q \vee r$	$p \wedge q \vee r$	The meaning is ambiguous.	?

1. Negation, $\sim$ — Least dominant
2. Conjunction, $\wedge$
   Disjunction, $\vee$ — Same level of dominance
3. Conditional, $\rightarrow$
4. Biconditional, $\leftrightarrow$ — Most dominant

Using the Dominance of Connectives

Grouping symbols must be given with this statement to determine whether it means $(p \wedge q) \vee r$, a disjunction, or $p \wedge (q \vee r)$, a conjunction.

Here are examples of using dominance connectives:

Let *p, q,* and *r* represent the following simple statements.

p: I fail the course.

q: I study hard.

r: I pass the final.

Write each compound statement in symbolic form:

a. I do not fail the course if and only if I study hard and I pass the final.

$\sim p \leftrightarrow (q \wedge r)$

b. I do not fail the course if and only if I study hard, and I pass the final.

$(\sim p \leftrightarrow q) \wedge r$

Write each compound statement below in symbolic form:

I do not fail the course if and only if I study hard and I pass the final.

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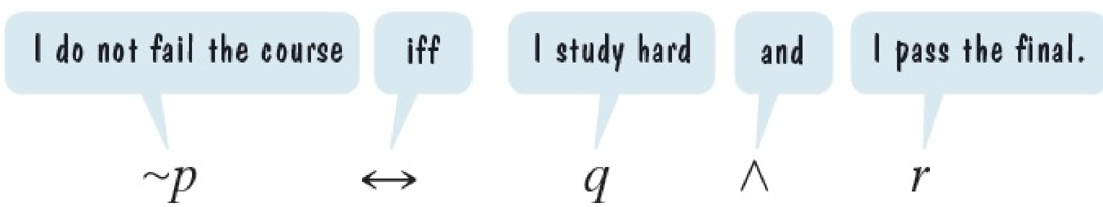
p: I fail the course.

q: I study hard.

r: I pass the final.



Because the most dominant connective that appears is $\leftrightarrow$, the symbolic form with parentheses is $\sim p \leftrightarrow (q \wedge r)$.

I do not fail the course iff I study hard, and I pass the final.

In this statement, the comma indicates the grouping, so it is not necessary to apply the dominance of connectives. The symbolic form of the statement is $(\sim p \leftrightarrow q) \wedge r$.