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Chapter 13 POLYNOMIALS: FACTORING

13.1 Introduction to Factoring

Learning Objectives

A Find the greatest common factor, the GCF, of monomials.

B Factor polynomials when the terms have a common factor, factoring out the greatest common factor.

C Factor certain expressions with four terms using factoring by grouping.

Key Terms

Use the vocabulary terms listed below to complete each statement in Exercises 1–4. Terms may be used more than once.

factor **factoring by grouping** **factorization**

1. To ____________________ a polynomial is to express it as a product.

2. A(n) ____________________ of a polynomial P is a polynomial that can be used to express P as a product.

3. A(n) ____________________ of a polynomial is an expression that names that polynomial as a product.

4. Certain polynomials with four terms can be factored using ____________________ .

GUIDED EXAMPLES AND PRACTICE

Objective A Find the greatest common factor, the GCF, of monomials.

Review this example for Objective A:

1. Find the GCF of $6pq^3$, $24p^2q^2$, and $18p^3q$.

$$\begin{aligned} 6pq^2 &= 2\cdot 3\cdot p\cdot q^3 \\ 24p^2q^2 &= 2\cdot 2\cdot 2\cdot 3\cdot p^2\cdot q^2 \\ 18p^3q &= 2\cdot 3\cdot 3\cdot p^3\cdot q \end{aligned}$$

The GCF is $2\cdot 3\cdot p\cdot q$, or $6pq$.

Practice this exercise:

1. Find the GCF of $16x^3y^2$, $24xy^3$, and $12x^2y^2$.

Objective B Factor polynomials when the terms have a common factor, factoring out the greatest common factor.

Review this example for Objective B:

2. Factor $4x^2+4x-20$.

$$\begin{aligned} 4x^2+4x-20 &= 4\cdot 4x+4\cdot x-4\cdot 5 \\ &= 4(4x+x-5) \end{aligned}$$

Practice this exercise:

2. Factor $3x^8-30x^6+15x^5$.

Objective C Factor certain expressions with four terms using factoring by grouping.

Review this example for Objective C:

3. Factor $3x^3 - 6x^2 - x + 2$.

$$\begin{aligned} 3x^3 - 6x^2 - x + 2 &= (3x^3 - 6x^2) + (-x + 2) \\ &= 3x^2(x-2) - (x-2) \\ &= (x-2)(3x^2 - 1) \end{aligned}$$

Practice this exercise:

3. Factor $x^3 + 3x^2 - 7x - 21$.

ADDITIONAL EXERCISES

Objective A Find the greatest common factor, the GCF, of monomials.

For extra help, see Examples 1–6 on pages 950–952 of your text and the Section 13.1 lecture video.

Find the GCF.

1. $x^3, -10x$

2. $4x^3,\ 20x^5,\ 12x^4$

3. $-11x^4y^3,\ 33x^3y^5,\ 55x^2y$

4. $-15x,\ 12x^2,\ 18x^7$

Objective B Factor polynomials when the terms have a common factor, factoring out the greatest common factor.

For extra help, see Examples 7–13 on pages 953–954 of your text and the Section 13.1 lecture video.

Factor. Check by multiplying.

5. $10x^5 - 5x^2$

6. $11x^4y^3 - 33x^3y^5 - 55x^2y$

7. $1.4x^4 - 3.5x^3 + 4.2x^2 + 7.0x$

8. $\frac{3}{2}x^7 + \frac{7}{2}x^5 - \frac{1}{2}x^3 + \frac{1}{2}x^2$

Objective C Factor certain expressions with four terms using factoring by grouping.

For extra help, see Examples 14–19 on pages 955–956 of your text and the Section 13.1 lecture video.

Factor.

9. $x^3(x+1) + 3(x+1)$

10. $7p^2(4p-5) - (4p-5)$

Factor by grouping.

11. $2z^3 + 2z^2 + 3z + 3$

12. $20x^3 - 15x^2 + 8x - 6$

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Chapter 13 POLYNOMIALS: FACTORING

13.2 Factoring Trinomials of the Type $x^2 + bx + c$

Learning Objectives
A Factor trinomials of the type $x^2 + bx + c$ by examining the constant term c.

Key Terms
Use the vocabulary terms listed below to complete each statement in Exercises 1–4.

leading coefficient **negative** **positive** **prime**

1. In the expression $ax^2 + bx + c$, a is called the ____________________ .

2. A(n) ____________________ polynomial cannot be factored further.

3. When the constant term of a trinomial is ____________________ , the constant terms of the binomial factors have the same sign.

4. When the constant term of a trinomial is ____________________ , the constant terms of the binomial factors have opposite signs.

GUIDED EXAMPLES AND PRACTICE

Objective A Factor trinomials of the type $x^2 + bx + c$ by examining the constant term c.

Review this example for Objective A:

1. Factor $x^2 - 2x - 15$.

Since the constant term, –15, is negative, we look for a factorization of –15 in which one factor is positive and one factor is negative. The sum of the factors must be the coefficient of the middle term, –2, so the negative factor must have the larger absolute value. The possible pairs of factors that meet these criteria are 1, –15 and 3, –5. The numbers we need are 3 and –5.

$$x^2 - 2x - 15 = (x+3)(x-5)$$

Practice this exercise:

1. Factor $x^2 - 9x + 8$.

ADDITIONAL EXERCISES

Objective A Factor trinomials of the type x^2+bx+c by examining the constant term c.

For extra help, see Examples 1–9 on pages 960–964 of your text and the Section 13.2 lecture video.

Factor. Remember that you can check by multiplying.

1. $p^2-5p-14$

2. x^2+3x+1

3. $x^2+22x+121$

4. $y^2-16y+60$

5. $x^2-0.1x-0.06$

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Chapter 13 POLYNOMIALS: FACTORING

13.3 Factoring $ax^2 + bx + c$: The FOIL Method

Learning Objectives
A Factor trinomials of the type $ax^2 + bx + c$, $a \neq 1$ using the FOIL method.

Key Terms
Use the vocabulary terms listed below to complete each statement in Exercises 1–4.

First **Inside** **Last** **Outside**

1. In the product $(x-3)(2x+1)$, x and $2x$ are the ____________________ terms.

2. In the product $(x-3)(2x+1)$, –3 and 1 are the ____________________ terms.

3. In the product $(x-3)(2x+1)$, –3 and $2x$ are the ____________________ terms.

4. In the product $(x-3)(2x+1)$, x and 1 are the ____________________ terms.

GUIDED EXAMPLES AND PRACTICE

Objective A Factor trinomials of the type $ax^2 + bx + c$, $a \neq 1$ using the FOIL method.

Review this example for Objective A:

1. Factor $2y^3 + 5y^2 - 3y$.

1. Factor out the largest common factor, y:
$$2y^3 + 5y^2 - 3y = y(2y^2 + 5y - 3)$$
2. Because $2y^2$ factors as $2y \cdot y$, we have this possibility for a factorization:
$$(2y+\quad)(y+\quad).$$
3. There are two pairs of factors of –3 and each can be written in two ways:
3, –1 –3, 1
and
–1, 3 1, –3.
4. From steps (2) and (3) we see that there are four possibilities for factorizations. We look for Outer and Inner products for which the sum is the middle term, $5y$. We try some possibilities.
$$(2y+3)(y-1) = 2y^2 + y - 3$$
$$(2y-1)(y+3) = 2y^2 + 5y - 3$$

Practice this exercise:

1. Factor $6z^2 + 14z + 4$.

The factorization of $2y^2+5y-3$ is

$$(2y-1)(y+3).$$

We must include the common factor to get a factorization of the original trinomial.

$$2y^3+5y^2-3y = y(2y-1)(y+3)$$

ADDITIONAL EXERCISES

Objective A Factor trinomials of the type $ax^2 + bx + c$, $a \neq 1$ using the FOIL method.

For extra help, see Examples 1–5 on pages 970–973 of your text and the Section 13.3 lecture video.

Factor.

1. $3x^2-2x-1$

2. $15x^2+55x+50$

3. $14a^4+23a^2+3$

4. $16x^2-8xy-8y^2$

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Chapter 13 POLYNOMIALS: FACTORING

13.4 Factoring $ax^2 + bx + c$: The *ac*-Method

Learning Objectives
A Factor trinomials of the type $ax^2 + bx + c$, $a \neq 1$ using the *ac*-method.

Key Terms
Use the vocabulary terms listed below to complete the steps for factoring using the *ac*-method in Exercises 1–6.

common factor	**grouping**	**leading coefficient**
multiplying	**split**	**sum**

1. Factor out a(n) ____________________ , if any.

2. Multiply the ____________________ a and the constant c.

3. Try to factor the product ac so that the ____________________ of the factors is b.

4. ____________________ the middle term.

5. Factor by ____________________ .

6. Check by ____________________ .

GUIDED EXAMPLES AND PRACTICE

Objective A Factor trinomials of the type $ax^2 + bx + c$, $a \neq 1$ using the *ac*-method.

Review this example for Objective A:

1. Factor $5x^2 + 7x - 6$ by grouping.

1. There is no common factor (other than 1 or –1).
2. Multiply the leading coefficient 5 and the constant, –6:
$$5(-6) = -30.$$
3. Look for a factorization of –30 in which the sum of the factors is the coefficient of the middle term, 7.
The numbers we need are 10 and –3.
4. Split the middle term, writing it as a sum or difference using the factors found in step (3).
$$7x = 10x - 3x$$

Practice this exercise:

1. Factor $8x^2 - 2x - 1$ by grouping.

5. Factor by grouping.

$$\begin{aligned} 5x^2+7x-6 &= 5x^2+10x-3x-6 \\ &= 5x(x+2)-3(x+2) \\ &= (x+2)(5x-3) \end{aligned}$$

6. Check: $(x+2)(5x-3)=5x^2+7x-6$

ADDITIONAL EXERCISES

Objective A Factor trinomials of the type $ax^2 + bx + c$, $a \neq 1$ using the *ac*-method.

For extra help, see Examples 1–2 on pages 977–978 of your text and the Section 13.4 lecture video.

Factor. Note that the middle term has already been split.

1. $6x^2+4x+15x+10$

Factor by grouping.

2. $4x^2+8x-21$

3. $14x^2-65x+9$

4. $12p^4-4p^3-5p^2$

5. $15y^2+10y+18$

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Chapter 13 POLYNOMIALS: FACTORING

13.5 Factoring Trinomials Squares and Differences of Squares

Learning Objectives
A Recognize trinomial squares.
B Factor trinomial squares.
C Recognize differences of squares.
D Factor differences of squares, being careful to factor completely.

Key Terms
Use the vocabulary terms listed below to complete each statement in Exercises 1–4.

difference of squares **factored completely**
sum of squares **trinomial square**

1. The expression $x^2+10x+25$ is a(n) ____________________ .

2. The expression $9x^2-16$ is a(n) ____________________ .

3. The expression y^2+81 is a(n) ____________________ .

4. When no factor can be factored further, we have ____________________ .

GUIDED EXAMPLES AND PRACTICE

Objective A Recognize trinomial squares.

Review this example for Objective A:

1. Determine whether each of the following is a trinomial square.

a) x^2-4x-4 b) $9x^2+1+6x$

a) x^2 and 4 are squares, but there is a minus sign before 4. This is not a trinomial square.

b) Write the trinomial in descending order: $9x^2+6x+1$. $9x^2$ and 1 are squares. There is no minus sign before $9x^2$ or 1. The middle term, $6x$, is $2\cdot 3x\cdot 1$. Thus $9x^2+1+6x$ is a trinomial square.

Practice this exercise:

1. Determine whether $y^2+32y+16$ is a trinomial square.

Objective B Factor trinomial squares.

Review this example for Objective B:

2. Factor $4x^2-12x+9$.

$$\begin{aligned}4x^2-12x+9&=(2x)^2-2\cdot 2x\cdot 3+3^2\\&=(2x-3)^2\end{aligned}$$

Practice this exercise:

2. Factor $16x^2+8x+1$.

Objective C Recognize differences of squares.

Review this example for Objective C:

3. Determine whether each of the following is a difference of squares.

a) $16t^4-9$ b) $25x^2-3$

a) The expressions $16t^4$ and 9 are squares:

$16t^4=(4t^2)^2$ and $9=3^2$

The expressions have different signs. Thus, $16t^4-9$ is a difference of squares.

b) 3 is not a square, so $25x^2-3$ is not a difference of squares.

Practice this exercise:

3. Determine whether $1-36y^6$ is a difference of squares.

Objective D Factor differences of squares, being careful to factor completely.

Review this example for Objective D:

4. Factor t^5-t.

$$\begin{aligned}t^5-t&=t(t^4-1)\\&=t(t^2+1)(t^2-1)\\&=t(t^2+1)(t+1)(t-1)\end{aligned}$$

Practice this exercise:

4. Factor $12-27x^2$ completely.

ADDITIONAL EXERCISES

Objective A Recognize trinomial squares.

For extra help, see Examples 1–3 on pages 985–986 of your text and the Section 13.5 lecture video.

Determine whether each of the following is a trinomial square. Answer "yes" or "no."

1. $x^2+10x-25$

2. $4x^2-12x+9$

3. x^2+4x+1

4. x^2-3x+8

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Objective B Factor trinomial squares.

For extra help, see Examples 4–9 pages 986–987 of your text and the Section 13.5 lecture video.

Factor completely. Remember to look first for a common factor and to check by multiplying.

5. $36+12x+x^2$

6. $64-80x+25x^2$

7. $12a^2-84a+147$

8. $x^2-8xy+16y^2$

Objective C Recognize differences of squares.

For extra help, see Examples 10–12 on page 988 of your text and the Section 13.5 lecture video.

Determine whether each of the following is a difference of squares. Answer "yes" or "no."

9. x^2-100

10. $4x^2-10y^2$

11. $-1+64x^2$

12. $25x^2-909$

Objective D Factor differences of squares, being careful to factor completely.

For extra help, see Examples 13–21 pages 988–990 of your text and the Section 13.5 lecture video.

Factor completely. Remember to look first for a common factor.

13. $64x^2 - y^2$

14. $4m^2 - 49n^2$

15. $3x^4 - 48$

16. $36p^4 - 1$

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Chapter 13 POLYNOMIALS: FACTORING

13.6 Factoring: A General Strategy

Learning Objectives
A Factor polynomials completely using any of the methods considered in this chapter.

Key Terms
Use the vocabulary terms listed below to complete each statement in Exercises 1–4.

completely **difference** **grouping** **perfect square**

1. When factoring a polynomial with two terms, determine whether you have a(n)

____________________ of squares.

2. When factoring a polynomial with three terms, determine whether the trinomial is a

____________________ .

3. When factoring a polynomial with four terms, try factoring by ____________________ .

4. Always factor ____________________ .

GUIDED EXAMPLES AND PRACTICE

Objective A Factor polynomials completely using any of the methods considered in this chapter.

Review this example for Objective A:

1. Factor $2y^3-12y^2+18y$ completely.

a) We look for a common factor.

$2y^3-12y^2+18y=2y\left(y^2-6y+9\right)$

b) The factor y^2-6y+9 has three terms and is a trinomial square. We factor it.

$$\begin{aligned}2y\left(y^2-6y+9\right)&=2y\left(y-2\cdot y\cdot 3+3^2\right)\\&=2y\left(y-3\right)^2\end{aligned}$$

Practice this exercise:

1. Factor $15x^2+5x-20$ completely.

ADDITIONAL EXERCISES

Objective A Factor polynomials completely using any of the methods considered in this chapter.

For extra help, see Examples 1–11 on pages 995–998 of your text and the Section 13.6 lecture video.

Factor completely.

1. $x^4 + 5x^2 - 4x^3 - 20x$

2. $9c^2 + 4d^2 - 12cd$

3. $4x\left(u^2 + 3v\right) - \left(u^2 + 3v\right)$

4. $25x^2z^2 + 40xyz + 16y^2$

5. $n^2 + 1$

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Chapter 13 POLYNOMIALS: FACTORING

13.7 Solving Quadratic Equations by Factoring

Learning Objectives
A Solve equations (already factored) using the principle of zero products.
B Solve quadratic equations by factoring and then using the principle of zero products.

Key Terms
Use the vocabulary terms listed below to complete each statement in Exercises 1–4.

factor **products** **quadratic equation** **zero**

1. $3x^2 - 5x + 3 = 0$ is an example of a(n) ____________________ .

2. The principle of zero ____________________ states that a product of two terms is 0 if and only if one or both of the factors is 0.

3. To use the principle of zero products, you must have ____________________ on one side of the equation.

4. To use the principle of zero products, we ____________________ a quadratic polynomial.

GUIDED EXAMPLES AND PRACTICE

Objective A Solve equations (already factored) using the principle of zero products.

Review this example for Objective A:

1. Solve: $(4x-3)(x+2)=0$.

$$(4x-3)(x+2)=0$$
$$4x-3=0 \quad or \quad x+2=0$$
$$x=\frac{3}{4} \quad or \quad x=-2$$

The solutions are $\frac{3}{4}$ and -2.

Practice this exercise:

1. Solve: $x(6x+5)=0$.

Objective B Solve quadratic equations by factoring and then using the principle of zero products.

Review this example for Objective B:

2. Solve: $x^2+2x=24$.

$$x^2+2x=24$$
$$x^2+2x-24=0$$
$$(x+6)(x-4)=0$$
$$x+6=0 \quad or \quad x-4=0$$
$$x=-6 \quad or \quad x=4$$

The solutions are -6 and 4.

Practice this exercise:

2. Solve: $16x^2=49$.

ADDITIONAL EXERCISES

Objective A Solve equations (already factored) using the principle of zero products.

For extra help, see Examples 1–3 on pages 1003–1004 of your text and the Section 13.7 lecture video.

Solve using the principle of zero products.

1. $(x+3)(x-10)=0$

2. $(4x+3)(2x-10)=0$

3. $\left(\frac{1}{2}-3x\right)\left(\frac{1}{3}-4x\right)=0$

4. $(0.1x+0.2)(0.5x-15)=0$

Objective B Solve quadratic equations by factoring and then using the principle of zero products.

For extra help, see Examples 4–10 on pages 1005–1007 of your text and the Section 13.7 lecture video.

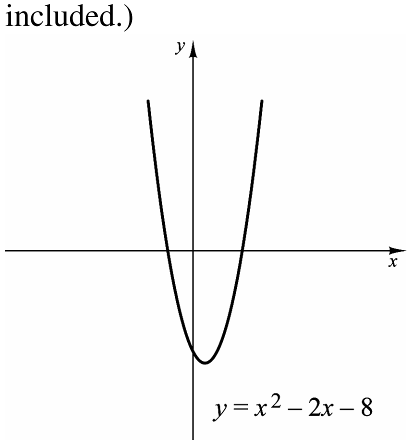
Solve by factoring and using the principle of zero products. Remember to check.

5. $x^2+4x-21=0$

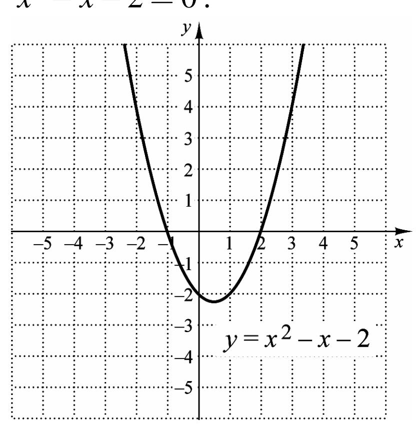
6. $3x^2+5x=2$

7. $2y^2+16y+30=0$

8. Find the x-intercepts for the graph of the equation. (The grid is intentionally not included.)



9. Use the following graph to solve $x^2-x-2=0$.



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Chapter 13 POLYNOMIALS: FACTORING

13.8 Applications of Quadratic Equations

Learning Objectives
A Solve applied problems involving quadratic equations that can be solved by factoring.

Key Terms
Use the vocabulary terms listed below to complete each statement in Exercises 1–4.

hypotenuse **legs** **Pythagorean** **right**

1. A triangle that has a 90° angle is a ____________________ triangle.

2. In a right triangle, the side opposite the 90° angle is the ____________________ .

3. The sides that form the 90° angle in a right triangle are called ____________________ .

4. The ____________________ theorem states that, in a right triangle, $a^2 + b^2 = c^2$.

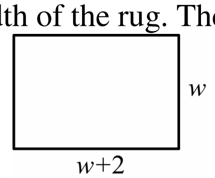
GUIDED EXAMPLES AND PRACTICE

Objective A Solve applied problems involving quadratic equations that can be solved by factoring.

Review this example for Objective A:

1. The length of a rectangular rug is 2 ft greater than the width. The area of the rug is 48 ft^2. Find the length and width.

1. *Familiarize*. We make a drawing. Let w = the width of the rug. Then the length is $w + 2$.



Recall that the area of a rectangle is length $\times$ width.

2. *Translate*. We reword the problem.

$$\begin{array}{ccccc} \text{Length} & \times & \text{width} & \text{is} & 48\text{ ft}^2. \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ (w+2) & \times & w & = & 48 \end{array}$$

Practice this exercise:

1. The height of a triangle is 4 cm greater than the base. The area is 30 cm^2. Find the height and the base.

3. *Solve*. We solve the equation.

$$(w+2)\times w = 48$$
$$w^2 + 2w = 48$$
$$w^2 + 2w - 48 = 0$$
$$(w+8)(w-6) = 0$$
$$w+8=0 \quad or \quad w-6=0$$
$$w=-8 \quad or \quad w=6$$

4. *Check*. The width of a rectangle cannot be negative, so –8 cannot be a solution. Suppose the width is 6 ft. Then the length is 6 + 2, or 8 ft and the area is $6\cdot 8,$ or 48 ft^2. These numbers check in the original problem.

5. *State*. The length is 8 ft and the width is 6 ft.

ADDITIONAL EXERCISES

Objective A Solve applied problems involving quadratic equations that can be solved by factoring.

For extra help, see Examples 1–6 on pages 1012–1017 of your text and the Section 13.8 lecture video.

Solve.

1. A rectangular serving tray is twice as long as it is wide. The area of the tray is 338 in^2. Find the dimensions of the tray.

2. A softball league plays a total of 210 games. How many teams are in the league if each team plays every other team twice? Use $x^2 - x = N,$ where *x* is the number of teams and *N* is the number of games played.

3. During a toast at a party, there were 66 “clinks” of glasses. How many people took part in the toast? Use $N = \frac{1}{2}(x^2 - x),$ where *N* is the number of clinks and *x* is the number of people.

4. The guy wire on a tower is 10 ft longer than the height of the tower. If the guy wire is anchored 50 ft from the foot of the tower, how tall is the tower?