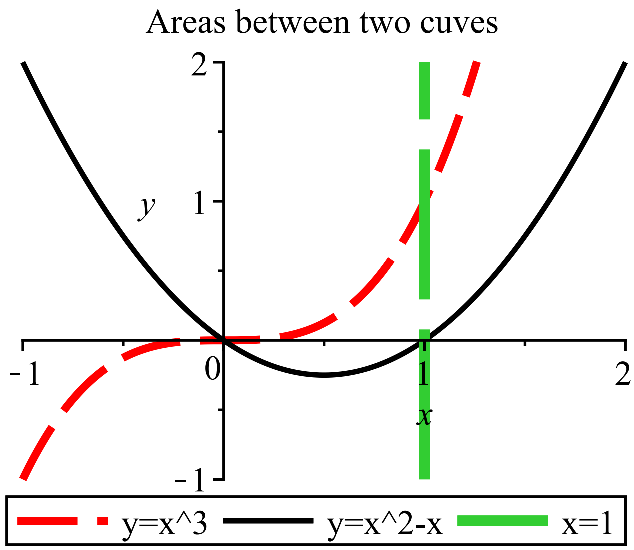
The Area Between Two Curves.

1. Find the area of the region between the curves $y = x^3$ and $y = x^2 - x$ on the interval $[0, 1]$.

$with(plots):$
$plot([x^3, x^2 - x], x = -1..2, y = -1..2, color = [red, black], thickness = 2);$

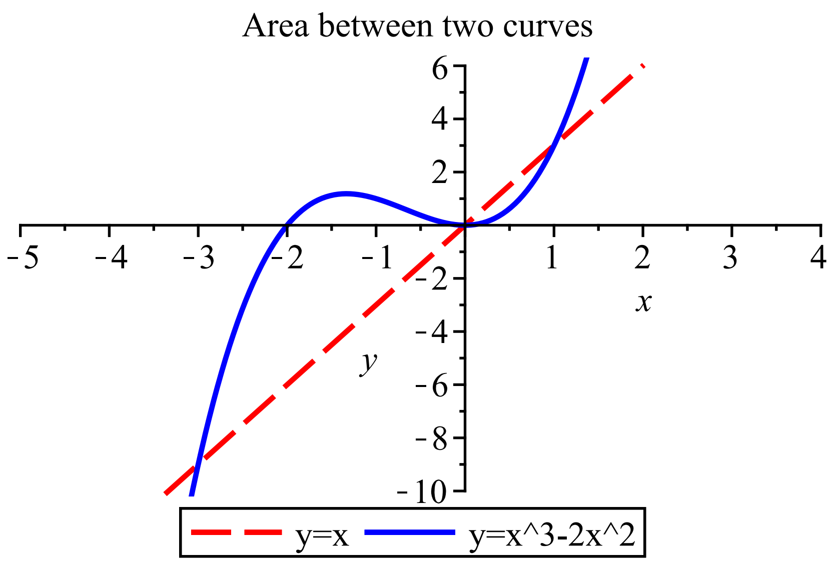


To find the area between the curves above we look at the upper function $y = x^3$ and the lower function $y = x^2 - x$ and subtract. So, the area A is found by :

$$A = \int_0^1 \left([x^3] - [x^2 - x] \right) \mathrm{d}x = \frac{5}{12}.$$

2. Find the area of the region bounded by the line $y = 3x$ and the curve $y = x^3 + 2x^2$.

$plot([3x, x^3 + 2x^2], x = -5..4, y = -10..6, color = [red, blue], linestyle = [dash, solid], thickness = 2);$

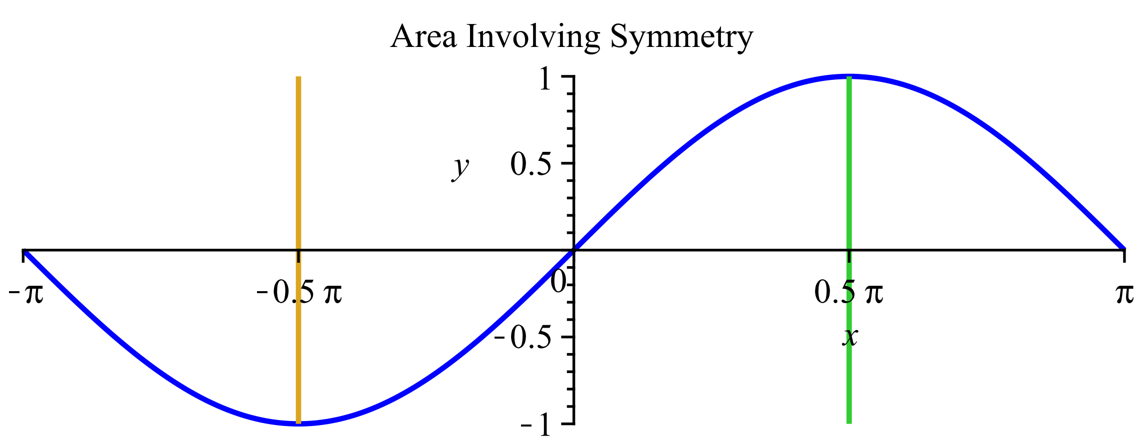


With the above graph we see that on the interval $(-3, 0)$ the curve is above the line so the area will be found by taking vertical strips such that the we subtract the line from the curve. On the other hand on the interval $(0, 1)$ the line is above the curve so the area will be found by taking vertical strips such that we subtract the curve from the line. So,

$$A = \int_{-3}^{0} \left[\left(x^3 - 2x^2\right) - (3x) \right] \mathrm{d}x + \int_{0}^{1} \left[(3x) - \left(x^3 + 2x^2\right) \right] \mathrm{d}x = \frac{71}{6}$$

3. Find the area bounded by the curve $y = \sin(x)$ and the x-axis between $x = -\frac{\pi}{2}$ and $\frac{\pi}{2}$.

$plot(\sin(x), x = -\mathrm{Pi}..\mathrm{Pi}, y = -1..1, color = blue, thickness = 2);$



In the above curve we see that we can find the area of the curve between - π/2 and π/2 by by finding the area between 0 and π/2 and doubling the result.

$$A = 2\int_0^{\frac{\pi}{2}} \sin(x)\ \mathrm{d}x = 2$$