# Houston Community College System <br> Department of Mathematics <br> MATH 1314-College Algebra <br> Final Exam Review 

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.
Solve the equation by using the square root property.

1) $\left(t-\frac{1}{6}\right)^{2}=-\frac{17}{36}$
2) $\qquad$

Solve the equation by using the quadratic formula.

$$
\text { 2) }(3 x-2)(x-1)=-3
$$

2) $\qquad$

Find the value of $\boldsymbol{n}$ so that the expression is a perfect square trinomial and then factor the trinomial.
3) Find the value of $n$ so that the expression is a perfect square trinomial and then
3) $\qquad$ factor the trinomial.

$$
t^{2}-\frac{14}{3} t+n
$$

Solve for the indicated variable.
4) $a t^{2}+u y=h \quad$ for $t$

Solve the problem.
5) The length of the longer leg of a right triangle is 14 ft longer than the length of
5) $\qquad$ the shorter leg $x$. The hypotenuse is 6 ft longer than twice the length of the shorter leg. Find the dimensions of the triangle.

Solve the following equations by the most convenient method. Please show all work.
6) $\frac{5}{x-4}-\frac{8}{x+1}=\frac{34}{x^{2}-3 x-4}$
6) $\qquad$
7) $5-\sqrt{x+10}=\sqrt{7-x}$
7) $\qquad$
8) $\qquad$
9) $\qquad$
9) $(x-1)^{4}-5(x-1)^{2}+4=0$

Solve the equation, then find $S$ the sum of the solutions:
10) $|5 x+4|=|x+9|$
10) $\qquad$

Solve the following inequalities by the most convenient method. Please show all work. Write the solution set in interval notation.
11) $x^{3}+4 x^{2}-9 x-36 \geq 0$
11) $\qquad$
12) $24 \leq 2+|-15 t+1|$
12) $\qquad$
13) $\frac{40-8 x}{x^{2}} \leq 0$
13) $\qquad$

The graph of $\boldsymbol{y}=\boldsymbol{f}(\boldsymbol{x})$ is given. Solve the inequality.
14) $f(x)>0$
14) $\qquad$


Write the given equation in the form $(\boldsymbol{x}-\boldsymbol{h})^{\mathbf{2}}+(\boldsymbol{y}-\boldsymbol{k})^{\mathbf{2}}=\boldsymbol{r}^{\mathbf{2}}$. Identify the center and radius.

$$
\text { 15) } 2 x^{2}+2 y^{2}+16 x-20 y+50=0
$$

15) $\qquad$
16) The endpoints of the diameter of a circke are $(-2,3)$ and $(-10,9)$. Write and equation
17) $\qquad$ of this circle in standard form and identify its center and radius.

Find an equation for the line, in the indicated form, with the given properties.
17) Containing the points $(8,-2)$ and $(-5,7)$; general form
17) $\qquad$

Given the equation $12 x=8-5 y$.
18) a)Write the equation in slope-intercept form
18)
b) Determine the slope and the $y$-intercept.
c) What is the $x$-intercept
d) Graph the line

Find an equation for the line with the given properties.
19) Perpendicular to the line $x-5 y=3$; containing the point $(5,3)$
19) $\qquad$

Determine whether the equation defines $y$ as a function of $x$.
20) $y= \pm \sqrt{1-5 x}$
20) $\qquad$
21) $y=\frac{5 x-1}{x+1}$
21) $\qquad$

Determine whether the graph is that of a function. If it is, use the graph to find its domain and range, the intercepts, if any, and any symmetry with respect to the $x$-axis, the $y$-axis, or the origin.
22)


## Solve the problem.

23) If $f(x)=\frac{x-B}{x-A}, f(4)=0$, and $f(6)$ is undefined, what are the values of $A$ and $B$ ?

Evaluate the function for the indicated value, then simplify.
24) $f(t)=t^{2}+5 t$; find $f(t-5)$, then simplify as much as possible.

Find $\frac{f(x+h)-f(x)}{h}$ for the given function.
25) $f(x)=x^{2}+8 x$.
22) $\qquad$
23) $\qquad$
24) $\qquad$
25) $\qquad$

## Graph the function.

26) $f(x)= \begin{cases}-x+3, & \text { for } x \leq 3 \\ \sqrt{x-3}, & \text { for } x>3\end{cases}$
27) $\qquad$

## Given

27) 
28) $\qquad$

$$
f(x)=\left\{\begin{array}{l}
-5 x+4, \text { for } x<-1 \\
x^{2}+3, \text { for }-1 \leq x<2 \\
1, \quad \text { for } x \geq 2
\end{array}\right.
$$

Evaluate $f(-1)-f(5)$

Find the domain of the following functions, then write it in interval notation
28) $f(x)=\frac{x}{\sqrt{x-6}}$
28) $\qquad$
29) $g(t)=\sqrt[3]{5-t}$
29) $\qquad$
30) $m(x)=\frac{5}{|x|+1}$
30) $\qquad$

Use the given graph of $f(x)$ to find the intervals on which the function is increasing or decreasing.
31)
31) $\qquad$


Find the indicated function and write its domain in interval notation.

$$
\text { 32) } s(x)=\frac{x-5}{x^{2}-49}, t(x)=\frac{x-7}{5-x},(s \cdot t)(x)=?
$$

32) $\qquad$

Evaluate the function for the given value of $\boldsymbol{x}$.
33) $f(x)=-2 x, g(x)=|x+2|,\left(\frac{f}{g}\right)(-5)=$ ?
33) $\qquad$

Find the indicated function and write its domain in interval notation.
34) $f(x)=\frac{9}{x^{2}-11}, g(x)=\sqrt{4-x},(f \circ g)(x)=$ ?
34) $\qquad$

Use transformations to graph the given function.
35) $f(x)=\sqrt{-x-4}$
35) $\qquad$

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.
A function $g$ is given. Identify the parent function. Then use the steps for graphing multiple transformations of functions to list, in order, the transformations applied to the parent function to obtain the graph of g.

$$
\text { 36) } g(x)=\frac{4}{x+5}-3
$$

A) Parent function: $f(x)=\frac{1}{x}$; Shift the graph of $f$ to the right 5 units, stretch the graph vertically by a factor of 4 , and shift the graph upward by 3 units.
B) Parent function: $f(x)=\frac{1}{x}$; Shift the graph of $f$ to the left 5 units, shrink the graph vertically by a factor of $\frac{1}{4}$, and shift the graph downward by 3 units.
C) Parent function: $f(x)=\frac{1}{x}$; Shift the graph of $f$ to the right 5 units, shrink the graph vertically by a factor of $\frac{1}{4}$, and shift the graph upward by 3 units.
D) Parent function: $f(x)=\frac{1}{x}$; Shift the graph of $f$ to the left 5 units, stretch the graph vertically by a factor of 4 , and shift the graph downward by 3 units.

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.
Write the function in the vertex form, then find the axis of symmetry, the intercepts of the graph the function, the domain and the range of the function.
37) $f(x)=-3 x^{2}+18 x-33$
37) $\qquad$

Solve the problem.
38) A trough at the end of a gutter spout is meant to direct water away from a house. $\qquad$
The homeowner makes the trough from a rectangular piece of aluminum that is 24 in . long and 8 in . wide. He makes a fold along the two long sides a distance of $x$ inches from the edge.

a. Write a function to represent the volume in terms of $x$.
b. What value of $x$ will maximize the volume of water that can be carried by the gutter?

Analyze the graph of the given function $f$ as follows:
(a) Determine the end behavior of the graph of the function.
(b) Find the $x$ - and $y$-intercepts of the graph.
(c) Determine whether the graph crosses or touches the $x$-axis at each $x$-intercept.
(d) Use the graph to determine the local maxima and local minima, if any exist. Round turning points to two decimal places.
(e) Use the information obtained in (a) - ( $d$ ) to draw a complete graph of $f$ by hand. Label all intercepts and turning points.
(f) Find the domain of $f$. Use the graph to find the range of $f$.
(g) Use the graph to determine where $f$ is increasing and where $f$ is decreasing. 39)
39) $\qquad$

$$
f(x)=x^{4}-8 x^{2}+16
$$

Form a polynomial $f(x)$ with real coefficients having the given degree and zeros.
40) Degree: 4; zeros: $-1,2$, and $1-2 \mathrm{i}$.
40) $\qquad$

Use the Rational Zeros Theorem to find all the real zeros of the polynomial function. Use the zeros to factor $f$ over the real numbers.

$$
\text { 41) } f(x)=4 x^{3}-11 x^{2}-6 x+9
$$

41) $\qquad$
List the potential rational zeros of the polynomial function. Do not find the zeros.
42) $f(x)=6 x^{4}+3 x^{3}-2 x^{2}+2$
43) $\qquad$

Solve the problem.
43) Find $m$ so that $x+4$ is a factor of $5 x^{3}+18 x^{2}+m x+16$
43) $\qquad$

Find the sum of all the zeros of the polynomial.

$$
\text { 44) } f(x)=3 x^{3}-x^{2}-18 x+6
$$

44) $\qquad$

Find all asymptotes, if any, of the function.
45) $f(x)=\frac{x^{2}+9 x+6}{x+5}$
45) $\qquad$

Write the domain in interval notation and identify any vertical asymptotes.
46) $f(x)=\frac{x^{2}+x-56}{x-7}$
46) $\qquad$

## Graph the function.

$$
\text { 47) } f(x)=\frac{-4 x^{2}}{x^{2}+4}
$$

47) $\qquad$

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.
Determine which of the rational functions given below has the following feature(s).
$48) x$-intercepts: $(4,0)$ and $(-3,0), y$-intercepts: none, vertical asymptotes: $x=0$ and $x=1$, horizontal $\qquad$ asymptote: $y=1$
A) $f(x)=\frac{(x-4)(x+3)}{x(x-1)}$
B) $f(x)=\frac{(x+4)(x-3)}{x(x+1)}$
C) $f(x)=\frac{(x+4)(x-3)}{(x+1)}$
D) $f(x)=\frac{(x-4)(x+3)}{(x-1)^{2}}$

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.
A one-to-one function is given. Write an expression for the inverse function.
49) $f(x)=4 x^{3}-9$

A one-to-one function is given
50) $f(x)=\sqrt{x+3}$
a. Graph $f(x)$
b. find then graph $f^{-1}(x)$
c. Write the domain of $f^{-1}$ in interval notation
a. Use transformations to graph the function.
b. Write the domain and range in interval notation.
c. Determine the vertical asymptote.
51) $y=\log _{5}(x+3)$
51) $\qquad$

Write as the sum or difference of logarithms and fully simplify, if possible. Assume the variable represents a positive real number.
52) $\log \left(\frac{\sqrt[3]{a b}}{c^{5}}\right)$
52) $\qquad$

Solve the problem.
53) Use the graph of $y=3^{x}$ to graph the function. Write the domain and range in
53) $\qquad$ interval notation.

$$
f(x)=3^{x+5}+2
$$

Solve the equation.
54) $2+\log _{3}(2 x+5)-\log _{3} x=4$. If the reciprocal of the solution is written as a
54) $\qquad$ reduced fraction $\frac{n}{m}$ (where $n$ and $m$ are integers whose greatest common factor is 1 ), what is the value of $m$.

Solve the exponential equation. Express the solution set in term of the natural logarithms.
55) $4 e^{(3 x+2)}=2$
55) $\qquad$

Solve the system by using any method. If a system does not have one unique solution, state whether the system is inconsistent or whether the equations are dependent.

$$
\text { 56) } \begin{aligned}
5 x+y & =-5 \\
-15 x-3 y & =9
\end{aligned}
$$

56) $\qquad$

Solve the nonlinear system . Provide the product of the $y$-values of the solutions

$$
\text { 57) } \begin{aligned}
& x-2 y=3 \\
& x^{2}-x y=20
\end{aligned}
$$

57) $\qquad$

## Solve the problem.

$$
\text { 58) } f(x)=4^{x+4} \text { and } g(x)=4^{-x+6}
$$

58) $\qquad$
Find the point of intersection of the graphs of $f$ and $g$ by solving $f(x)=g(x)$.

Solve the system of equations using Gauss-Jordan elimination.

$$
\text { 59) } \begin{aligned}
& x=5-y-z \\
& x-y+3 z=23 \\
& 2 x+y=9-z
\end{aligned}
$$

59) $\qquad$

Solve for $x$.
60)
60) $\qquad$

Use the given matrices to compute the given expression.
61) Let $A=\left[\begin{array}{rrr}9 & 8 & -3 \\ 3 & -7 & 5 \\ -1 & -2 & 2\end{array}\right]$ and $B=\left[\begin{array}{rrr}8 & 1 & 1 \\ -2 & 4 & -2 \\ -5 & -8 & 8\end{array}\right]$. Find $-3 A+4 B$.
61)

1) $\left\{\frac{1}{6} \pm \frac{\sqrt{17}}{6} i\right\}$
2) $\left\{\frac{5}{6}-\frac{\sqrt{35}}{6} i, \frac{5}{6}+\frac{\sqrt{35}}{6} i\right\}$
3) $n=\frac{49}{9} ;\left(t-\frac{7}{3}\right)^{2}$
4) $t= \pm \frac{\sqrt{a(h-u y)}}{a}$
5) Short leg $=10$, long leg $=24$, hypotenuse $=26$
6) $\{1\}$
7) $\{6,-9\}$
8) $\left\{-\frac{27}{64}, 27\right\}$
9) $\{-1,0,2,3\}$
10) $\{-11 / 12\}$
11) $[-4,-3] \cup[3, \infty)$
12) $\left(-\infty,-\frac{7}{5}\right] \cup\left[\frac{23}{15}, \infty\right)$
13) $[5, \infty)$
14) $(3,4)$
15) $(x+4)^{2}+(y-5)^{2}=16$

Center: $(-4,5) ; r=4$
16) $x^{2}+y^{2}+12 x-12 y+47=0$
17) $9 x+13 y=46$
18) $y=-\frac{12}{5} x+\frac{8}{5}$
19) $y=-5 x+28$
20) not a function
21) function
22) function
domain: $\{x \mid x \geq-2\}$
range: $\{y \mid y \geq 0\}$
intercepts: $(-2,0),(0,2),(2,0)$
symmetry: none
23) $\mathrm{A}=6, \mathrm{~B}=4$
24) $t^{2}-5 t$
25) $2 x+h+8$
26)

27) $\mathbf{3}$
28) $(6, \infty)$
29) $(-\infty, \infty)$
30) $(-\infty, \infty)$
31) The function is increasing on $(-3,3)$, and it is decreasing on $(-\infty,-3) \cup(3, \infty)$
32) $-\frac{1}{x+7} ;(-\infty,-7) \cup(-7,5) \cup(5,7) \cup(7, \infty)$
33) $\left(\frac{f}{g}\right)(-5)=\frac{10}{3}$
34) $-\frac{9}{x+7} ;(-\infty,-7) \cup(-7,4]$
35)

36) D
37) $f(x)=-3(x-3)^{2}-6$

Domain: $(-\infty, \infty)$.
Range: $(-\infty,-6]$
Axis of symmetry: $\mathrm{x}=3$
$x$ - intercept : None
$y$ - intercept : (0, -33)
38) a. $V(x)=-48 x^{2}+192 x$; b. $x=2$
39) (a) As $x \rightarrow-\infty, f(x) \rightarrow \infty$; As $x \rightarrow \infty, f(x) \rightarrow \infty$
(b) y-intercept: $(0,16)$; $x$-intercepts: $(-2,0)$, and $(2,0)$
(c) The graph of $\mathrm{f}(x)$ touches the x -axis at $(-2,0)$, and at $(2,0)$, and it bounces without crossing the $x$-axis.
(d) Local maxima at $(0,16)$; Local minimum at $(-2,0)$, and at $(2,0)$
(e)

(f) Domain of $\mathrm{f}(x)$ : all real numbers; range of $\mathrm{f}(x)$ : $[0, \infty)$
(g) $f$ is increasing on $(-2,0) U(2, \infty)$; $f$ is decreasing on $(-\infty,-2) U(0,2)$
40) $f(x)=x^{4}-3 x^{3}+5 x^{2}-x-10$
41) $-1, \frac{3}{4}, 3 ; f(x)=(4 x-3)(x-3)(x+1)$
42) $\pm \frac{1}{6}, \pm \frac{1}{3}, \pm \frac{1}{2}, \pm \frac{2}{3}, \pm 1, \pm 2$
43) $m=-4$
44) $\left\{\frac{1}{3}\right\}$
45) Vertical asymptote : $x=-5$

Horizontal asymptote: none
Oblique (slant) asymptote: $y=x+4$
46) $(-\infty, 7) \cup(7, \infty)$; None
47)

48) A
49) $f^{-1}(x)=\sqrt[3]{\frac{x+9}{4}}$
50) a.

b. $f^{-1}(x)=x^{2}-3 ; x \geq 0$;
c. $[0, \infty)$
51) a.

b. domain: $(-3, \infty)$, range $(-\infty, \infty)$
c. vertical asymptote: $x=-3$
52) $\frac{1}{3} \log a+\frac{1}{3} \log b-5 \log c$
53)


Domain: $(-\infty, \infty)$
Range: $(2, \infty)$
54) $\{5\}$
55) $\{-1 / 3[\ln (2)+2]\}$
56) $\}$; The system is inconsistent.
57) $\left\{-\frac{11}{2}\right\}$
58) $(1,1024)$
59) $\{(4,-4,5)\}$
60) -2
61) $\left[\begin{array}{rrr}5 & -20 & 13 \\ -17 & 37 & -23 \\ -17 & -26 & 26\end{array}\right]$

