CHAPTER 13 | *THE TRANSFER OF HEAT*

CONCEPTUAL QUESTIONS

---

1. ***REASONING AND SOLUTION*** Convection is the process in which heat is carried from one place to another by the bulk movement of the medium. In liquids and gases, the molecules are free to move; hence, convection occurs as a result of bulk molecular motion. In solids, however, the molecules are generally bound to specific locations (lattice sites). While the molecules in a solid can vibrate about their equilibrium locations, they are not free to move from place to place within the solid. Therefore, convection does not generally occur in solids.

---

2. ***REASONING AND SOLUTION*** A heavy drape, hung close to a cold window, reduces heat loss through the window by interfering with the process of convection. Without the drape, convection currents bring the warm air of the room into contact with the cold window. With the drape, convection currents are less prominent, and less room air is circulated directly past the cold surface of the window.

---

3. ***REASONING AND SOLUTION*** Forced convection plays the principal role in the wind chill factor. The wind mixes the cold ambient air with the warm layer of air that immediately surrounds the exposed portions of your body. The forced convection removes heat from your exposed body surfaces, thereby making you feel colder than you would otherwise feel if there were no wind.

---

4. ***REASONING AND SOLUTION*** A road surface is exposed to the air on its upper surface and to the earth on its lower surface. Even when the air temperature is at the freezing point, the road surface may be above this temperature as heat flows through the road from the earth. In order for a road to freeze, sufficient heat must be lost from the earth by conduction through the road surface. The temperature of the earth under the road must be reduced at least to the freezing point. A bridge is exposed to the air on both its upper and lower surfaces. It will, therefore, lose heat from both surfaces and reach thermal equilibrium with the air much more quickly than an ordinary roadbed. It is reasonable, then, that the bridge surface will usually freeze before the road surface.

---

5. **SSM** ***REASONING AND SOLUTION*** A piece of Styrofoam and a piece of wood are sandwiched together to form a layered slab. The two pieces have the same thickness and cross-sectional area. The exposed surfaces have constant temperatures. The temperature of the exposed Styrofoam surface is greater than the temperature of the exposed wood surface. The rate of heat flow through either layer can be determined from Equation 13.1: $Q/t = kA\Delta T/L$, where k is the thermal conductivity of the layer, A and L are the cross-sectional area and thickness of the layer, respectively, and ΔT is the temperature difference between the ends of the layer. Since heat is not trapped within the sandwich, the rate at

which heat flows through the sandwich, Q/t, must be uniform throughout both layers. Therefore, $(kA\Delta T / L)_{\text{Styrofoam}} = (kA\Delta T / L)_{\text{wood}}$. Since both layers have the same cross-sectional area and thickness, A and L are the same for both layers. Therefore, $k_{\text{Styrofoam}}\Delta T_{\text{Styrofoam}} = k_{\text{wood}}\Delta T_{\text{wood}}$. From Table 13.1, we see that the thermal conductivity of Styrofoam is less than the thermal conductivity of wood; therefore, the temperature difference between the two ends of the wood layer must be smaller than the temperature difference between the two ends of the Styrofoam layer. From this, we can conclude that the temperature at the Styrofoam-wood interface must be closer to the lower temperature of the exposed wood surface.

---

6. ***REASONING AND SOLUTION*** When heat is transferred from place to place inside the human body by the flow of blood, the main method of heat transfer is forced convection, similar to that illustrated for the radiator fluid in Figure 13.7. The heart is analogous to the water pump in the figure.

---

7. ***REASONING AND SOLUTION*** Some animals have hair, the strands of which are hollow, air-filled tubes. Other animals have hair that is composed of solid, tubular strands. For animals that live in very cold climates, hair that is composed of hollow air-filled tubes would be advantageous for survival. Since air has a small thermal conductivity, hair shafts composed of hollow air-filled tubes would reduce the loss of body heat by conduction. Since hair shafts are small, no appreciable convection would occur within them. Thus, the hollow air-filled structure of the hair shaft inhibits the loss of heat by conduction.

---

8. ***REASONING AND SOLUTION*** A poker used in a fireplace is held at one end, while the other end is in the fire. Such pokers are made of iron rather than copper because the thermal conductivity of iron is roughly smaller by a factor of five than the thermal conductivity of copper. Therefore, the transfer of heat along the poker by conduction is considerably reduced by using iron. Hence, one end of the poker can be placed in the fire, and the other end will remain cool enough to be comfortably handled.

---

9. ***REASONING AND SOLUTION*** Snow, with air trapped within it, is a thermal insulator, because air has a relatively low thermal conductivity and the small, dead-air spaces inhibit heat transfer by convection. Therefore, a lack of snow allows the ground to freeze at depths greater than normal.

---

10. ***REASONING AND SOLUTION*** Table 13.1 indicates that the thermal conductivity of steel is 14 $\text{J/(s}\cdot\text{m}\cdot\text{C}^\circ)$, while that of concrete is 1.1 $\text{J/(s}\cdot\text{m}\cdot\text{C}^\circ)$. According to Equation 13.1, $Q = kA\Delta Tt / L$, this implies that heat will flow more readily through a volume of steel than it will through an identically shaped volume of concrete. Therefore, while steel reinforcement bars can enhance the structural stability of concrete walls, they degrade the insulating value of the concrete.

---

11. ***REASONING AND SOLUTION*** A potato will bake faster if a nail is driven into it before it is placed in the oven. Since the nail is metal, we can assume that the thermal conductivity of the nail is greater than the thermal conductivity of the potato. The nail conducts more heat from the oven to the interior of the potato than does the flesh of the potato, thereby causing the potato to bake faster.

---

12. ***REASONING AND SOLUTION*** Several days after a snowstorm, the roof on a house is uniformly covered with snow. On a neighboring house, the snow on the roof has completely melted. Since one of the houses still has snow on the roof, it is reasonable to conclude that the ambient temperature is still below the freezing point of water. Since the snow has melted from the roof of the neighboring house, we can conclude that the heat required to melt the snow must have come through the attic and the roof by conduction. Hence, the house which has the uniform layer of snow on the roof is probably better insulated. The better the insulation, the smaller is the amount of heat conducted through the roof to melt the snow.

---

13. **SSM** ***REASONING AND SOLUTION*** One car has a metal body, while another car has a plastic body. On a cold winter day, these cars are parked side by side. The metal car feels colder to the touch of your bare hand even though both cars are at the same temperature. This is because your fingers are sensitive to the rate at which heat is transferred to or from them, rather than to the temperature itself. The metal car feels colder than the plastic car at the same temperature, because heat flows from your bare hand into the metal car more readily than it flows into the plastic car. The flow occurs into the metal more readily, because the thermal conductivity of the metal is greater than that of the plastic.

---

14. ***REASONING AND SOLUTION*** Many high-quality pots have copper bases and polished stainless steel sides. Since copper has a high thermal conductivity, heat can readily enter the bottom of the pot by means of conduction. Since the temperature of the pot is greater than the temperature of its environment, the pot will lose heat by means of radiation. Polished stainless steel has a low emissivity; that is, it is a poor emitter of radiant energy. Hence, by making the sides of the pot polished stainless steel, the amount of heat that would be lost by radiation is minimized. This design is optimal. If the pot were constructed entirely of copper, the bottom would efficiently conduct heat into the pan; however, heat would also be conducted efficiently into the sides of the pot, raising their temperature and increasing the loss from the sides via radiation. If, on the other hand, the pot were constructed entirely of stainless steel, the loss of heat through radiant energy would be minimized; however, since stainless steel has a low thermal conductivity, heat would not efficiently enter the bottom of the pot through conduction.

---

15. ***REASONING AND SOLUTION*** The radiant energy Q emitted in a time t by an object that has a Kelvin temperature T, a surface area A, and an emissivity e, is given by Equation 13.2, $Q = e\sigma T^4 At$, where σ is the Stefan-Boltzmann constant.

We now consider two objects that have the same size and shape. Object A has an emissivity of 0.3, and object B has an emissivity of 0.6. Since each object radiates the same power, $e_A \sigma T_A^4 A_A = e_B \sigma T_B^4 A_B$. The Stefan-Boltzmann constant is a universal constant, and since the objects have the same size and shape, $A_A = A_B$; therefore, $e_A T_A^4 = e_B T_B^4$, or $T_A / T_B = \sqrt[4]{e_B / e_A} = \sqrt[4]{2}$. Hence, the Kelvin temperature of A is $\sqrt[4]{2}$ or 1.19 times the Kelvin temperature of B, not twice the temperature of B.

---

16. **SSM** ***REASONING AND SOLUTION***

a. The highly reflective paint reduces the ability of the so-called "radiator" to deliver heat into the room via the mechanism of radiation. This is because the paint allows the surface of the device to reflect more radiation than it otherwise would. Being a better reflector means that the device has become a poorer absorber of radiation, and poor absorbers are also poor emitters. Since the painted "radiator" loses less heat by the mechanism of radiation, it becomes hotter than it would if it were unpainted.

b. Since painting the device reduces its ability to radiate electromagnetic waves, we dismiss radiation as the primary mechanism by which "radiators" deliver heat. We also dismiss conduction, since air is not a good conductor of heat. That leaves convection. "Radiators" indeed function primarily via convection. The fact that the paint enables the device to become hotter for a given supply of hot water or steam is beneficial for convection. The hotter the device becomes, the more effectively it can generate the convection currents that distribute the heat around the room.

---

17. ***REASONING AND SOLUTION*** Two strips of material, A and B, are identical except that they have emissivities of 0.4 and 0.7, respectively. The strips are heated to the same temperature and have a bright glow. The emissivity is the ratio of the energy that an object actually radiates to the energy that the object would radiate if it were a perfect emitter. The strip with the higher emissivity will radiate more energy per second than the strip with the lower emissivity, other things being equal. Therefore, strip B will have the brighter glow.

---

18. ***REASONING AND SOLUTION*** The thermal conductivity of the bottom of the pot is greater than the thermal conductivity of air; therefore, the portion of the heating element beneath the pot loses heat by conduction through the bottom of the pot. The exposed portion of the heating element loses some heat through convection, but the convective process is not as efficient as the conductive process through the bottom of the pot. The exposed portion of the heating element will, therefore, lose less heat and be at a higher temperature than the portion of the heating element beneath the pot. Thus, the exposed portion glows cherry red.

---

19. ***REASONING AND SOLUTION*** If we consider a glove and a mitten, each of the same "size" and made of the same material, we can deduce that the mitten has less surface area A exposed to the cold winter air. Thus, according to Equation 13.1, $Q = kA\Delta Tt / L$, we can conclude that the mitten will conduct less heat per unit time from the hand to the winter air. Therefore, to keep your hands as warm as possible during skiing, you should wear mittens as opposed to gloves.

---

20. ***REASONING AND SOLUTION*** Two identical hot cups of cocoa are sitting on a kitchen table. One has a metal spoon in it and one does not. After five minutes, the cocoa with the metal spoon in it will be cooler. The metal spoon conducts heat from the cocoa to the handle of the spoon. Convection currents in the air and radiation then remove the heat from the spoon handle. The conduction-convection-radiation process removes heat from the cocoa, thereby cooling it faster than the cocoa that does not have a spoon in it.

---

21. ***REASONING AND SOLUTION*** The radiant energy Q emitted in a time t by an object that has a Kelvin temperature T, a surface area A, and an emissivity e, is given by Equation 13.2: $Q = e\sigma T^4 At$, where σ is the Stefan-Boltzmann constant.

a. A hot solid cube will cool more rapidly if it is cut in half, rather than if it is left intact. Since the cube is warmer than its environment, it will lose heat primarily through radiation. Convection currents will also remove some heat from the surface of the cube. When the cube has been cut in half, the surface area of the solid has been increased. If the length of one edge of the original cube is L, then cutting the cube in half increases the surface area from $6L^2$ to $8L^2$. From Equation 13.2, the amount of heat Q radiated in a time t is proportional to the surface area of the cube; therefore, the cube will radiate more rapidly and cool more rapidly if it is cut in half.

b. One pound of spaghetti noodles has a larger effective surface area than one pound of lasagna noodles. Imagine cutting many spaghetti noodles from one large lasagna noodle, in a way similar to what was done to the cube in part (a). Since the effective surface area of the spaghetti noodles is greater than that of the lasagna, heat will be radiated from the surface of the spaghetti noodles more effectively than heat will be radiated from the surface of the lasagna noodles. Therefore, the spaghetti noodles will cool more rapidly from the same initial temperature than the lasagna noodles.

---

22. ***REASONING AND SOLUTION*** The black asphalt is a better absorber than the cement; the black asphalt will absorb more of the sun's radiant energy than the cement. Since the sun has been shining all day, the asphalt will be at a higher temperature than the cement. The temperature of the asphalt is apparently above the freezing point of water, while the temperature of the cement playground is below the freezing point of water. Therefore, when snow hits the asphalt, it melts immediately, while the snow collects on the cement.

---

23. ***REASONING AND SOLUTION*** The radiant energy Q emitted in a time t by an object that has a Kelvin temperature T, a surface area A, and an emissivity e, is given by Equation 13.2: $Q = e\sigma T^4 At$, where σ is the Stefan-Boltzmann constant.

If you are stranded in the mountains in bitter cold weather, you could minimize energy losses from your body by curling up into the tightest possible ball. In doing so, you minimize your effective surface area. Therefore, A in Equation 13.2 is made smaller, and you would radiate less heat.

CHAPTER 13 | *THE TRANSFER OF HEAT*

PROBLEMS

---

1. **SSM** ***REASONING*** The heat conducted through the iron poker is given by Equation 13.1, $Q = (kA\,\Delta T)\,t\,/\,L$. If we assume that the poker has a circular cross-section, then its cross-sectional area is $A = \pi r^2$. Table 13.1 gives the thermal conductivity of iron as $79\text{ J}/(\text{s}\cdot\text{m}\cdot\text{C}^\circ)$.

SOLUTION The amount of heat conducted from one end of the poker to the other in 5.0 s is, therefore,

$$Q = \frac{(k\,A\Delta T)\,t}{L} = \frac{\left[79\text{ J}/\left(\text{s}\cdot\text{m}\cdot\text{C}^\circ\right)\right]\pi\left(5.0\times10^{-3}\text{ m}\right)^2\left(502\text{ °C}-26\text{ °C}\right)\left(5.0\text{ s}\right)}{1.2\text{ m}} = \boxed{12\text{ J}}$$

---

2. ***REASONING AND SOLUTION*** The rate at which energy is gained through the refrigerator walls is

$$\frac{Q}{t} = \frac{kA\Delta T}{L} = \frac{\left[0.030\text{ J/(s}\cdot\text{m}\cdot\text{C}^\circ)\right]\left(5.3\text{ m}^2\right)\left(25\text{ °C}-5\text{ °C}\right)}{0.075\text{ m}} = 42\text{ J/s}$$

Therefore, the amount of heat per second that must be removed from the unit to keep it cool is $\boxed{42\text{ J/s}}$.

---

3. ***REASONING*** Since heat Q is conducted from the blood capillaries to the skin, we can use the relation $Q = \dfrac{\left(kA\Delta T\right)t}{L}$ (Equation 13.1) to describe how the conduction process depends on the various factors. We can determine the temperature difference between the capillaries and the skin by solving this equation for ΔT and noting that the heat conducted per second is Q/t.

SOLUTION Solving Equation 13.1 for the temperature difference, and using the fact that Q/t = 240 J/s, yields

$$\Delta T = \frac{\left(Q/t\right)L}{kA} = \frac{\left(240\text{ J/s}\right)\left(2.0\times10^{-3}\text{m}\right)}{\left[0.20\text{ J/}\left(\text{s}\cdot\text{m}\cdot\text{C}^\circ\right)\right]\left(1.6\text{ m}^2\right)} = \boxed{1.5\text{ C}^\circ}$$

We have taken the thermal conductivity of body fat from Table 13.1.

---

4. ***REASONING AND SOLUTION*** The heat lost in each case is given by $Q = (kA\Delta T)t/L$. For the goose down jacket

$$Q_g = \frac{[0.025\text{ J/(s}\cdot\text{m}\cdot\text{C}°)](A\Delta T)t}{1.5\times10^{-2}\text{ m}}$$

For the wool jacket

$$Q_w = \frac{[0.040\text{ J/(s}\cdot\text{m}\cdot\text{C}°)](A\Delta T)t}{5.0\times10^{-3}\text{ m}}$$

Now

$$Q_w / Q_g = \boxed{4.8}$$

---

5. **SSM** ***REASONING*** The heat transferred in a time t is given by Equation 13.1, $Q = (kA\,\Delta T)t/L$. If the same amount of heat per second is conducted through the two plates, then $(Q/t)_{al} = (Q/t)_{st}$. Using Equation 13.1, this becomes

$$\frac{k_{al}A\Delta T}{L_{al}} = \frac{k_{st}A\Delta T}{L_{st}}$$

This expression can be solved for L_{st}.

SOLUTION Solving for L_{st} gives

$$L_{st} = \frac{k_{st}}{k_{al}}L_{al} = \frac{14\text{ J/(s}\cdot\text{m}\cdot\text{C}°)}{240\text{ J/(s}\cdot\text{m}\cdot\text{C}°)}(0.035\text{ m}) = \boxed{2.0\times10^{-3}\text{ m}}$$

---

6. ***REASONING*** The heat Q conducted along the bar is given by the relation $Q = \frac{(kA\Delta T)t}{L}$ (Equation 13.1). We can determine the temperature difference between the hot end of the bar and a point 0.15 m from that end by solving this equation for ΔT and noting that the heat conducted per second is Q/t and that $L = 0.15$ m.

SOLUTION Solving Equation 13.1 for the temperature difference, using the fact that $Q/t = 3.6$ J/s, and taking the thermal conductivity of brass from Table 13.1, yield

$$\Delta T = \frac{(Q/t)L}{kA} = \frac{(3.6\text{ J/s})(0.15\text{ m})}{[110\text{ J/(s}\cdot\text{m}\cdot\text{C}°)](2.6\times10^{-4}\text{ m}^2)} = 19\text{ C}°$$

The temperature at a distance of 0.15 m from the hot end of the bar is

$$T = 306\text{ °C} - 19\text{ C}° = \boxed{287\text{ °C}}$$

---

7. SSM WWW ***REASONING AND SOLUTION*** Values for the thermal conductivities of Styrofoam and air are given in Table 11.1. The conductance of an 0.080 mm thick sample of Styrofoam of cross-sectional area A is

$$\frac{k_s A}{L_s} = \frac{\left[0.010\ \text{J/(s}\cdot\text{m}\cdot\text{C}^\circ)\right] A}{0.080 \times 10^{-3}\ \text{m}} = [125\ \text{J/(s}\cdot\text{m}^2\cdot\text{C}^\circ)]\, A$$

The conductance of a 3.5 mm thick sample of air of cross-sectional area A is

$$\frac{k_a A}{L_a} = \frac{\left[0.0256\ \text{J/(s}\cdot\text{m}\cdot\text{C}^\circ)\right] A}{3.5 \times 10^{-3}\ \text{m}} = [7.3\ \text{J/(s}\cdot\text{m}^2\cdot\text{C}^\circ)]\, A$$

Dividing the conductance of Styrofoam by the conductance of air for samples of the same cross-sectional area A, gives

$$\frac{[125\ \text{J/(s}\cdot\text{m}^2\cdot\text{C}^\circ)]\, A}{[7.3\ \text{J/(s}\cdot\text{m}^2\cdot\text{C}^\circ)]\, A} = 17$$

Therefore, the body can adjust the conductance of the tissues beneath the skin by $\boxed{\text{a factor of 17}}$.

---

8. ***REASONING*** To find the total heat conducted, we will apply Equation 13.1 to the steel portion and the iron portion of the rod. In so doing, we use the area of a square for the cross section of the steel. The area of the iron is the area of the circle minus the area of the square. The radius of the circle is one half the length of the diagonal of the square.

SOLUTION In preparation for applying Equation 13.1, we need the area of the steel and the area of the iron. For the steel, the area is simply $A_{\text{Steel}} = L^2$, where L is the length of a side of the square. For the iron, the area is $A_{\text{Iron}} = \pi R^2 - L^2$. To find the radius R, we use the Pythagorean theorem, which indicates that the length D of the diagonal is related to the length of the sides according to $D^2 = L^2 + L^2$. Therefore, the radius of the circle is $R = D/2 = \sqrt{2}L/2$. For the iron, then, the area is

$$A_{\text{Iron}} = \pi R^2 - L^2 = \pi\left(\frac{\sqrt{2}L}{2}\right)^2 - L^2 = \left(\frac{\pi}{2} - 1\right)L^2$$

Taking values for the thermal conductivities of steel and iron from Table 13.1 and applying Equation 13.1, we find

$$Q_{\text{Total}} = Q_{\text{Steel}} + Q_{\text{Iron}}$$

$$= \left[\frac{(kA\Delta T)t}{L}\right]_{\text{Steel}} + \left[\frac{(kA\Delta T)t}{L}\right]_{\text{Iron}} = \left[k_{\text{Steel}}L^2 + k_{\text{Iron}}\left(\frac{\pi}{2}-1\right)L^2\right]\frac{(\Delta T)t}{L}$$

$$= \left[\left(14\ \frac{\text{J}}{\text{s}\cdot\text{m}\cdot\text{C}^\circ}\right)(0.010\ \text{m})^2 + \left(79\ \frac{\text{J}}{\text{s}\cdot\text{m}\cdot\text{C}^\circ}\right)\left(\frac{\pi}{2}-1\right)(0.010\ \text{m})^2\right]$$

$$\times\frac{(78\ ^\circ\text{C} - 18\ ^\circ\text{C})(120\ \text{s})}{0.50\ \text{m}} = \boxed{85\ \text{J}}$$

---

9. ***REASONING AND SOLUTION*** Using Equation 13.1, $Q = (kA\,\Delta T)t / L$, we obtain

$$\left(\frac{Q}{At}\right) = \frac{k\Delta T}{L} \tag{1}$$

Before Equation (1) can be applied to the ice-aluminum combination, the temperature T at the interface must be determined. We find the temperature at the interface by noting that the heat conducted through the ice must be equal to the heat conducted through the aluminum: $Q_{\text{ice}} = Q_{\text{aluminum}}$. Applying Equation 13.1 to this condition, we have

$$\left(\frac{kA\Delta Tt}{L}\right)_{\text{ice}} = \left(\frac{kA\Delta Tt}{L}\right)_{\text{aluminum}} \tag{2}$$

or

$$\frac{[2.2\ \text{J/(s}\cdot\text{m}\cdot\text{C}^\circ)]\,A[(-10.0\ ^\circ\text{C}) - \text{T}]\,t}{0.0050\ \text{m}} = \frac{[240\ \text{J/(s}\cdot\text{m}\cdot\text{C}^\circ)]\,A[T - (-25.0\ ^\circ\text{C})]\,t}{0.0015\ \text{m}}$$

The factors A and t can be eliminated algebraically. Solving for T gives $T = -24.959\ ^\circ\text{C}$ for the temperature at the interface.

a. Applying Equation (1) to the ice leads to

$$\left(\frac{Q}{At}\right)_{\text{ice}} = \frac{[2.2\ \text{J/(s}\cdot\text{m}\cdot\text{C}^\circ)][(-10.0\ ^\circ\text{C}) - (-24.959\ ^\circ\text{C})]}{0.0050\ \text{m}} = \boxed{6.58\times10^3\ \text{J/(s}\cdot\text{m}^2)}$$

Since heat is not building up in the materials, the rate of heat transfer per unit area is the same throughout the ice-aluminum combination. Thus, this must be the heat per second per square meter that is conducted through the ice-aluminum combination.

b. Applying Equation (1) to the aluminum in the absence of any ice gives:

$$\left(\frac{Q}{At}\right)_{\text{Al}} = \frac{[240\ \text{J/(s}\cdot\text{m}\cdot\text{C°)}]\left[(-10.0\ \text{°C})-(-25.0\ \text{°C})\right]}{0.0015\ \text{m}} = \boxed{2.40\times10^{6}\ \text{J/(s}\cdot\text{m}^{2})}$$

---

10. ***REASONING*** The water in both pots is boiling away at the same rate. This means that the heat per second $\frac{Q}{t}$ being delivered to the water through the bottom of the pot is the same in each case. The heat passes through the bottom of either pot via conduction. Therefore, we know that Equation 13.1 applies, so that $\frac{Q}{t}=\frac{kA\Delta T}{L}$, where k is the thermal conductivity of the material from which the pot bottom is made, A is the cross-sectional area of the bottom, ΔT is the difference in temperature between the inner and outer surface of the pot bottom, and L is the thickness of the bottom. We will apply this relation to the aluminum and to the copper bottom.

SOLUTION Applying Equation 13.1, we obtain

$$\left(\frac{Q}{t}\right)_{\text{copper}} = \left(\frac{Q}{t}\right)_{\text{aluminum}} \quad \text{or} \quad \frac{k_{\text{copper}}\,A(\Delta T)_{\text{copper}}}{L} = \frac{k_{\text{aluminum}}\,A(\Delta T)_{\text{aluminum}}}{L}$$

Note that the area A and thickness L are the same for each pot. Algebraically eliminating these terms gives

$$k_{\text{copper}}\,(\Delta T)_{\text{copper}} = k_{\text{aluminum}}\,(\Delta T)_{\text{aluminum}}$$

Solving for $(\Delta T)_{\text{copper}}$, we find that

$$(\Delta T)_{\text{copper}} = T_{\text{heating element}} - T_{\text{water}} = \frac{k_{\text{aluminum}}\,(\Delta T)_{\text{aluminum}}}{k_{\text{copper}}}$$

Thus, the heating element on which the copper bottom rests has a temperature of

$$T_{\text{heating element}} = T_{\text{water}} + \frac{k_{\text{aluminum}}\,(\Delta T)_{\text{aluminum}}}{k_{\text{copper}}}$$

$$= 100.0\ \text{°C} + \frac{\left[240\ \text{J/}\left(\text{s}\cdot\text{m}\cdot\text{C°}\right)\right]\left(155.0\ \text{°C} - 100.0\ \text{°C}\right)}{390\ \text{J/}\left(\text{s}\cdot\text{m}\cdot\text{C°}\right)} = \boxed{134\ \text{°C}}$$

---

11. SSM ***REASONING*** The heat lost per second due to conduction through the glass is given by Equation 13.1 as $Q/t = (kA\Delta T)/L$. In this expression, we have no information for the thermal conductivity k, the cross-sectional area A, or the length L. Nevertheless, we can apply the equation to the initial situation and again to the situation where the outside temperature has fallen. This will allow us to eliminate the unknown variables from the calculation.

SOLUTION Applying Equation 13.1 to the initial situation and to the situation after the outside temperature has fallen, we obtain

$$\left(\frac{Q}{t}\right)_{\text{Initial}} = \frac{kA\left(T_{\text{In}} - T_{\text{Out, initial}}\right)}{L} \quad \text{and} \quad \left(\frac{Q}{t}\right)_{\text{Colder}} = \frac{kA\left(T_{\text{In}} - T_{\text{Out, colder}}\right)}{L}$$

Dividing these two equations to eliminate the common variables gives

$$\frac{(Q/t)_{\text{Colder}}}{(Q/t)_{\text{Initial}}} = \frac{\dfrac{kA\left(T_{\text{In}} - T_{\text{Out, colder}}\right)}{L}}{\dfrac{kA\left(T_{\text{In}} - T_{\text{Out, initial}}\right)}{L}} = \frac{T_{\text{In}} - T_{\text{Out, colder}}}{T_{\text{In}} - T_{\text{Out, initial}}}$$

Remembering that twice as much heat is lost per second when the outside is colder, we find

$$\frac{2(Q/t)_{\text{Initial}}}{(Q/t)_{\text{Initial}}} = 2 = \frac{T_{\text{In}} - T_{\text{Out, colder}}}{T_{\text{In}} - T_{\text{Out, initial}}}$$

Solving for the colder outside temperature gives

$$T_{\text{Out, colder}} = 2T_{\text{Out, initial}} - T_{\text{In}} = 2(5.0\ ^\circ\text{C}) - (25\ ^\circ\text{C}) = \boxed{-15\ ^\circ\text{C}}$$

---

12. ***REASONING*** Heat Q flows along the length L of the bar via conduction, so that Equation 13.1 applies: $Q = \dfrac{(kA\Delta T)t}{L}$, where k is the thermal conductivity of the material from which the bar is made, A is the cross-sectional area of the bar, ΔT is the difference in temperature between the ends of the bar, and t is the time during which the heat flows. We will apply this expression twice in determining the length of the bar.

SOLUTION Solving Equation 13.1 for the length L of the bar gives

$$L = \frac{(kA\Delta T)t}{Q} = \frac{kA\left(T_{\text{W}} - T_{\text{C}}\right)t}{Q} \tag{1}$$

where T_W and T_C, respectively are the temperatures at the warmer and cooler ends of the bar. In this result, we do not know the terms *k*, *A*, *t*, or *Q*. However, we can evaluate the heat *Q* by recognizing that it flows through the entire length of the bar. This means that we can also apply Equation 13.1 to the 0.13 m of the bar at its cooler end and thereby obtain an expression for *Q*:

$$Q = \frac{kA\left(T - T_C\right)t}{D}$$

where the length of the bar through which the heat flows is $D = 0.13$ m and the temperature at the 0.13-m point is $T = 23$ °C, so that $\Delta T = T - T_C$. Substituting this result into Equation (1) and noting that the terms *k*, *A*, and *t* can be eliminated algebraically, we find

$$L = \frac{kA\left(T_W - T_C\right)t}{Q} = \frac{kA\left(T_W - T_C\right)t}{\dfrac{kA\left(T - T_C\right)t}{D}} = \frac{\cancel{k}\cancel{A}\left(T_W - T_C\right)\cancel{t}D}{\cancel{k}\cancel{A}\left(T - T_C\right)\cancel{t}}$$

$$= \frac{\left(T_W - T_C\right)D}{\left(T - T_C\right)} = \frac{\left(48\text{ °C} - 11\text{ °C}\right)\left(0.13\text{ m}\right)}{23\text{ °C} - 11\text{ °C}} = \boxed{0.40\text{ m}}$$

---

13. ***REASONING*** The heat *Q* required to change liquid water at 100.0 °C into steam at 100.0 °C is given by the relation $Q = mL_v$ (Equation 12.5), where *m* is the mass of the water and L_v is the latent heat of vaporization. The heat required to vaporize the water is conducted through the bottom of the pot and the stainless steel plate. The amount of heat conducted in a time *t* is given by $Q = \dfrac{\left(kA\Delta T\right)t}{L}$ (Equation 13.1), where k is the thermal conductivity, *A* and *L* are the cross-sectional area and length, and ΔT is the temperature difference. We will use these two relations to find the temperatures at the aluminum-steel interface and at the steel surface in contact with the heating element.

SOLUTION

a. Substituting Equation 12.5 into Equation 13.1 and solving for ΔT, we have

$$\Delta T = \frac{QL}{kAt} = \frac{\left(mL_v\right)L}{kAt}$$

The thermal conductivity k_{Al} of aluminum can be found in Table 13.1, and the latent heat of vaporization for water can be found in Table 12.3. The temperature difference ΔT_{Al} between the aluminum surfaces is

$$\Delta T_{Al} = \frac{\left(mL_v\right)L}{k_{Al}At} = \frac{\left(0.15\text{ kg}\right)\left(22.6\times10^5\text{ J/kg}\right)\left(3.1\times10^{-3}\text{m}\right)}{\left[240\text{ J/}\left(\text{s}\cdot\text{m}\cdot\text{C°}\right)\right]\left(0.015\text{ m}^2\right)\left(240\text{ s}\right)} = 1.2\text{ C°}$$

The temperature at the aluminum-steel interface is $T_{\text{Al-Steel}}$ = 100.0 °C + ΔT_{Al} = $\boxed{101.2\ °\text{C}}$.

b. Using the thermal conductivity k_{ss} of stainless steel from Table 13.1, we find that the temperature difference ΔT_{ss} between the stainless steel surfaces is

$$\Delta T_{\text{ss}} = \frac{\left(mL_{\text{v}}\right)L}{k_{\text{ss}}At} = \frac{(0.15\text{ kg})\left(22.6\times10^{5}\text{ J/kg}\right)\left(1.4\times10^{-3}\text{m}\right)}{\left[14\text{ J/}\left(\text{s}\cdot\text{m}\cdot\text{C}°\right)\right]\left(0.015\text{ m}^{2}\right)(240\text{ s})} = 9.4\text{ C}°$$

The temperature at the steel-burner interface is T = 101.2 °C + ΔT_{ss} = $\boxed{110.6\ °\text{C}}$.

---

14. ***REASONING*** Heat flows along the rods via conduction, so that Equation 13.1 applies: $Q = \frac{(kA\Delta T)t}{L}$, where Q is the amount of heat that flows in a time t, k is the thermal conductivity of the material from which a rod is made, A is the cross-sectional area of the rod, and ΔT is the difference in temperature between the ends of a rod. In arrangement *a*, this expression applies to each rod and ΔT has the same value of $\Delta T = T_{\text{W}} - T_{\text{C}}$. The total heat Q' is the sum of the heats through each rod. In arrangement *b*, the situation is more complicated. We will use the fact that the same heat flows through each rod to determine the temperature at the interface between the rods and then use this temperature to determine ΔT and the heat flow through either rod.

SOLUTION For arrangement *a*, we apply Equation 13.1 to each rod and obtain for the total heat that

$$Q' = Q_1 + Q_2 = \frac{k_1 A\left(T_{\text{W}} - T_{\text{C}}\right)t}{L} + \frac{k_2 A\left(T_{\text{W}} - T_{\text{C}}\right)t}{L} = \frac{\left(k_1 + k_2\right)A\left(T_{\text{W}} - T_{\text{C}}\right)t}{L} \qquad (1)$$

For arrangement *b*, we use T to denote the temperature at the interface between the rods and note that the same heat flows through each rod. Thus, using Equation 13.1 to express the heat flowing in each rod, we have

$$\underbrace{\frac{k_1 \cancel{A}\left(T_{\text{W}} - T\right)\cancel{t}}{\cancel{L}}}_{\substack{\text{Heat flowing}\\ \text{through rod 1}}} = \underbrace{\frac{k_2 \cancel{A}\left(T - T_{\text{C}}\right)\cancel{t}}{\cancel{L}}}_{\substack{\text{Heat flowing}\\ \text{through rod 2}}} \quad \text{or} \quad k_1\left(T_{\text{W}} - T\right) = k_2\left(T - T_{\text{C}}\right)$$

Solving this expression for the temperature T gives

$$T = \frac{k_1 T_{\text{W}} + k_2 T_{\text{C}}}{k_1 + k_2} \qquad (2)$$

Applying Equation 13.1 to either rod in arrangement *b* and using Equation (2) for the interface temperature, we can determine the heat Q that is flowing. Choosing rod 2, we find that

$$Q=\frac{k_2 A\left(T-T_\mathrm{C}\right)t}{L}=\frac{k_2 A\left(\dfrac{k_1 T_\mathrm{W}+k_2 T_\mathrm{C}}{k_1+k_2}-T_\mathrm{C}\right)t}{L}$$

$$=\frac{k_2 A\left(\dfrac{k_1 T_\mathrm{W}-k_1 T_\mathrm{C}}{k_1+k_2}\right)t}{L}=\frac{k_2 A k_1\left(T_\mathrm{W}-T_\mathrm{C}\right)t}{L\left(k_1+k_2\right)} \qquad (3)$$

Using Equations (1) and (3), we obtain for the desired ratio that

$$\frac{Q'}{Q}=\frac{\dfrac{\left(k_1+k_2\right)A\left(T_\mathrm{W}-T_\mathrm{C}\right)t}{L}}{\dfrac{k_2 A k_1\left(T_\mathrm{W}-T_\mathrm{C}\right)t}{L\left(k_1+k_2\right)}}=\frac{\left(k_1+k_2\right)\cancel{A}\,\cancel{\left(T_\mathrm{W}-T_\mathrm{C}\right)}\,\cancel{t}\,\cancel{L}\left(k_1+k_2\right)}{\cancel{L}\,k_2\,\cancel{A}\,k_1\cancel{\left(T_\mathrm{W}-T_\mathrm{C}\right)}\,\cancel{t}}=\frac{\left(k_1+k_2\right)^2}{k_2\,k_1}$$

Using the fact that $k_2=2k_1$, we obtain

$$\frac{Q'}{Q}=\frac{\left(k_1+k_2\right)^2}{k_2\,k_1}=\frac{\left(k_1+2k_1\right)^2}{2k_1\,k_1}=\boxed{4.5}$$

---

15. **SSM** **WWW** ***REASONING*** If the cylindrical rod were made of solid copper, the amount of heat it would conduct in a time t is, according to Equation 13.1, $Q_\mathrm{copper}=(k_\mathrm{copper}A_2\ \Delta T/L)t$. Similarly, the amount of heat conducted by the lead-copper combination is the sum of the heat conducted through the copper portion of the rod and the heat conducted through the lead portion:

$$Q_\mathrm{combination}=\left[k_\mathrm{copper}(A_2-A_1)\Delta T/L+k_\mathrm{lead}A_1\Delta T/L\right]\ t\ .$$

Since the lead-copper combination conducts one-half the amount of heat than does the solid copper rod, $Q_\mathrm{combination}=\frac{1}{2}Q_\mathrm{copper}$, or

$$\frac{k_\mathrm{copper}(A_2-A_1)\Delta T}{L}+\frac{k_\mathrm{lead}A_1\Delta T}{L}=\frac{1}{2}\left(\frac{k_\mathrm{copper}A_2\ \Delta T}{L}\right)$$

This expression can be solved for A_1/A_2, the ratio of the cross-sectional areas. Since the cross-sectional area of a cylinder is circular, $A=\pi r^2$. Thus, once the ratio of the areas is known, the ratio of the radii can be determined.

SOLUTION Solving for the ratio of the areas, we have

$$\frac{A_1}{A_2}=\frac{k_\mathrm{copper}}{2\left(k_\mathrm{copper}-k_\mathrm{lead}\right)}$$

The cross-sectional areas are circular so that $A_1 / A_2 = (\pi r_1^2)/(\pi r_2^2) = (r_1 / r_2)^2$; therefore,

$$\frac{r_1}{r_2} = \sqrt{\frac{k_{\text{copper}}}{2(k_{\text{copper}} - k_{\text{lead}})}} = \sqrt{\frac{390\ \text{J/(s}\cdot\text{m}\cdot\text{C°)}}{2[390\ \text{J/(s}\cdot\text{m}\cdot\text{C°)} - 35\ \text{J/(s}\cdot\text{m}\cdot\text{C°)}]}} = \boxed{0.74}$$

where we have taken the thermal conductivities of copper and lead from Table 13.1.

---

16. ***REASONING*** The radiant energy Q absorbed by the person's head is given by $Q = e\sigma T^4 At$ (Equation 13.2), where e is the emissivity, σ is the Stefan-Boltzmann constant, T is the Kelvin temperature of the environment surrounding the person (T = 28 °C + 273 = 301 K), A is the area of the head that is absorbing the energy, and t is the time. The radiant energy absorbed per second is $Q/t = e\sigma T^4 A$.

SOLUTION
a. The radiant energy absorbed per second by the person's head when it is covered with hair (e = 0.85) is

$$\frac{Q}{t} = e\sigma T^4 A = (0.85)\left[5.67\times10^{-8}\ \text{J/}\left(\text{s}\cdot\text{m}^2\cdot\text{K}^4\right)\right](301\ \text{K})^4\left(160\times10^{-4}\ \text{m}^2\right) = \boxed{6.3\ \text{J/s}}$$

b. The radiant energy absorbed per second by a bald person's head (e = 0.65) is

$$\frac{Q}{t} = e\sigma T^4 A = (0.65)\left[5.67\times10^{-8}\ \text{J/}\left(\text{s}\cdot\text{m}^2\cdot\text{K}^4\right)\right](301\ \text{K})^4\left(160\times10^{-4}\ \text{m}^2\right) = \boxed{4.8\ \text{J/s}}$$

---

17. **SSM** **WWW** ***REASONING AND SOLUTION*** Solving the Stefan-Boltzmann law, Equation 13.2, for the time t, and using the fact that $Q_{\text{blackbody}} = Q_{\text{bulb}}$, we have

$$t_{\text{blackbody}} = \frac{Q_{\text{blackbody}}}{\sigma T^4 A} = \frac{Q_{\text{bulb}}}{\sigma T^4 A} = \frac{P_{\text{bulb}}\ t_{\text{bulb}}}{\sigma T^4 A}$$

where P_{bulb} is the power rating of the light bulb. Therefore,

$$t_{\text{blackbody}} = \frac{(100.0\ \text{J/s})\ (3600\ \text{s})}{\left[5.67\times10^{-8}\ \text{J/(s}\cdot\text{m}^2\cdot\text{K}^4)\right]\ (303\ \text{K})^4\ \left[(6\ \text{sides})(0.0100\ \text{m})^2/\text{side}\right]}$$

$$\times\left(\frac{1\ \text{h}}{3600\ \text{s}}\right)\left(\frac{1\ \text{d}}{24\ \text{h}}\right) = \boxed{14.5\ \text{d}}$$

---

18. ***REASONING*** According to the Stefan-Boltzmann law, the radiant power emitted by the "radiator" is $\frac{Q}{t} = e\sigma T^4 A$ (Equation 13.2), where Q is the energy radiated in a time t, e is the emissivity of the surface, σ is the Stefan-Boltzmann constant, T is the temperature in Kelvins, and A is the area of the surface from which the radiant energy is emitted. We will apply this law to the "radiator" before and after it is painted. In either case, the same radiant power is emitted.

SOLUTION Applying the Stefan-Boltzmann law, we obtain the following:

$$\left(\frac{Q}{t}\right)_{\text{after}} = e_{\text{after}}\,\sigma T_{\text{after}}^4 A \quad \text{and} \quad \left(\frac{Q}{t}\right)_{\text{before}} = e_{\text{before}}\,\sigma T_{\text{before}}^4 A$$

Since the same radiant power is emitted before and after the "radiator" is painted, we have

$$\left(\frac{Q}{t}\right)_{\text{after}} = \left(\frac{Q}{t}\right)_{\text{before}} \quad \text{or} \quad e_{\text{after}}\,\sigma T_{\text{after}}^4 A = e_{\text{before}}\,\sigma T_{\text{before}}^4 A$$

The terms σ and A can be eliminated algebraically, so this result becomes

$$e_{\text{after}}\,\cancel{\sigma} T_{\text{after}}^4 \cancel{A} = e_{\text{before}}\,\cancel{\sigma} T_{\text{before}}^4 \cancel{A} \quad \text{or} \quad e_{\text{after}}\, T_{\text{after}}^4 = e_{\text{before}}\, T_{\text{before}}^4$$

Remembering that the temperature in the Stefan-Boltzmann law must be expressed in Kelvins, so that T_{before} = 62 °C +273 = 335 K (see Section 12.2), we find that

$$T_{\text{after}}^4 = \frac{e_{\text{before}}\, T_{\text{before}}^4}{e_{\text{after}}} \quad \text{or} \quad T_{\text{after}} = \sqrt[4]{\frac{e_{\text{before}}}{e_{\text{after}}}}\left(T_{\text{before}}\right) = \sqrt[4]{\frac{0.75}{0.50}}\left(335\text{ K}\right) = 371\text{ K}$$

On the Celsius scale, this temperature is 371 K – 273 = $\boxed{98\ °\text{C}}$.

---

19. ***REASONING AND SOLUTION*** We know from Equation 13.2 that

$$A = \frac{Q/t}{e\sigma T^4} = \frac{6.0\times 10^1\text{ W}}{(0.36)\left[5.67\times 10^{-8}\text{ J/(s}\cdot\text{m}^2\cdot\text{K}^4)\right](3273\text{ K})^4} = \boxed{2.6\times 10^{-5}\text{ m}^2}$$

---

20. ***REASONING*** The radiant energy Q radiated by the sun is given by $Q = e\sigma T^4 At$ (Equation 13.2), where e is the emissivity, σ is the Stefan-Boltzmann constant, T is its temperature (in Kelvins), A is the surface area of the sun, and t is the time. The radiant energy emitted per second is $Q/t = e\sigma T^4 A$. Solving this equation for T gives the surface temperature of the sun.

SOLUTION The radiant power produced by the sun is $Q/t = 3.9 \times 10^{26}$ W. The surface area of a sphere of radius r is $A = 4\pi r^2$. Since the sun is a perfect blackbody, $e = 1$. Solving Equation 13.2 for the surface temperature of the sun gives

$$T = \sqrt[4]{\frac{Q/t}{e\sigma 4\pi r^2}} = \sqrt[4]{\frac{3.9\times10^{26}\ \text{W}}{(1)\left[5.67\times10^{-8}\ \text{J/}\left(\text{s}\cdot\text{m}^2\cdot\text{K}^4\right)\right]4\pi\left(6.96\times10^{8}\ \text{m}\right)^2}} = \boxed{5800\ \text{K}}$$

---

21. **SSM** ***REASONING AND SOLUTION*** The net power generated by the stove is given by Equation 13.3, $P_{\text{net}} = e\sigma A(T^4 - T_0^4)$. Solving for T gives

$$T = \left(\frac{P_{\text{net}}}{e\sigma A} + T_0^4\right)^{1/4}$$

$$= \left\{\frac{7300\ \text{W}}{(0.900)[5.67\times10^{-8}\ \text{J}/(\text{s}\cdot\text{m}^2\cdot\text{K}^4)](2.00\ \text{m}^2)} + (302\ \text{K})^4\right\}^{1/4} = \boxed{532\ \text{K}}$$

---

22. ***REASONING*** The net rate at which energy is being lost via radiation can not exceed the production rate of 115 J/s, if the body temperature is to remain constant. The net rate at which an object at temperature T radiates energy in a room where the temperature is T_0 is given by Equation 13.3 as $P_{\text{net}} = e\sigma A(T^4 - T_0^4)$. P_{net} is the net energy per second radiated. We need only set P_{net} equal to 115 J/s and solve for T_0. We note that the temperatures in this equation must be expressed in Kelvins, not degrees Celsius.

SOLUTION According to Equation 13.3, we have

$$P_{\text{net}} = e\sigma A\left(T^4 - T_0^4\right) \quad \text{or} \quad T_0^4 = T^4 - \frac{P_{\text{net}}}{e\sigma A}$$

Using Equation 12.1 to convert from degrees Celsius to Kelvins, we have
$T = 34 + 273 = 307$ K. Using this value, it follows that

$$T_0 = \sqrt[4]{T^4 - \frac{P_{\text{net}}}{e\sigma A}}$$

$$= \sqrt[4]{(307\ \text{K})^4 - \frac{115\ \text{J/s}}{0.700\left[5.67\times10^{-8}\ \text{J/}\left(\text{s}\cdot\text{m}^2\cdot\text{K}^4\right)\right]\left(1.40\ \text{m}^2\right)}} = \boxed{287\ \text{K}\ (14\ ^\circ\text{C})}$$

---

23. ***REASONING AND SOLUTION*** The heat Q conducted during a time t through a wall of thickness L and cross sectional area A is given by Equation 13.1:

$$Q = \frac{kA\Delta T\, t}{L}$$

The radiant energy Q, emitted in a time t by a wall that has a Kelvin temperature T, surface area A, and emissivity e is given by Equation (13.2):

$$Q = e\sigma T^4 At$$

If the amount of radiant energy emitted per second per square meter at 0 °C is the same as the heat lost per second per square meter due to conduction, then

$$\left(\frac{Q}{tA}\right)_{\text{conduction}} = \left(\frac{Q}{tA}\right)_{\text{radiation}}$$

Making use of Equations 13.1 and 13.2, the equation above becomes

$$\frac{k\Delta T}{L} = e\sigma T^4$$

Solving for the emissivity e gives:

$$e = \frac{k\Delta T}{L\sigma T^4} = \frac{[1.1\ \text{J/(s}\cdot\text{m}\cdot\text{K)}](293.0\ \text{K} - 273.0\ \text{K})}{(0.10\ \text{m})[5.67\times10^{-8}\ \text{J/(s}\cdot\text{m}^2\cdot\text{K}^4)]\ (273.0\ \text{K})^4} = \boxed{0.70}$$

Remark on units: Notice that the units for the thermal conductivity were expressed as J/(s·m·K) even though they are given in Table 13.1 as J/(s·m·C°). The two units are equivalent since the "size" of a Celsius degree is the same as the "size" of a Kelvin; that is, 1 C° = 1 K. Kelvins were used, rather than Celsius degrees, to ensure consistency of units. However, Kelvins must be used in Equation 13.2 or any equation that is derived from it.

---

24. ***REASONING*** The heat Q necessary to vaporize a mass m of any substance at its boiling point is $Q = mL_\text{v}$ where L_v is the latent heat of vaporization. Therefore, the mass vaporized by an amount of heat Q is $m = Q/L_\text{v}$.

For the liquid helium system in question, it continually absorbs heat through radiation. The net power absorbed is given by Equation 13.3, $P_\text{net} = e\sigma A(T^4 - T_0^4)$ where T_0 is the temperature of the liquid helium, and T is the temperature maintained by the shield. Since the container is a perfect blackbody radiator, $e = 1$. Thus, the rate at which the mass of liquid helium boils away through the venting value is

$$\frac{m}{t}=\frac{(Q/t)}{L_v}=\frac{P_{net}}{L_v}=\frac{e\sigma A(T^4-T_0^4)}{L_v}$$

This expression can be multiplied by the time t to determine the mass vaporized during that time.

SOLUTION The rate at which liquid helium mass boils away is

$$\frac{m}{t}=\frac{(1)[5.67\times10^{-8}\ \text{J/(s}\cdot\text{m}^2\cdot\text{K}^4)]4\pi(0.30\ \text{m})^2[(77\ \text{K})^4-(4.2\ \text{K})^4]}{2.1\times10^4\ \text{J/kg}}=1.07\times10^{-4}\ \text{kg/s}$$

The mass of liquid helium that boils away in one hour is, therefore,

$$\left(1.07\times10^{-4}\ \frac{\text{kg}}{\text{s}}\right)(1.0\ \text{h})\left(\frac{3600\ \text{s}}{1.0\ \text{h}}\right)=\boxed{0.39\ \text{kg}}$$

---

25. **SSM** ***REASONING*** The total radiant power emitted by an object that has a Kelvin temperature T, surface area A, and emissivity e can be found by rearranging Equation 13.2, the Stefan-Boltzmann law: $Q=e\sigma T^4At$. The emitted power is $P=Q/t=e\sigma T^4A$. Therefore, when the original cylinder is cut perpendicular to its axis into N smaller cylinders, the ratio of the power radiated by the pieces to that radiated by the original cylinder is

$$\frac{P_{pieces}}{P_{original}}=\frac{e\sigma T^4A_2}{e\sigma T^4A_1} \qquad (1)$$

where A_1 is the surface area of the original cylinder, and A_2 is the sum of the surface areas of all N smaller cylinders. The surface area of the original cylinder is the sum of the surface area of the ends and the surface area of the cylinder body; therefore, if L and r represent the length and cross-sectional radius of the original cylinder, with $L=10r$,

$$A_1=(\text{area of ends})+(\text{area of cylinder body})$$

$$=2(\pi r^2)+(2\pi r)L=2(\pi r^2)+(2\pi r)(10r)=22\pi r^2$$

When the original cylinder is cut perpendicular to its axis into N smaller cylinders, the total surface area A_2 is

$$A_2=N2(\pi r^2)+(2\pi r)L=N2(\pi r^2)+(2\pi r)(10r)=(2N+20)\pi r^2$$

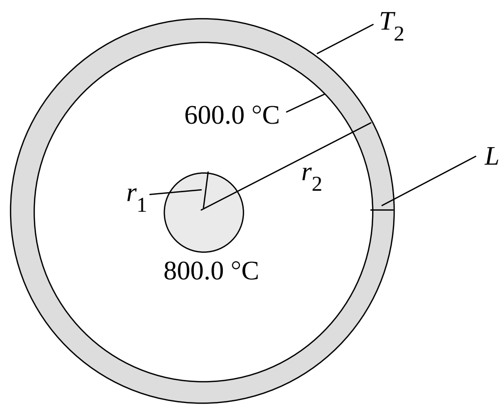
Substituting the expressions for A_1 and A_2 into Equation (1), we obtain the following expression for the ratio of the power radiated by the N pieces to that radiated by the original cylinder

$$\frac{P_{\text{pieces}}}{P_{\text{original}}} = \frac{e\sigma T^4 A_2}{e\sigma T^4 A_1} = \frac{(2N+20)\pi r^2}{22\pi r^2} = \frac{N+10}{11}$$

SOLUTION Since the total radiant power emitted by the N pieces is twice that emitted by the original cylinder, $P_{\text{pieces}}/P_{\text{original}} = 2$, we have ($N$ + 10)/11 = 2. Solving this expression for N gives $N = 12$. Therefore, $\boxed{\text{there are 12 smaller cylinders}}$.

---

26. ***REASONING*** The drawing shows a cross-sectional view of the small sphere inside the larger spherical asbestos shell. The small sphere produces a net radiant energy, because its temperature (800.0 °C) is greater than that of its environment (600.0 °C). This energy is then conducted through the thin asbestos shell (thickness = L). By setting the net radiant energy produced by the small sphere equal to the energy conducted through the asbestos shell, we will be able to obtain the temperature T_2 of the outer surface of the shell.



SOLUTION The heat Q conducted during a time through the thin asbestos shell is given by Equation 13.1 as $Q = \frac{(k_{\text{asbestos}} A_2 \,\Delta T)t}{L}$, where k_{asbestos} is the thermal conductivity of asbestos (see Table 13.1), A_2 is the area of the spherical shell $\left(A_2 = \pi r_2^2\right)$, ΔT is the temperature difference between the inner and outer surfaces of the shell (ΔT = 600.0 °C − T_2), and L is the thickness of the shell. Solving this equation for the T_2 yields

$$T_2 = \frac{Q\,L}{k_{\text{asbestos}}\left(\pi r_2^2\right)t} + \ 600.0\ ^\circ\text{C}$$

The heat Q is produced by the net radiant energy generated by the small sphere inside the asbestos shell. According to Equation 13.3, the net radiant energy is $Q = P_{\text{net}}t = e\sigma A_1\left(T^4 - T_0^4\right)t$, where e is the emissivity, σ is the Stefan-Boltzmann constant, A_1 is the spherical area of the sphere $\left(A_1 = \pi r_1^2\right)$, T is the temperature of the sphere (T = 800.0 °C = 1073.2 K) and T_0 is the temperature of the environment that surrounds the sphere (T_0 = 600.0 °C = 873.2 K). Substituting this expression for Q into the expression above for T_2, and algebraically eliminating the time t and the factors of π, gives

$$T_2 = 600.0\ ^\circ\text{C} - \frac{\left[e\sigma\left(T^4 - T_0^4\right)\right]L}{k_{\text{asbestos}}\left(\frac{r_2}{r_1}\right)^2}$$

$$= 600.0\ ^\circ\text{C} - \frac{\left\{(0.90)\left(5.67\times10^{-8}\ \frac{\text{J}}{\text{s}\cdot\text{m}^2\cdot\text{K}^4}\right)\left[(1073.2\ \text{K})^4 - (873.2\ \text{K})^4\right]\right\}\left(1.00\times10^{-2}\ \text{m}\right)}{\left[0.090\ \frac{\text{J}}{\text{s}\cdot\text{m}\cdot\text{C}^\circ}\right](10)^2}$$

$$= \boxed{557.7\ ^\circ\text{C}}$$

---

27. SSM ***REASONING AND SOLUTION*** According to Equation 13.1, the heat per second lost is

$$\frac{Q}{t} = \frac{kA\,\Delta T}{L} = \frac{[0.040\ \text{J/(s}\cdot\text{m}\cdot\text{C}^\text{o})]\ (1.6\ \text{m}^2)(25\ \text{C}^\text{o})}{2.0\times10^{-3}\ \text{m}} = \boxed{8.0\times10^2\ \text{J/s}}$$

where the value for the thermal conductivity k of wool has been taken from Table 13.1.

---

28. ***REASONING AND SOLUTION***

a. The heat lost by the oven is

$$Q = \frac{(kA\Delta T)t}{L} = \frac{[0.045\ \text{J}/(\text{s}\cdot\text{m}\cdot\text{C}^\circ)](1.6\ \text{m}^2)(160\ ^\circ\text{C} - 50\ ^\circ\text{C})(6.0\ \text{h})\left(\frac{3600\ \text{s}}{1\ \text{h}}\right)}{0.020\ \text{m}}$$

$$= \boxed{8.6\times10^6\ \text{J}}$$

b. As indicated on the page facing the inside of the front cover, 3.600×10^6 J = 1 kWh, so that 1 J = 2.78×10^{-7} kWh. Therefore, Q = 2.4 kWh. At \$ 0.10 per kWh, the cost is $\boxed{\$\ 0.24}$.

---

29. ***REASONING*** According to the discussion in Section 13.3, the net power P_{net} radiated by the person is $P_{\text{net}} = e\sigma A\left(T^4 - T_0^4\right)$, where e is the emissivity, σ is the Stefan-Boltzmann constant, A is the surface area, and T and T_0 are the temperatures of the person and the environment, respectively. Since power is the change in energy per unit time (see Equation 6.10b), the time t required for the person to emit the energy Q contained in the dessert is $t = Q/P_{\text{net}}$.

SOLUTION The time required to emit the energy from the dessert is

$$t=\frac{Q}{P_{\text{net}}}=\frac{Q}{e\sigma A\left(T^4-T_0^4\right)}$$

The energy is $Q=(260\text{ Calories})\left(\dfrac{4186\text{ J}}{1\text{ Calorie}}\right)$, and the Kelvin temperatures are T = 36 °C + 273 = 309 K and T_0 = 21 °C + 273 = 294 K. The time is

$$t=\frac{(260\text{ Calories})\left(\dfrac{4186\text{ J}}{1\text{ Calorie}}\right)}{(0.75)\left[5.67\times10^{-8}\text{ J/}\left(\text{s}\cdot\text{m}^2\cdot\text{K}^4\right)\right]\left(1.3\text{ m}^2\right)\left[(309\text{ K})^4-(294\text{ K})^4\right]}=\boxed{1.2\times10^4\text{ s}}$$

---

30. ***REASONING AND SOLUTION***

a. The radiant power lost by the body is

$$P_{\text{L}}=e\sigma T^4A=(0.80)[5.67\times10^{-8}\text{ J/(s}\cdot\text{m}^2\cdot\text{K}^4)](307\text{ K})^4(1.5\text{ m}^2)=604\text{ W}$$

The radiant power gained by the body from the room is

$$P_{\text{g}}=(0.80)[5.67\times10^{-8}\text{ J/(s}\cdot\text{m}^2\cdot\text{K}^4)](298\text{ K})^4(1.5\text{ m}^2)=537\text{ W}$$

The net loss of radiant power is $P=P_{\text{L}}-P_{\text{g}}=\boxed{67\text{ W}}$

b. The net energy lost by the body is

$$Q=Pt=(67\text{ W})(3600\text{ s})\left(\frac{1\text{ Calorie}}{4186\text{ J}}\right)=\boxed{58\text{ Calories}}$$

---

31. **SSM** ***REASONING AND SOLUTION*** The power radiated per square meter by the car when it has reached a temperature T is given by the Stefan-Boltzmann law, Equation 13.2, $P_{\text{radiated}}/A=e\sigma T^4$, where $P_{\text{radiated}}=Q/t$. Solving for T we have

$$T=\left[\frac{(P_{\text{radiated}}/A)}{e\sigma}\right]^{1/4}=\left\{\frac{560\text{ W/m}^2}{(1.00)\left[5.67\times10^{-8}\text{ J/(s}\cdot\text{m}^2\cdot\text{K}^4)\right]}\right\}^{1/4}=\boxed{320\text{ K}}$$

---

32. ***REASONING AND SOLUTION*** According to Equation 13.2, for the sphere we have $Q/t = e\sigma A_s T_s^4$, and for the cube $Q/t = e\sigma A_c T_c^4$. Equating and solving we get

$$T_c^4 = (A_s/A_c)T_s^4$$

Now

$$A_s/A_c = (4\pi R^2)/(6L^2)$$

The volume of the sphere and the cube are the same, (4/3) $\pi R^3 = L^3$, so $R = \left(\frac{3}{4\pi}\right)^{1/3} L$.

The ratio of the areas is $\frac{A_s}{A_c} = \frac{4\pi R^2}{6L^2} = \frac{4\pi}{6}\left(\frac{3}{4\pi}\right)^{2/3} = 0.806$. The temperature of the cube is, then

$$T_c = \left(\frac{A_s}{A_c}\right)^{1/4} T_s = (0.806)^{1/4}(773\text{ K}) = \boxed{732\text{ K}}$$

---

33. ***REASONING*** The heat Q required to melt ice at 0 °C into water at 0 °C is given by the relation $Q = mL_f$ (Equation 12.5), where m is the mass of the ice and L_f is the latent heat of fusion. We divide both sides of this equation by the time t and solve for the mass of ice per second (m/t) that melts:

$$\frac{m}{t} = \frac{\left(\frac{Q}{t}\right)}{L_f} \qquad (1)$$

The heat needed to melt the ice is conducted through the copper bar, from the hot end to the cool end. The amount of heat conducted in a time t is given by $Q = \frac{(kA\Delta T)t}{L}$ (Equation 13.1), where k is the thermal conductivity of the bar, A and L are its cross-sectional area and length, and ΔT is the temperature difference between the ends. We will use these two relations to find the mass of ice per second that melts.

SOLUTION Solving Equation 13.1 for Q/t and substituting the result into Equation (1) gives

$$\frac{m}{t} = \frac{\frac{kA\Delta T}{L}}{L_f} = \frac{kA\Delta T}{L\,L_f}$$

The thermal conductivity of copper can be found in Table 13.1, and the latent heat of fusion for water can be found in Table 12.3. The temperature difference between the ends of the rod is ΔT = 100 C°, since the hot end is in boiling water (100 °C) and the cool end is in ice (0 °C). Thus,

$$\frac{m}{t}=\frac{k\,A\,\Delta T}{L\,L_{\mathrm{f}}}=\frac{\left[390\ \mathrm{J/(s\cdot m\cdot C^\circ)}\right]\left(4.0\times10^{-4}\,\mathrm{m^2}\right)(100\ \mathrm{C^\circ})}{(1.5\ \mathrm{m})\left(33.5\times10^{4}\ \mathrm{J/kg}\right)}=\boxed{3.1\times10^{-5}\ \mathrm{kg/s}}$$

---

34. ***REASONING AND SOLUTION*** The rate of heat transfer is the same for all three materials so

$$Q/t\ =\ k_{\mathrm{p}}A\Delta T_{\mathrm{p}}/L\ =\ k_{\mathrm{b}}A\Delta T_{\mathrm{b}}/L\ =\ k_{\mathrm{w}}A\Delta T_{\mathrm{w}}/L$$

Let T_{i} be the inside temperature, T_1 be the temperature at the plasterboard-brick interface, T_2 be the temperature at the brick-wood interface, and T_{o} be the outside temperature. Then

$$k_{\mathrm{p}}T_{\mathrm{i}}-k_{\mathrm{p}}T_1=k_{\mathrm{b}}T_1-k_{\mathrm{b}}T_2 \qquad (1)$$

and

$$k_{\mathrm{b}}T_1-k_{\mathrm{b}}T_2=k_{\mathrm{w}}T_2-k_{\mathrm{w}}T_{\mathrm{o}} \qquad (2)$$

Solving (1) for T_2 gives

$$T_2=(k_{\mathrm{p}}+k_{\mathrm{b}})T_1/k_{\mathrm{b}}-(k_{\mathrm{p}}/k_{\mathrm{b}})T_{\mathrm{i}}$$

a. Substituting this into (2) and solving for T_1 yields

$$T_1=\frac{\left(k_{\mathrm{p}}/k_{\mathrm{b}}\right)\left(1+k_{\mathrm{w}}/k_{\mathrm{b}}\right)T_{\mathrm{i}}\ +\ \left(k_{\mathrm{w}}/k_{\mathrm{b}}\right)T_0}{\left(1+k_{\mathrm{w}}/k_{\mathrm{b}}\right)\left(1+k_{\mathrm{p}}/k_{\mathrm{b}}\right)\ -\ 1}=\boxed{21\,^\circ\mathrm{C}}$$

b. Using this value in (1) yields

$$\boxed{T_2=18\ ^\circ\mathrm{C}}$$

---

35. **SSM** **WWW** ***REASONING*** The rate at which heat is conducted along either rod is given by Equation 13.1, $Q/t=\left(k\,A\,\Delta T\right)/L$. Since both rods conduct the same amount of heat per second, we have

$$\frac{k_{\mathrm{s}}A_{\mathrm{s}}\ \Delta T}{L_{\mathrm{s}}}=\frac{k_{\mathrm{i}}A_{\mathrm{i}}\ \Delta T}{L_{\mathrm{i}}} \qquad (1)$$

Since the same temperature difference is maintained across both rods, we can algebraically cancel the ΔT terms. Because both rods have the same mass, $m_{\mathrm{s}}=m_{\mathrm{i}}$; in terms of the densities of silver and iron, the statement about the equality of the masses becomes $\rho_s\left(L_s A_s\right) = \rho_i\left(L_i A_i\right)$, or

$$\frac{A_s}{A_i} = \frac{\rho_i L_i}{\rho_s L_s} \tag{2}$$

Equations (1) and (2) may be combined to find the ratio of the lengths of the rods. Once the ratio of the lengths is known, Equation (2) can be used to find the ratio of the cross-sectional areas of the rods. If we assume that the rods have circular cross sections, then each has an area of $A = \pi r^2$. Hence, the ratio of the cross-sectional areas can be used to find the ratio of the radii of the rods.

SOLUTION
a. Solving Equation (1) for the ratio of the lengths and substituting the right hand side of Equation (2) for the ratio of the areas, we have

$$\frac{L_s}{L_i} = \frac{k_s A_s}{k_i A_i} = \frac{k_s\left(\rho_i L_i\right)}{k_i\left(\rho_s L_s\right)} \quad \text{or} \quad \left(\frac{L_s}{L_i}\right)^2 = \frac{k_s \rho_i}{k_i \rho_s}$$

Solving for the ratio of the lengths, we have

$$\frac{L_s}{L_i} = \sqrt{\frac{k_s \rho_i}{k_i \rho_s}} = \sqrt{\frac{[420\ \text{J/(s}\cdot\text{m}\cdot\text{C°)}](7860\ \text{kg/m}^3)}{[79\ \text{J/(s}\cdot\text{m}\cdot\text{C°)}](10\,500\ \text{kg/m}^3)}} = \boxed{2.0}$$

b. From Equation (2) we have

$$\frac{\pi r_s^2}{\pi r_i^2} = \frac{\rho_i L_i}{\rho_s L_s} \quad \text{or} \quad \left(\frac{r_s}{r_i}\right)^2 = \frac{\rho_i L_i}{\rho_s L_s}$$

Solving for the ratio of the radii, we have

$$\frac{r_s}{r_i} = \sqrt{\frac{\rho_i}{\rho_s}\left(\frac{L_i}{L_s}\right)} = \sqrt{\frac{7860\ \text{kg/m}^3}{10\,500\ \text{kg/m}^3}\left(\frac{1}{2.0}\right)} = \boxed{0.61}$$

---

36. ***REASONING AND SOLUTION*** The heat which must be removed to form a volume V of ice is

$$Q = mL_f = \rho V L_f = \rho A h L_f$$

The heat is conducted through the ice to the air, so Q is $Q = kA(\Delta T)t/L$. Thus, we have

$$h=\frac{k\,\Delta T\,t}{\rho\,L_{\mathrm{f}}\,L}=\frac{[2.2\text{ J/(s}\cdot\text{m}\cdot\text{C°)}](15\text{ C°})\left(3.0\times10^{2}\text{ s}\right)}{\left(917\text{ kg/m}^{3}\right)\left(3.35\times10^{5}\text{ J/kg}\right)(0.30\text{ m})}=1.1\times10^{-4}\text{ m}=\boxed{0.11\text{ mm}}$$

where the values for the thermal conductivity k, the density ρ, and the heat of fusion L_f have been taken from Table 13.1, Table 12.3, and Table 11.1, respectively.

---

37. ***CONCEPT QUESTIONS*** a. The temperature is 100.0 °C, because water boils at 100.0 °C under one atmosphere of pressure. The temperature remains at 100.0 °C until all the water is gone.

b. When water boils, it changes from the liquid to the vapor phase. The heat needed to make the water change phase is $Q = mL_v$, according to Equation 12.5, where m is the mass and L_v is the latent heat of vaporization of water.

c. The temperature of the heating element must be greater than 100.0 °C. This is because heat flows via conduction from a higher to a lower temperature and the temperature of the boiling water is 100.0 °C.

SOLUTION Applying Equation 13.1 to the heat conduction and using Equation 12.5 to express the heat needed to boil away the water, we have

$$Q=\frac{\left(k_{\text{copper}}A\Delta T\right)t}{L}=mL_{\text{v}}$$

The thermal conductivity of copper can be found in Table 13.1 $\left[k_{\text{copper}}=390\text{ J/}\left(\text{s}\cdot\text{m}\cdot\text{C°}\right)\right]$, and the latent heat of vaporization for water can be found in Table 12.4 ($L_v = 22.6 \times 10^5$ J/kg). The area A is the area of a circle or $A = \pi R^2$. Finally, the temperature difference is $\Delta T = T_E - 100.0$ °C. Using this expression for ΔT in the heat-conduction equation and solving for T_E gives

$$\frac{k_{\text{copper}}A\left(T_{\text{E}}-100.0\text{ °C}\right)t}{L}=mL_{\text{v}}\quad\text{or}\quad T_{\text{E}}=100.0\text{ °C}+\frac{LmL_{\text{v}}}{k_{\text{Copper}}At}$$

$$T_{\text{E}}=100.0\text{ °C}+\frac{\left(2.0\times10^{-3}\text{ m}\right)(0.45\text{ kg})\left(22.6\times10^{5}\text{ J/kg}\right)}{\left[390\text{ J/}\left(\text{s}\cdot\text{m}\cdot\text{C°}\right)\right]\pi(0.065\text{ m})^{2}(120\text{ s})}=\boxed{103.3\text{ °C}}$$

---

38. ***CONCEPT QUESTIONS*** a. According to Equation 13.1 less heat is lost when the area through which the heat flows is smaller. Since the window has the smaller area, it would lose less heat than the wall, other things being equal.

b. According to Equation 13.1 more heat is lost when the thickness through which the heat flows is smaller. Since the window has the smaller thickness, it would lose more heat than the wall, other things being equal.

c. According to Equation 13.1 more heat is lost when the thermal conductivity of the material through which the heat flows is greater. According to Table 13.1 the thermal conductivity of glass is $k_G = 0.80 \text{ J/}(\text{s}\cdot\text{m}\cdot\text{C}°)$, while the value for Styrofoam is $k_S = 0.010 \text{ J/}(\text{s}\cdot\text{m}\cdot\text{C}°)$. Therefore, the window would lose more heat than the wall, other things being equal.

SOLUTION The percentage of the heat lost by the window is

$$\text{Percentage} = \left(\frac{Q_{\text{window}}}{Q_{\text{wall}} + Q_{\text{window}}}\right)\times 100$$

$$= \left[\frac{\dfrac{k_G A_G (\Delta T)t}{L_G}}{\dfrac{k_S A_S (\Delta T)t}{L_S} + \dfrac{k_G A_G (\Delta T)t}{L_G}}\right]\times 100 = \left(\frac{\dfrac{k_G A_G}{L_G}}{\dfrac{k_S A_S}{L_S} + \dfrac{k_G A_G}{L_G}}\right)\times 100$$

Here, we algebraically eliminated the time t and the temperature difference ΔT, since they are the same in each term. The percentage is

$$\text{Percentage} = \left(\frac{\dfrac{k_G A_G}{L_G}}{\dfrac{k_S A_S}{L_S} + \dfrac{k_G A_G}{L_G}}\right)\times 100$$

$$= \left\{\frac{\dfrac{\left[0.80 \text{ J/}(\text{s}\cdot\text{m}\cdot\text{C}°)\right]\left(0.16 \text{ m}^2\right)}{2.0\times 10^{-3} \text{ m}}}{\dfrac{\left[0.010 \text{ J/}(\text{s}\cdot\text{m}\cdot\text{C}°)\right]\left(18 \text{ m}^2\right)}{0.10 \text{ m}} + \dfrac{\left[0.80 \text{ J/}(\text{s}\cdot\text{m}\cdot\text{C}°)\right]\left(0.16 \text{ m}^2\right)}{2.0\times 10^{-3} \text{ m}}}\right\}\times 100$$

$$= \boxed{97\ \%}$$

39. ***CONCEPT QUESTIONS*** a. The cross-sectional area *A* through which the heat flows is greater for arrangement *b*; the cross-sectional area in *b* is twice that in *a*.

b. The thickness *L* of the material through which the heat flows is greater for arrangement *a*; the thickness in *a* is twice that in *b*.

c. For two reasons, Q_a is less than Q_b. First, the area in arrangement *a* is smaller, and the heat flows in direct proportion to the area. A smaller area means less heat. Second, the thickness in arrangement *a* is greater, and the heat flows in inverse proportion to the thickness. A greater thickness means less heat.

SOLUTION Applying Equation 13.1 for the conduction of heat to both arrangements gives

$$Q_a = \frac{kA_a(\Delta T)t}{L_a} \quad \text{and} \quad Q_b = \frac{kA_b(\Delta T)t}{L_b}$$

Note that the thermal conductivity *k*, the temperature difference ΔT, and the time *t* are the same in both arrangements. Dividing Q_a by Q_b gives

$$\frac{Q_a}{Q_b} = \frac{\dfrac{kA_a(\Delta T)t}{L_a}}{\dfrac{kA_b(\Delta T)t}{L_b}} = \frac{A_a L_b}{A_b L_a}$$

Remember that $A_b = 2A_a$ and that $L_a = 2L_b$. As expected then, we find that

$$\frac{Q_a}{Q_b} = \frac{A_a L_b}{A_b L_a} = \frac{A_a L_b}{(2A_a)(2L_b)} = \boxed{\frac{1}{4}}$$

---

40. ***CONCEPT QUESTIONS*** a. According to Equation 6.10b, power is the change in energy divided by the time during which the change occurs. In this case, then, the power is $P = Q/t$.

b. According to the Stefan-Boltzmann law (Equation 13.2), the power radiated is $Q/t = e\sigma T^4 A$. The power is proportional to the fourth power of the temperature *T* (in Kelvins). Thus, a higher temperature promotes more radiated power.

c. According to the Stefan-Boltzmann law (Equation 13.2), the power radiated is $Q/t = e\sigma T^4 A$. The power is proportional to the area *A*. Thus, a smaller area generates less radiated power.

d. The higher temperature of bulb #1 promotes a greater radiated power. The only way for both bulbs to radiate the same power, then, is for the filament area of bulb #1 to be smaller than that of bulb #2, in order to offset the effect of the higher temperature.

SOLUTION Using the Stefan-Boltzmann law (Equation 13.2) for both bulbs, we have

$$P_1 = Q_1/t_1 = e\sigma T_1^4 A_1 \quad \text{and} \quad P_2 = Q_2/t_2 = e\sigma T_2^4 A_2$$

Since $P_1 = P_2$, we see that

$$e\sigma T_1^4 A_1 = e\sigma T_2^4 A_2 \quad \text{or} \quad T_1^4 A_1 = T_2^4 A_2$$

As expected, solving for the ratio A_1/A_2 gives a value less than one:

$$\frac{A_1}{A_2} = \frac{T_2^4}{T_1^4} = \frac{(2100\text{ K})^4}{(2700\text{ K})^4} = \boxed{0.37}$$

---

41. ***CONCEPT QUESTIONS*** a. The power radiated by the object is given by $Q/t = e\sigma T^4 A$ (Equation 13.2), where e is the emissivity of the object, σ is the Stefan-Boltzmann constant, T is the Kelvin temperature of the object, and A is the surface area of the object.

b. The power that the object absorbs from the room is given by $Q/t = e\sigma T_0{}^4 A$. Except for the temperature T_0 of the room, this expression has the same form as that for the power radiated by the object. Note especially that the area A is the radiation area for the ***object, not the room***. Review part b of the solution to Example 8 in the text to understand this important point.

c. The power emitted by the object is proportional to the fourth power of the its temperature, and the power absorbed is proportional to the fourth power of the room's temperature. Since the object emits more power than it absorbs, its temperature T must be greater than the room's temperature T_0.

SOLUTION The object emits three times more power than it absorbs from the room, so it follow that $(Q/t)_{\text{emit}} = 3(Q/t)_{\text{absorb}}$. Using the Stefan-Boltzmann law for each of the powers, we find

$$e\sigma T^4 A = 3e\sigma T_0^4 A$$

Solving for T gives

$$T = \sqrt[4]{3}\,T_0 = \sqrt[4]{3}\,(293\text{ K}) = \boxed{386\text{ K}}$$

---

42. ***CONCEPT QUESTIONS*** a. According to the Stefan-Boltzmann law, the power radiated by an object is $Q/t = e\sigma T^4 A$, where A is the area from which the radiation is emitted. The power radiated is proportional to the fourth power of the temperature T. Therefore, other things being equal, the greater surface temperature of Sirius B would imply that its radiated power is greater than that of our sun.

b. The fact that Sirius B radiates less power than our sun, means that something is offsetting the effect of the greater surface temperature in the Stefan-Boltzmann law. This can only be the surface area A. The power radiated is proportional to A, according to the law. A smaller area means a smaller radiated power. Therefore, the surface area of Sirius B must be less than the surface area of our sun.

c. The surface area of a sphere is $4\pi R^2$, where R is the radius. Therefore, having less surface area, Sirius B must also have a radius that is less than the radius of our sun.

SOLUTION Writing the Stefan-Boltzmann law (Equation 13.2) for both stars, we have

$$Q_{\text{Sirius}} / t_{\text{Sirius}} = e\sigma T_{\text{Sirius}}^4 A_{\text{Sirius}} \qquad \text{and} \qquad Q_{\text{Sun}} / t_{\text{Sun}} = e\sigma T_{\text{Sun}}^4 A_{\text{Sun}}$$

Dividing the equation for Sirius B by the equation for our sun and remembering that $Q_{\text{Sirius}}/t_{\text{Sirius}} = (0.040)\ Q_{\text{Sun}}/t_{\text{Sun}}$, we obtain

$$\frac{Q_{\text{Sirius}} / t_{\text{Sirius}}}{Q_{\text{Sun}} / t_{\text{Sun}}} = \frac{e\sigma T_{\text{Sirius}}^4 A_{\text{Sirius}}}{e\sigma T_{\text{Sun}}^4 A_{\text{Sun}}} \qquad \text{or} \qquad \frac{(0.040) Q_{\text{Sun}} / t_{\text{Sun}}}{Q_{\text{Sun}} / t_{\text{Sun}}} = \frac{T_{\text{Sirius}}^4 A_{\text{Sirius}}}{T_{\text{Sun}}^4 A_{\text{Sun}}}$$

Simplifying this result and using the fact that the surface area of a sphere is $4\pi R^2$ gives

$$0.040 = \frac{T_{\text{Sirius}}^4 \pi R_{\text{Sirius}}^2}{T_{\text{Sun}}^4 \pi R_{\text{Sun}}^2}$$

Solving for the radius of Sirius B gives

$$R_{\text{Sirius}} = \sqrt{0.040} \left(\frac{T_{\text{Sun}}}{T_{\text{Sirius}}} \right)^2 R_{\text{Sun}} = \sqrt{0.040} \left(\frac{T_{\text{Sun}}}{4T_{\text{Sun}}} \right)^2 \left(6.96 \times 10^8 \text{ m} \right) = \boxed{8.7 \times 10^6 \text{ m}}$$

As expected, the radius of Sirius B is less than that of our sun, so much so that it is called a white dwarf star.

---

43. ***CONCEPT QUESTIONS*** a. The heat Q conducted during a time t through a bar of length L and cross-sectional area A is $Q = \dfrac{(kA\Delta T)t}{L}$ (Equation 13.1). The heat depends on two

geometrical factors, the cross-sectional area and the length. Even though the cross-sectional area for heat conduction through the block in C is greater than that in A, it does not necessarily mean that more heat is conducted in C, because the lengths of the conduction paths are different.

b. Even though the length of material through which heat is conducted in block A is greater than that in B, it does not necessarily follow that less heat is conducted in A, because the blocks have different cross-sectional areas.

c. The heat Q conducted during a time t through a bar of length L and cross-sectional area A is $Q = \dfrac{(kA\Delta T)t}{L}$ (Equation 13.1). The cross-sectional area and length of each block are: $A_{\text{A}} = 2L_0^2$ and $L_{\text{A}} = 3L_0$, $A_{\text{B}} = 3L_0^2$ and $L_{\text{B}} = 2L_0$, $A_{\text{C}} = 6L_0^2$ and $L_{\text{C}} = L_0$. The heat conducted through each block is

$$Q_{\text{A}} = \left(\tfrac{2}{3}L_0\right)k\Delta Tt \qquad Q_{\text{B}} = \left(\tfrac{3}{2}L_0\right)k\Delta Tt \qquad Q_{\text{C}} = \left(6L_0\right)k\Delta Tt$$

Therefore, the ranking of the heat conduction is (highest to lowest): C, B, A

SOLUTION From the result of part c in the Concept Questions, the heat conducted in each case is:

Case A

$$Q_{\text{A}} = \left(\tfrac{2}{3}L_0\right)k\Delta Tt = \tfrac{2}{3}(0.30\text{ m})\left[250\text{ J/}(\text{s}\cdot\text{m}\cdot\text{C}^\circ)\right](35\ ^\circ\text{C} - 19\ ^\circ\text{C})(5.0\text{ s}) = \boxed{4.0\times10^3\text{ J}}$$

Case B

$$Q_{\text{B}} = \left(\tfrac{3}{2}L_0\right)k\Delta Tt = \tfrac{3}{2}(0.30\text{ m})\left[250\text{ J/}(\text{s}\cdot\text{m}\cdot\text{C}^\circ)\right](35\ ^\circ\text{C} - 19\ ^\circ\text{C})(5.0\text{ s}) = \boxed{9.0\times10^3\text{ J}}$$

Case C

$$Q_{\text{C}} = \left(6L_0\right)k\Delta Tt = 6(0.30\text{ m})\left[250\text{ J/}(\text{s}\cdot\text{m}\cdot\text{C}^\circ)\right](35\ ^\circ\text{C} - 19\ ^\circ\text{C})(5.0\text{ s}) = \boxed{3.6\times10^4\text{ J}}$$

44. ***CONCEPT QUESTIONS*** a. The net radiant power emitted by the bar in part (*a*) of the drawing is zero. The reason is that the temperature of the bar is the same as that of the room, and this temperature does not change. Therefore, the bar emits the same power into the room as it absorbs from the room, so the net radiant power emitted by the bar is zero.

b. The two bars in part (*b*) of the drawing emit more power. According to Equation 13.2, the radiant power (or energy per unit time) emitted by an object is $Q/t = e\sigma T^4 A$, which is directly proportional to its surface area *A*. The two bars in part (*b*) have a greater total surface area than the single bar in part (*a*).

c. The two bars in part (*b*) of the drawing also absorb more power. From the results of Concept Question b, we know that the two bars emit more power because of their greater surface area. However, since their temperature does not change, the two bars must absorb as much power as they emit. Thus, they absorb more power from the room than the single bar in part (*a*).

SOLUTION
a. The power (or energy per unit time) absorbed by the two bars in part (*b*) of the drawing is given by $Q/t = e\sigma T^4 A_2$, where A_2 is the total surface area of the two bars: $A_2 = 28L_0^2$. The power absorbed by the single bar in A is $Q/t = e\sigma T^4 A_1$, where A_1 is the total surface area of the single bar: $A_1 = 22L_0^2$.The ratio of the power P_2 absorbed by the two bars in part (*b*) to the power P_1absorbed by the single bar in part (*a*) is

$$\frac{P_2}{P_1} = \frac{e\sigma T^4\left(28\ L_0^2\right)}{e\sigma T^4\left(22\ L_0^2\right)} = \boxed{1.27}$$

b. If the power absorbed by the two bars in part (*b*) of the drawing is the same as that absorbed by the single bar in part (*a*), then

$$\underbrace{e\sigma T_1^4 A_1}_{\substack{\text{Power absorbed}\\ \text{by single bar}\\ \text{in part }(a)}} = \underbrace{e\sigma T_2^4 A_2}_{\substack{\text{Power absorbed}\\ \text{by two bars}\\ \text{in part }(b)}}$$

Solving for the temperature of the room and the bars in part (*b*) gives

$$T_2 = T_1 \sqrt[4]{\frac{A_1}{A_2}} = (450.0\ \text{K}) \sqrt[4]{\frac{22L_0^2}{28L_0^2}} = \boxed{424\ \text{K}}$$