1. **REASONING AND SOLUTION**  The car will accelerate if its velocity changes in magnitude, in direction, or both. If a car is traveling at a constant speed of 35 m/s, it can be accelerating if its direction of motion is changing.

2. **REASONING AND SOLUTION**  Consider two people, one on the earth's surface at the equator, and the other at the north pole. If we combine Equations 5.1 and 5.2, we see that the centripetal acceleration of an object moving in a circle of radius \( r \) with period \( T \) can be written as \( a_c = \left( \frac{4 \pi^2 r}{T^2} \right) \). The earth rotates about an axis that passes approximately through the north pole and is perpendicular to the plane of the equator. Since both people are moving on the earth's surface, they have the same period \( T \). The person at the equator moves in a larger circle so that \( r \) is larger for the person at the equator. Therefore, the person at the equator has a larger centripetal acceleration than the person at the north pole.

3. **REASONING AND SOLUTION**  The equations of kinematics (Equations 3.3 - 3.6) cannot be applied to uniform circular motion because an object in uniform circular motion does not have a constant acceleration. While the acceleration vector is constant in magnitude \( (a = v^2 / r) \), its direction changes constantly -- it always points toward the center of the circle. As the object moves around the circle the direction of the acceleration must constantly change. Because of this changing direction, the condition of constant acceleration that is required by Equations 3.3 – 3.6 is violated.

4. **REASONING AND SOLUTION**  Acceleration is the rate of change of velocity. In order to have an acceleration, the velocity vector must change either in magnitude or direction, or both. Therefore, if the velocity of the object is constant, the acceleration must be zero. On the other hand, if the speed of the object is constant, the object could be accelerating if the direction of the velocity is changing.

5. **REASONING AND SOLUTION**  When the car is moving at constant speed along the straight segments (i.e., \( AB \) and \( DE \)), the acceleration is zero. Along the curved segments, the magnitude of the acceleration is given by \( v^2 / r \). Since the speed of the car is constant, the magnitude of the acceleration is largest where the radius \( r \) is smallest. Ranked from smallest to largest the magnitudes of the accelerations in each of the four sections are: \( AB \) or \( DE \), \( CD \), \( BC \).
6. **REASONING AND SOLUTION** From Example 7, the maximum safe speed with which a car can round an unbanked horizontal curve of radius $r$ is given by $v = \sqrt{\mu gr}$. Since the acceleration due to gravity on the moon is roughly one sixth that on earth, the safe speed for the same curve on the moon would be less than that on earth. In other words, other things being equal, it would be more difficult to drive at high speed around an unbanked curve on the moon as compared to driving around the same curve on the earth.

7. **SSM REASONING AND SOLUTION** A bug lands on a windshield wiper. The wipers are turned on. Since the wipers move along the arc of a circle, the bug will experience a centripetal acceleration, and hence, a centripetal force must be present. The magnitude of the centripetal force is given by $F_c = \frac{mv^2}{r}$. In order for the bug to remain at rest on the wiper blade, the force of static friction between the bug and the wiper blade must contribute in a major way to the centripetal force. Without the centripetal force, the bug will be dislodged. When the wipers are turned on at a higher setting, $v$ is larger, and the centripetal force required to keep the bug moving along the arc of the circle is larger than if the wipers are turned on the low setting. Since the high setting requires a larger centripetal force to keep the bug on the wiper, the bug is more likely to be dislodged at that setting than at the low setting.

8. **REASONING AND SOLUTION** From Example 7, the maximum safe speed with which a car can round an unbanked curve of radius $r$ is given by $v = \sqrt{\mu gr}$. This expression is independent of the mass (and therefore the weight) of the car. Thus, the chance of a light car safely rounding an unbanked curve on an icy road is the same as that for a heavier car (assuming that all other factors are the same).

9. **SSM REASONING AND SOLUTION** Since the speed and radius of the circle are constant, the centripetal acceleration is constant. As the water leaks out, however, the mass of the object undergoing the uniform circular motion decreases. Centripetal force is mass times the centripetal acceleration, so that the centripetal force applied to the container must be decreasing. It is the tension in the rope that provides the centripetal force. You are holding the free end of the rope and pulling on it in order to create the tension. Therefore, you must be reducing your pull as the water leaks out. In turn, according to Newton’s third law, the rope must be pulling back on your hand with a force of decreasing magnitude, and you feel this pull weakening as time passes.

10. **REASONING AND SOLUTION** As the propeller rotates faster, the centripetal acceleration of the parts of the propeller increase. As the centripetal acceleration increases, the centripetal force required to cause the various parts of the propeller to rotate in the circle also increases. When the necessary centripetal force exceeds the mechanical forces that hold the propeller together, the propeller will come apart.
11. **REASONING AND SOLUTION** The centripetal force on the penny is given by 
\[ F_c = \frac{mv^2}{r}, \] 
where \( v = 2\pi r / T \) and \( T \) is the constant period of the turntable. Therefore, the centripetal force on the penny is given by 
\[ F_c = 4\pi^2 m r / T^2. \] 
Clearly, the penny will require the largest centripetal force to remain in place when located at the largest value of \( r \); that is, at the edge of the turntable.

12. **REASONING AND SOLUTION** A model airplane on a guideline can fly in a circle because the tension in the guideline provides the horizontal centripetal force necessary to pull the plane into a horizontal circle. A real airplane has no such horizontal forces. The air on the wings on a real plane exerts an upward lifting force that is perpendicular to the wings. The plane must bank so that a component of the lifting force can be oriented horizontally, thereby providing the required centripetal force to cause the plane to fly in a circle.

13. [SSM] **REASONING AND SOLUTION**

a. Referring to Figure 5.10 in the text, we can see that the centripetal force on the plane is 
\[ L \sin \theta = \frac{mv^2}{r}, \] 
where \( L \) is the magnitude of the lifting force. In addition, the vertical component of the lifting force must balance the weight of the plane, so that 
\[ L \cos \theta = mg. \]
Dividing these two equations reveals that 
\[ \tan \theta = \frac{v^2}{rg}. \]

b. The banking condition for a car traveling at speed \( v \) around a curve of radius \( r \), banked at angle \( \theta \) is 
\[ \tan \theta = \frac{v^2}{rg}, \] 
according to Equation 5.4 in the text.

c. The speed \( v \) of a satellite in a circular orbit of radius \( r \) about the earth is given by 
\[ v = \sqrt{\frac{GM_E}{r}}, \] 
according to Equation 5.5 in the text.

d. The minimum speed required for a loop-the-loop trick around a loop of radius \( r \) is 
\[ v = \sqrt{rg}, \] 
according to the discussion in Section 5.7 of the text. According to Equations 4.4 and 4.5, 
\[ g = \frac{GM_E}{r^2}. \] 
Thus, any expression that depends on \( g \) also depends on \( ME \) and would be affected by a change in the earth's mass. Such is the case for each of the four situations discussed above.

14. **REASONING AND SOLUTION** When the string is whirled in a horizontal circle, the tension in the string, \( F_T \), provides the centripetal force which causes the stone to move in a circle. Since the speed of the stone is constant, 
\[ mv^2 / r = F_T \] 
and the tension in the string is constant.

When the string is whirled in a vertical circle, the tension in the string and the weight of the stone both contribute to the centripetal force, depending on where the stone is on the circle. Now, however, the tension increases and decreases as the stone traverses the vertical circle. When the stone is at the lowest point in its swing, the tension in the string pulls the stone upward, while the weight of the stone acts downward. Therefore, the centripetal force
is $mv^2/r = F_r - mg$. Solving for the tension shows that $F_r = mv^2/r + mg$. This tension is larger than in the horizontal case. Therefore, the string has a greater chance of breaking when the stone is whirled in a vertical circle.

15. **REASONING AND SOLUTION** A fighter pilot pulls out of a dive on a vertical circle and begins to climb upward. As the pilot moves along the circle, all parts of his body, including the blood in his head, must experience a centripetal force in order to remain on the circle. The blood, however, is not rigidly attached to the body and does not experience the requisite centripetal force until it flows out of the head, away from the circle's center, and collects in the lower body parts, which ultimately push on it enough to keep it on the circular path.

16. **REASONING AND SOLUTION** When a car moves around an unbanked horizontal curve, the centripetal force that keeps the car on the road so that it can negotiate the curve comes from the force of static friction. If car A cannot negotiate the curve, then the force of static friction between the treads of car A's tires and the road is not great enough to provide the centripetal force. The coefficient of static friction between the tires and the road are less for car A than for car B, since car B can negotiate the turn.
CHAPTER  5 | DYNAMICS OF UNIFORM CIRCULAR MOTION

PROBLEMS

1. **REASONING** The speed of the plane is given by Equation 5.1: \( v = \frac{2\pi r}{T} \), where \( T \) is the period or the time required for the plane to complete one revolution.

**SOLUTION** Solving Equation 5.1 for \( T \) we have

\[
T = \frac{2\pi r}{v} = \frac{2\pi (2850 \text{ m})}{110 \text{ m/s}} = 160 \text{ s}
\]

2. **REASONING AND SOLUTION** Since the speed of the object on and off the circle remains constant at the same value, the object always travels the same distance in equal time intervals, both on and off the circle. Furthermore since the object travels the distance \( OA \) in the same time it would have moved from \( O \) to \( P \) on the circle, we know that the distance \( OA \) is equal to the distance along the arc of the circle from \( O \) to \( P \).

The circumference of the circle is \( 2\pi r = 2\pi (3.6 \text{ m}) = 22.6 \text{ m} \). The arc \( OP \) subtends an angle of \( \theta = 25^\circ \); therefore, since any circle contains 360°, the arc \( OP \) is 25/360 or 6.9 per cent of the circumference of the circle. Thus,

\[
OP = (22.6 \text{ m})(0.069) = 1.6 \text{ m}
\]

and, from the argument given above, we conclude that the distance \( OA \) is \( 1.6 \text{ m} \).

3. **REASONING** Since the tip of the blade moves on a circular path, it experiences a centripetal acceleration whose magnitude \( a_c \) is given by Equation 5.2 as, \( a_c = \frac{v^2}{r} \), where \( v \) is the speed of blade tip and \( r \) is the radius of the circular path. The radius is known, and the speed can be obtained by dividing the distance that the tip travels by the time \( t \) of travel. Since an angle of 90° corresponds to one fourth of the circumference of a circle, the distance is \( \frac{1}{4}(2\pi r) \).

**SOLUTION** Since \( a_c = \frac{v^2}{r} \) and \( v = \frac{1}{4}(2\pi r)/t = \pi r/(2t) \), the magnitude of the centripetal acceleration of the blade tip is

\[
a_c = \frac{v^2}{r} = \frac{(\frac{\pi r}{2t})^2}{r} = \frac{\pi^2 r^2}{4t^2} = \frac{\pi^2 (0.45 \text{ m})}{4(0.40 \text{ s})^2} = 6.9 \text{ m/s}^2
\]
4. **REASONING** The magnitude $a_c$ of the car’s centripetal acceleration is given by Equation 5.2 as $a_c = v^2 / r$, where $v$ is the speed of the car and $r$ is the radius of the track. The radius is $r = 2.6 \times 10^2$ m. The speed can be obtained from Equation 5.1 as the circumference $(2\pi r)$ of the track divided by the period $T$ of the motion. The period is the time for the car to go once around the track ($T = 360$ s).

**SOLUTION** Since $a_c = v^2 / r$ and $v = (2\pi r) / T$, the magnitude of the car’s centripetal acceleration is

$$a_c = \frac{v^2}{r} = \frac{\left(\frac{2\pi r}{T}\right)^2}{r} = \frac{4\pi^2 r}{T^2} = \frac{4\pi^2 \left(2.6 \times 10^3 \text{ m}\right)}{(360 \text{ s})^2} = 0.79 \text{ m/s}^2$$

5. **SSM REASONING AND SOLUTION** In each case, the magnitude of the centripetal acceleration is given by Equation 5.2, $a_c = v^2 / r$. Therefore,

$$\frac{a_{cA}}{a_{cB}} = \frac{v_{A}^2 / r_{A}}{v_{B}^2 / r_{B}}$$

Since each boat experiences the same centripetal acceleration, $a_{cA} / a_{cB} = 1$. Solving for the ratio of the speeds gives

$$\frac{v_{A}}{v_{B}} = \sqrt{\frac{r_{A}}{r_{B}}} = \sqrt{\frac{120 \text{ m}}{240 \text{ m}}} = \frac{0.71}{1}$$

6. **REASONING** The astronaut in the chamber is subjected to a centripetal acceleration $a_c$ that is given by $a_c = v^2 / r$ (Equation 5.2). In this expression $v$ is the speed at which the astronaut in the chamber moves on the circular path of radius $r$. We can solve this relation for the speed.

**SOLUTION** Using Equation 5.2, we have

$$a_c = \frac{v^2}{r} \quad \text{or} \quad v = \sqrt{a_c r} = \sqrt{\left[7.5 \left(9.80 \text{ m/s}^2\right)\right] (15 \text{ m})} = 33 \text{ m/s}$$

7. **REASONING** The centripetal acceleration is given by Equation 5.2 as $a_c = v^2 / r$. The value of the radius $r$ is given, so to determine $a_c$ we need information about the speed $v$. But the speed is related to the period $T$ by $v = (2\pi r) / T$, according to Equation 5.1. We can substitute this expression for the speed into Equation 5.2 and see that
\[ a_c = \frac{v^2}{r} = \frac{(2\pi r / T)^2}{r} = \frac{4\pi^2 r}{T^2} \]

**SOLUTION** To use the expression obtained in the reasoning, we need a value for the period \( T \). The period is the time for one revolution. Since the container is turning at 2.0 revolutions per second, the period is \( T = (1 \text{ s})/(2.0 \text{ revolutions}) = 0.50 \text{ s} \). Thus, we find that the centripetal acceleration is

\[ a_c = \frac{4\pi^2 r}{T^2} = \frac{4\pi^2 (0.12 \text{ m})}{(0.50 \text{ s})^2} = 19 \text{ m/s}^2 \]

8. **REASONING AND SOLUTION** The centripetal acceleration for any point on the blade a distance \( r \) from center of the circle, according to Equation 5.2, is \( a_c = \frac{v^2}{r} \). From Equation 5.1, we know that \( v = \frac{2\pi r}{T} \) where \( T \) is the period of the motion. Combining these two equations, we obtain

\[ a_c = \frac{(2\pi r / T)^2}{r} = \frac{4\pi^2 r}{T^2} \]

a. Since the turbine blades rotate at 617 rev/s, all points on the blades rotate with a period of \( T = (1/617) \text{ s} = 1.62 \times 10^{-3} \text{ s} \). Therefore, for a point with \( r = 0.020 \text{ m} \), the magnitude of the centripetal acceleration is

\[ a_c = \frac{4\pi^2 (0.020 \text{ m})}{(1.62 \times 10^{-3} \text{ s})^2} = 3.0 \times 10^5 \text{ m/s}^2 \]

b. Expressed as a multiple of \( g \), this centripetal acceleration is

\[ a_c = \left(3.0 \times 10^5 \text{ m/s}^2\right) \left(\frac{1.00 g}{9.80 \text{ m/s}^2}\right) = 3.1 \times 10^4 g \]

9. **SSM REASONING** The magnitude of the centripetal acceleration of any point on the helicopter blade is given by Equation 5.2, \( a_c = \frac{v^2}{r} \), where \( r \) is the radius of the circle on which that point moves. From Equation 5.1: \( v = \frac{2\pi r}{T} \). Combining these two expressions, we obtain

\[ a_c = \frac{4\pi^2 r}{T^2} \]

All points on the blade move with the same period \( T \).
**SOLUTION** The ratio of the centripetal acceleration at the end of the blade (point 1) to that which exists at a point located 3.0 m from the center of the circle (point 2) is

\[
\frac{a_{c1}}{a_{c2}} = \frac{4\pi^2 r_1 / T^2}{4\pi^2 r_2 / T^2} = \frac{r_1}{r_2} = \frac{6.7 \text{ m}}{3.0 \text{ m}} = 2.2
\]

10. **REASONING** The centripetal acceleration for any point that is a distance \( r \) from the center of the disc is, according to Equation 5.2, \( a_c = \frac{v^2}{r} \). From Equation 5.1, we know that \( v = \frac{2\pi r}{T} \) where \( T \) is the period of the motion. Combining these two equations, we obtain

\[
a_c = \frac{(2\pi r / T)^2}{r} = \frac{4\pi^2 r}{T^2}
\]

**SOLUTION** Using the above expression for \( a_c \), the ratio of the centripetal accelerations of the two points in question is

\[
\frac{a_2}{a_1} = \frac{4\pi^2 r_2 / T_2^2}{4\pi^2 r_1 / T_1^2} = \frac{r_2 / T_2^2}{r_1 / T_1^2}
\]

Since the disc is rigid, all points on the disc must move with the same period, so \( T_1 = T_2 \). Making this cancellation and solving for \( a_2 \), we obtain

\[
a_2 = a_1 \frac{r_2}{r_1} = \left(120 \text{ m/s}^2\right) \left(\frac{0.050 \text{ m}}{0.030 \text{ m}}\right) = 2.0 \times 10^2 \text{ m/s}^2
\]

Note that even though \( T_1 = T_2 \), it is not true that \( v_1 = v_2 \). Thus, the simplest way to approach this problem is to express the centripetal acceleration in terms of the period \( T \) which cancels in the final step.

11. **SSM REASONING** In Example 3, it was shown that the magnitudes of the centripetal acceleration for the two cases are

\[
\text{[Radius = 33 m]} \quad a_c = 35 \text{ m/s}^2
\]
\[
\text{[Radius = 24 m]} \quad a_c = 48 \text{ m/s}^2
\]

According to Newton's second law, the centripetal force is \( F_c = ma_c \) (see Equation 5.3).

**SOLUTION** a. Therefore, when the sled undergoes the turn of radius 33 m,

\[
F_c = ma_c = (350 \text{ kg})(35 \text{ m/s}^2) = 1.2 \times 10^4 \text{ N}
\]
b. Similarly, when the radius of the turn is 24 m,

\[ F_c = ma_c = (350 \text{ kg})(48 \text{ m/s}^2) = 1.7 \times 10^4 \text{ N} \]

12. **REASONING** The magnitude \( F_c \) of the centripetal force that acts on the skater is given by Equation 5.3 as \( F_c = \frac{mv^2}{r} \), where \( m \) and \( v \) are the mass and speed of the skater, and \( r \) is the distance of the skater from the pivot. Since all of these variables are known, we can find the magnitude of the centripetal force.

**SOLUTION** The magnitude of the centripetal force is

\[ F_c = \frac{mv^2}{r} = \frac{(80.0 \text{ kg})(6.80 \text{ m/s})^2}{6.10 \text{ m}} = 606 \text{ N} \]

13. **REASONING AND SOLUTION**

a. In terms of the period of the motion, the centripetal force is written as

\[ F_c = 4\pi^2 mr/T^2 = 4\pi^2 \frac{(0.0120 \text{ kg})(0.100 \text{ m})}{(0.500 \text{ s})^2} = 0.189 \text{ N} \]

b. The centripetal force varies as the square of the speed. Thus, doubling the speed would increase the centripetal force by a factor of \( 2^2 = 4 \).

14. **REASONING** At the maximum speed, the maximum centripetal force acts on the tires, and static friction supplies it. The magnitude of the maximum force of static friction is specified by Equation 4.7 as \( f_s^{\text{MAX}} = \mu_s F_N \), where \( \mu_s \) is the coefficient of static friction and \( F_N \) is the magnitude of the normal force. Our strategy, then, is to find the normal force, substitute it into the expression for the maximum frictional force, and then equate the result to the centripetal force, which is \( F_c = \frac{mv^2}{r} \), according to Equation 5.3. This will lead us to an expression for the maximum speed that we can apply to each car.

**SOLUTION** Since neither car accelerates in the vertical direction, we can conclude that the car’s weight \( mg \) is balanced by the normal force, so \( F_N = mg \). From Equations 4.7 and 5.3 it follows that

\[ f_s^{\text{MAX}} = \mu_s F_N = \mu_s mg = F_c = \frac{mv^2}{r} \]

Thus, we find that

\[ \mu_s mg = \frac{mv^2}{r} \quad \text{or} \quad v = \sqrt{\mu_s gr} \]
Applying this result to car A and car B gives

\[ v_A = \sqrt{\mu_{s, A} g r} \quad \text{and} \quad v_B = \sqrt{\mu_{s, B} g r} \]

In these two equations, the radius \( r \) does not have a subscript, since the radius is the same for either car. Dividing the two equations and noting that the terms \( g \) and \( r \) are eliminated algebraically, we see that

\[ \frac{v_B}{v_A} = \sqrt{\frac{\mu_{s, B}}{\mu_{s, A}}} \quad \text{or} \quad v_B = v_A \sqrt{\frac{\mu_{s, B}}{\mu_{s, A}}} = (25 \text{ m/s}) \sqrt{\frac{0.85}{1.1}} = 22 \text{ m/s} \]

15. **REASONING** The person feels the centripetal force acting on his back. This force is \( F_c = m v^2 / r \), according to Equation 5.3. This expression can be solved directly to determine the radius \( r \) of the chamber.

**SOLUTION** Solving Equation 5.3 for the radius \( r \) gives

\[ r = \frac{m v^2}{F_c} = \frac{(83 \text{ kg})(3.2 \text{ m/s})^2}{560 \text{ N}} = 1.5 \text{ m} \]

16. **REASONING AND SOLUTION** Initially, the stone executes uniform circular motion in a circle of radius \( r \) which is equal to the radius of the tire. At the instant that the stone flies out of the tire, the force of static friction just exceeds its maximum value \( f_s^{\text{MAX}} = \mu_s F_N \) (see Equation 4.7). The force of static friction that acts on the stone from one side of the tread channel is, therefore,

\[ f_s^{\text{MAX}} = 0.90(1.8 \text{ N}) = 1.6 \text{ N} \]

and the magnitude of the total frictional force that acts on the stone just before it flies out is \( 2 \times 1.6 \text{ N} = 3.2 \text{ N} \). If we assume that only static friction supplies the centripetal force, then, \( F_c = 3.2 \text{ N} \). Solving Equation 5.3 \( (F_c = m v^2 / r) \) for the radius \( r \), we have

\[ r = \frac{m v^2}{F_c} = \frac{(6.0 \times 10^{-3} \text{ kg})(13 \text{ m/s})^2}{3.2 \text{ N}} = 0.31 \text{ m} \]

17. **REASONING** Let \( v_0 \) be the initial speed of the ball as it begins its projectile motion. Then, the centripetal force is given by Equation 5.3: \( F_c = m v_0^2 / r \). We are given the values for \( m \) and \( r \); however, we must determine the value of \( v_0 \) from the details of the projectile motion after the ball is released.
In the absence of air resistance, the \( x \) component of the projectile motion has zero acceleration, while the \( y \) component of the motion is subject to the acceleration due to gravity. The horizontal distance traveled by the ball is given by Equation 3.5a (with \( a_x = 0 \)):

\[
x = v_{0x} t = (v_0 \cos \theta) t
\]

with \( t \) equal to the flight time of the ball while it exhibits projectile motion. The time \( t \) can be found by considering the vertical motion. From Equation 3.3b,

\[
v_y = v_{0y} + a_y t
\]

After a time \( t \), \( v_y = -v_{0y} \). Assuming that up and to the right are the positive directions, we have

\[
t = \frac{-2v_{0y}}{a_y} = \frac{-2v_0 \sin \theta}{a_y}
\]

and

\[
x = (v_0 \cos \theta) \left( \frac{-2v_0 \sin \theta}{a_y} \right)
\]

Using the fact that \( 2 \sin \theta \cos \theta = \sin 2\theta \), we have

\[
x = -\frac{2v_0^2 \cos \theta \sin \theta}{a_y} = -\frac{v_0^2 \sin 2\theta}{a_y}
\]

Equation (1) (with upward and to the right chosen as the positive directions) can be used to determine the speed \( v_0 \) with which the ball begins its projectile motion. Then Equation 5.3 can be used to find the centripetal force.

**SOLUTION** Solving equation (1) for \( v_0 \), we have

\[
v_0 = \sqrt{\frac{-x a_y}{\sin 2\theta}} = \sqrt{\frac{-(86.75 \text{ m})(-9.80 \text{ m/s}^2)}{\sin 2(41^\circ)}} = 29.3 \text{ m/s}
\]

Then, from Equation 5.3,

\[
F_c = \frac{mv_0^2}{r} = \frac{(7.3 \text{ kg})(29.3 \text{ m/s})^2}{1.8 \text{ m}} = 3500 \text{ N}
\]

18. **REASONING AND SOLUTION** The centripetal acceleration of the block is

\[
a_c = v^2/r = (28 \text{ m/s})^2/(150 \text{ m}) = 5.2 \text{ m/s}^2
\]
The angle $\theta$ can be obtained from

$$\theta = \tan^{-1}\left(\frac{a_c}{g}\right) = \tan^{-1}\left(\frac{5.2 \text{ m/s}^2}{9.80 \text{ m/s}^2}\right) = 28^\circ$$

19. **REASONING AND SOLUTION** If $F$ is the net force on mass 2, then $2F$ is the net force on mass 1, and we have

For mass 1: $2F = m_1v_1^2/r_1$ \hspace{1cm} For mass 2: $F = m_2v_2^2/r_2$

Dividing the equations and rearranging gives

$$\frac{m_2}{m_1} = \frac{1}{2}\left(\frac{r_2}{r_1}\right)\left(\frac{v_1}{v_2}\right)^2 \text{ since } r_2 = 2r_1$$

The period of revolution is the same for both masses so $v_1 = 2\pi r_1/T$ and $v_2 = 2\pi r_2/T$. Dividing these gives $v_1/v_2 = r_1/r_2 = 1/2$. Now

$$\frac{m_2}{m_1} = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

20. **REASONING** The relation $\tan \theta = \frac{v^2}{rg}$ (Equation 5.4) determines the banking angle $\theta$ that a banked curve of radius $r$ must have if a car is to travel around it at a speed $v$ without relying on friction. In this expression $g$ is the magnitude of the acceleration due to gravity. We will solve for $v$ and apply the result to each curve. The fact that the radius of each curve is the same will allow us to determine the unknown speed.

**SOLUTION** According to Equation 5.4, we have

$$\tan \theta = \frac{v^2}{rg} \hspace{1cm} \text{or} \hspace{1cm} v = \sqrt{rg \tan \theta}$$

Applying this result for the speed to each curve gives

$$v_A = \sqrt{rg \tan \theta_A} \hspace{1cm} \text{and} \hspace{1cm} v_B = \sqrt{rg \tan \theta_B}$$

Note that the terms $r$ and $g$ are the same for each curve. Therefore, these terms are eliminated algebraically when we divide the two equations. We find, then, that

$$\frac{v_B}{v_A} = \sqrt{r \tan \theta_B \tan \theta_A} = \sqrt{\tan \theta_B \tan \theta_A} \hspace{1cm} \text{or} \hspace{1cm} v_B = v_A \sqrt{\tan \theta_B \tan \theta_A} = (18 \text{ m/s}) \sqrt{\tan 19^\circ \tan 13^\circ} = 22 \text{ m/s}$$
21. **REASONING AND SOLUTION** 
Equation 5.4 gives the relationship between the speed \( v \) the angle of banking, and the radius of curvature. Solving for \( v \), we obtain 

\[
v = \sqrt{rg \tan \theta} = \sqrt{(120 \text{ m})(9.80 \text{ m/s}^2) \tan 18^\circ} = 2.0 \times 10^1 \text{ m/s}
\]

22. **REASONING** 
The angle \( \theta \) at which a friction-free curve is banked depends on the radius \( r \) of the curve and the speed \( v \) with which the curve is to be negotiated, according to Equation 5.4: \( \tan \theta = \frac{v^2}{rg} \). For known values of \( \theta \) and \( r \), the safe speed is 

\[
v = \sqrt{rg \tan \theta}
\]

Before we can use this result, we must determine \( \tan \theta \) for the banking of the track.

**SOLUTION** 
The drawing at the right shows a cross-section of the track. From the drawing we have

\[
\tan \theta = \frac{18 \text{ m}}{53 \text{ m}} = 0.34
\]

a. Therefore, the smallest speed at which cars can move on this track without relying on friction is

\[
v_{\text{min}} = \sqrt{(112 \text{ m})(9.80 \text{ m/s}^2)(0.34)} = 19 \text{ m/s}
\]

b. Similarly, the largest speed is

\[
v_{\text{max}} = \sqrt{(165 \text{ m})(9.80 \text{ m/s}^2)(0.34)} = 23 \text{ m/s}
\]

23. **REASONING** 
From the discussion on banked curves in Section 5.4, we know that a car can safely round a banked curve without the aid of static friction if the angle \( \theta \) of the banked curve is given by \( \tan \theta = \frac{v_0^2}{rg} \), where \( v_0 \) is the speed of the car and \( r \) is the radius of the curve (see Equation 5.4). The maximum speed that a car can have when rounding an unbanked curve is \( v_0 = \sqrt{\mu gr} \) (see Example 7). By combining these two relations, we can find the angle \( \theta \).

**SOLUTION** 
The angle of the banked curve is \( \theta = \tan^{-1} \left( \frac{v_0^2}{rg} \right) \). Substituting the expression \( v_0 = \sqrt{\mu gr} \) into this equation gives

\[
\theta = \tan^{-1} \left( \frac{v_0^2}{rg} \right) = \tan^{-1} \left( \frac{\mu gr}{rg} \right) = \tan^{-1} (\mu_\mu) = \tan^{-1} (0.81) = 39^\circ
\]
24. **REASONING** The distance \( d \) is related to the radius \( r \) of the circle on which the car travels by \( d = r/\sin 50.0^\circ \) (see the drawing).

We can obtain the radius by noting that the car experiences a centripetal force that is directed toward the center of the circular path. This force is provided by the component, \( F_N \cos 50.0^\circ \), of the normal force that is parallel to the radius. Setting this force equal to the mass \( m \) of the car times the centripetal acceleration \( (a_c = v^2/r) \) gives
\[
F_N \cos 50.0^\circ = ma_c = mv^2/r.
\]
Solving for the radius \( r \) and substituting it into the relation \( d = r/\sin 50.0^\circ \) gives
\[
d = \frac{r}{\sin 50.0^\circ} = \frac{F_N \cos 50.0^\circ}{\sin 50.0^\circ} = \frac{mv^2}{(F_N \cos 50.0^\circ)(\sin 50.0^\circ)}
\]
(1)

The magnitude \( F_N \) of the normal force can be obtained by observing that the car has no vertical acceleration, so the net force in the vertical direction must be zero, \( \sum F_y = 0 \). The net force consists of the upward vertical component of the normal force and the downward weight of the car. The vertical component of the normal force is \( +F_N \sin 50.0^\circ \), and the weight is \( -mg \), where we have chosen the “up” direction as the + direction. Thus, we have that
\[
\sum F_y = +F_N \sin 50.0^\circ - mg = 0
\]
(2)

Solving this equation for \( F_N \) and substituting it into the equation above will yield the distance \( d \).

**SOLUTION** Solving Equation (2) for \( F_N \) and substituting the result into Equation (1) gives
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\[ d = \frac{mv^2}{(F_N \cos 50.0^\circ)(\sin 50.0^\circ)} = \frac{mv^2}{\left(\frac{mg}{\sin 50.0^\circ}\right)(\cos 50.0^\circ)(\sin 50.0^\circ)} \]

\[ = \frac{v^2}{g \cos 50.0^\circ} = \frac{(34.0 \text{ m/s})^2}{(9.80 \text{ m/s}^2) \cos 50.0^\circ} = 184 \text{ m} \]

25. **REASONING** Refer to Figure 5.10 in the text. The horizontal component of the lift \( L \) is the centripetal force that holds the plane in the circle. Thus,

\[ L \sin \theta = \frac{mv^2}{r} \quad (1) \]

The vertical component of the lift supports the weight of the plane; therefore,

\[ L \cos \theta = mg \quad (2) \]

Dividing the first equation by the second gives

\[ \tan \theta = \frac{v^2}{rg} \]

(3)

Equation (3) can be used to determine the angle \( \theta \) of banking. Once \( \theta \) is known, then the magnitude of \( L \) can be found from either equation (1) or equation (2).

**SOLUTION** Solving equation (3) for \( \theta \) gives

\[ \theta = \tan^{-1} \left[ \frac{(123 \text{ m/s})^2}{(3810 \text{ m})(9.80 \text{ m/s}^2)} \right] = 22.1^\circ \]

The lifting force is, from equation (2),

\[ L = \frac{mg}{\cos \theta} = \frac{(2.00 \times 10^5 \text{ kg})(9.80 \text{ m/s}^2)}{\cos 22.1^\circ} = 2.12 \times 10^6 \text{ N} \]

26. **REASONING** The centripetal force \( F_c \) required to keep an object of mass \( m \) that moves with speed \( v \) on a circle of radius \( r \) is \( F_c = \frac{mv^2}{r} \) (Equation 5.3). From Equation 5.1, we know that \( v = 2\pi r / T \), where \( T \) is the period or the time for the suitcase to go around once. Therefore, the centripetal force can be written as
This expression can be solved for $T$. However, we must first find the centripetal force that acts on the suitcase.

**SOLUTION**  Three forces act on the suitcase. They are the weight $mg$ of the suitcase, the force of static friction $f_{s}^{MAX}$, and the normal force $F_N$ exerted on the suitcase by the surface of the carousel. The following figure shows the free body diagram for the suitcase.

In this diagram, the $y$ axis is along the vertical direction. The force of gravity acts, then, in the $-y$ direction. The centripetal force that causes the suitcase to move on its circular path is provided by the net force in the $+x$ direction in the diagram. From the diagram, we can see that only the forces $F_N$ and $f_{s}^{MAX}$ have horizontal components. Thus, we have

$$F_c = f_{s}^{MAX} \cos \theta - F_N \sin \theta,$$

where the minus sign indicates that the $x$ component of $F_N$ points to the left in the diagram. Using Equation 4.7 for the maximum static frictional force, we can write this result as in equation (2).

$$F_c = \mu_s F_N \cos \theta - F_N \sin \theta = F_N (\mu_s \cos \theta - \sin \theta) \quad (2)$$

If we apply Newton's second law in the $y$ direction, we see from the diagram that

$$F_N \cos \theta + f_{s}^{MAX} \sin \theta - mg = ma_y = 0 \quad \text{or} \quad F_N \cos \theta + \mu_s F_N \sin \theta - mg = 0$$

where we again have used Equation 4.7 for the maximum static frictional force. Solving for the normal force, we find

$$F_N = \frac{mg}{\cos \theta + \mu_s \sin \theta}$$

Using this result in equation (2), we obtain the magnitude of the centripetal force that acts on the suitcase:

$$F_c = F_N (\mu_s \cos \theta - \sin \theta) = \frac{mg(\mu_s \cos \theta - \sin \theta)}{\cos \theta + \mu_s \sin \theta}$$

With this expression for the centripetal force, equation (1) becomes

$$\frac{mg(\mu_s \cos \theta - \sin \theta)}{\cos \theta + \mu_s \sin \theta} = \frac{4m\pi^2 r}{T^2}$$
Solving for the period \( T \), we find

\[
T = \sqrt{\frac{4\pi^2 r (\cos \theta + \mu_z \sin \theta)}{g (\mu_z \cos \theta - \sin \theta)}} = \sqrt{\frac{4\pi^2 (11.0 \text{ m}) (\cos 36.0^\circ + 0.760 \sin 36.0^\circ)}{(9.80 \text{ m/s}^2) (0.760 \cos 36.0^\circ - \sin 36.0^\circ)}} = 45 \text{ s}
\]

27. **SSM WWW REASONING** Equation 5.5 gives the orbital speed for a satellite in a circular orbit around the earth. It can be modified to determine the orbital speed around any planet \( P \) by replacing the mass of the earth \( M_E \) by the mass of the planet \( M_P \):

\[
v = \sqrt{\frac{GM_P}{r}}.
\]

**SOLUTION** The ratio of the orbital speeds is, therefore,

\[
\frac{v_2}{v_1} = \sqrt{\frac{GM_p/r_2}{GM_p/r_1}} = \sqrt{\frac{r_1}{r_2}}
\]

Solving for \( v_2 \) gives

\[
v_2 = v_1 \sqrt{\frac{r_1}{r_2}} = (1.70 \times 10^4 \text{ m/s}) \sqrt{\frac{5.25 \times 10^6 \text{ m}}{8.60 \times 10^6 \text{ m}}} = 1.33 \times 10^4 \text{ m/s}
\]

28. **REASONING** Two pieces of information are provided. One is the fact that the magnitude of the centripetal acceleration \( a_c \) is 9.80 m/s\(^2\). The other is that the space station should not rotate faster than two revolutions per minute. This rate of twice per minute corresponds to thirty seconds per revolution, which is the minimum value for the period \( T \) of the motion. With these data in mind, we will base our solution on Equation 5.2, which gives the centripetal acceleration as \( a_c = \frac{v^2}{r} \), and on Equation 5.1, which specifies that the speed \( v \) on a circular path of radius \( r \) is \( v = 2\pi r/T \).

**SOLUTION** From Equation 5.2, we have

\[
a_c = \frac{v^2}{r} \quad \text{or} \quad r = \frac{v^2}{a_c}
\]

Substituting \( v = 2\pi r / T \) into this result and solving for the radius gives

\[
r = \frac{v^2}{a_c} = \left(\frac{2\pi r / T}{a_c}\right)^2 \quad \text{or} \quad r = \frac{a_c T^2}{4\pi^2} = \frac{(9.80 \text{ m/s}^2) (30.0 \text{ s})^2}{4\pi^2} = 223 \text{ m}
\]

29. **REASONING AND SOLUTION** We have for Jupiter \( v^2 = GM_J/r \), where
\[ r = 6.00 \times 10^5 \text{ m} + 7.14 \times 10^7 \text{ m} = 7.20 \times 10^7 \text{ m} \]

Thus,

\[ v = \sqrt{\frac{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(1.90 \times 10^{27} \text{ kg})}{7.20 \times 10^7 \text{ m}}} = 4.20 \times 10^4 \text{ m/s} \]

30. **REASONING** The speed of the satellite is given by Equation 5.1 as \[ v = \frac{2\pi r}{T} \]. Since we are given that the period is \( T = 1.20 \times 10^4 \text{ s} \), it will be possible to determine the speed from Equation 5.1 if we can determine the radius \( r \) of the orbit. To find the radius, we will use Equation 5.6, which relates the period to the radius according to \( T = \frac{2\pi r^{3/2}}{\sqrt{GM_E}} \), where \( G \) is the universal gravitational constant and \( M_E \) is the mass of the earth.

**SOLUTION** According to Equation 5.1, the orbital speed is

\[ v = \frac{2\pi r}{T} \]

To find a value for the radius, we begin with Equation 5.6:

\[ T = \frac{2\pi r^{3/2}}{\sqrt{GM_E}} \quad \text{or} \quad r^{3/2} = \frac{T\sqrt{GM_E}}{2\pi} \]

Next, we square both sides of the result for \( r^{3/2} \):

\[ \left( r^{3/2} \right)^2 = \left( \frac{T\sqrt{GM_E}}{2\pi} \right)^2 \quad \text{or} \quad r^3 = \frac{T^2GM_E}{4\pi^2} \]

We can now take the cube root of both sides of the expression for \( r^3 \) in order to determine \( r \):

\[ r = \frac{3}{4\pi^2} \left( \frac{T^2GM_E}{4\pi^2} \right)^{1/3} = \left( \frac{(1.20 \times 10^4 \text{ s})^2(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(5.98 \times 10^{24} \text{ kg})}{4\pi^2} \right)^{1/3} = 1.13 \times 10^7 \text{ m} \]

With this value for the radius, we can use Equation 5.1 to obtain the speed:

\[ v = \frac{2\pi r}{T} = \frac{2\pi(1.13 \times 10^7 \text{ m})}{1.20 \times 10^4 \text{ s}} = 5.92 \times 10^3 \text{ m/s} \]

31. **REASONING** In Section 5.5 it is shown that the period \( T \) of a satellite in a circular orbit about the earth is given by (see Equation 5.6)
where $r$ is the radius of the orbit, $G$ is the universal gravitational constant, and $M_E$ is the mass of the earth. The ratio of the periods of satellites A and B is, then,

\[
\frac{T_A}{T_B} = \frac{\sqrt{GM_E}}{2\pi r_A^{3/2}} = \frac{\sqrt{GM_E}}{2\pi r_B^{3/2}}
\]

We do not know the radii $r_A$ and $r_B$. However we do know that the speed $v$ of a satellite is equal to the circumference $(2\pi r)$ of its orbit divided by the period $T$, so $v = 2\pi r/T$.

**SOLUTION** Solving the relation $v = 2\pi r/T$ for $r$ gives $r = vT/(2\pi)$. Substituting this value for $r$ into Equation (1) yields

\[
\frac{T_A}{T_B} = \frac{\sqrt{GM_E}}{2\pi \left(\frac{vA T_A}{2\pi}\right)^{3/2}} = \frac{\left(\frac{vA T_A}{2\pi}\right)^{3/2}}{\left(\frac{vB T_B}{2\pi}\right)^{3/2}}
\]

Squaring both sides of this equation, algebraically solving for the ratio $T_A/T_B$, and using the fact that $v_A = 3v_B$ gives

\[
\frac{T_A}{T_B} = \frac{v_B^3}{v_A^3} = \frac{v_B^3}{(3v_B)^3} = \frac{1}{27}
\]

32. **REASONING AND SOLUTION** The period of rotation is given by $T^2 = 4\pi^2 r^3/GM$. Comparing the orbital periods for Earth and Venus yields

\[
(T_V/T_E)^2 = (r_V/r_E)^3 \quad \text{so that} \quad T_V/T_E = 0.611
\]

The earth's orbital period is 365 days so

\[
T_V = (0.611)(365 \text{ days}) = 223 \text{ days}
\]

33. **SSM REASONING** The true weight of the satellite when it is at rest on the planet's surface can be found from Equation 4.4: $W = (GM_p m)/r^2$ where $M_p$ and $m$ are the masses
of the planet and the satellite, respectively, and \( r \) is the radius of the planet. However, before we can use Equation 4.4, we must determine the mass \( M_p \) of the planet.

The mass of the planet can be found by replacing \( M_E \) by \( M_p \) in Equation 5.6 and solving for \( M_p \). When using Equation 5.6, we note that \( r \) corresponds to the radius of the circular orbit relative to the center of the planet.

**SOLUTION** The period of the satellite is \( T = 2.00 \text{ h} = 7.20 \times 10^3 \text{ s} \). From Equation 5.6,

\[
M_p = \frac{4\pi^2 r^3}{GT^2} = \frac{4\pi^2 [(4.15 \times 10^6 \text{ m}) + (4.1 \times 10^5 \text{ m})]^3}{(6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2)(7.20 \times 10^3 \text{ s})^2} = 1.08 \times 10^{24} \text{ kg}
\]

Using Equation 4.4, we have

\[
W = \frac{GM_p m}{r^2} = \frac{(6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2)(1.08 \times 10^{24} \text{ kg})(5850 \text{ kg})}{(4.15 \times 10^6 \text{ m})^2} = 2.45 \times 10^4 \text{ N}
\]

34. **REASONING** Equation 5.2 for the centripetal acceleration applies to both the plane and the satellite, and the centripetal acceleration is the same for each. Thus, we have

\[
a_c = \frac{v^2_{\text{plane}}}{r_{\text{plane}}} = \frac{v^2_{\text{satellite}}}{r_{\text{satellite}}} \quad \text{or} \quad v_{\text{plane}} = \sqrt{\frac{r_{\text{plane}}}{r_{\text{satellite}}} \cdot \frac{Gm_E}{r_{\text{satellite}}}}
\]

The speed of the satellite can be obtained directly from Equation 5.5.

**SOLUTION** Using Equation 5.5, we can express the speed of the satellite as

\[
v_{\text{satellite}} = \sqrt{\frac{Gm_E}{r_{\text{satellite}}}}
\]

Substituting this expression into the expression obtained in the reasoning for the speed of the plane gives

\[
v_{\text{plane}} = \sqrt{\frac{r_{\text{plane}}}{r_{\text{satellite}}} \cdot \frac{Gm_E}{r_{\text{satellite}}}} = \sqrt{\frac{r_{\text{plane}}}{r_{\text{satellite}}} \cdot \frac{Gm_E}{r_{\text{satellite}}}} = \sqrt{\frac{6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2 \cdot 5.98 \times 10^{24} \text{ kg}}{6.7 \times 10^6 \text{ m}}} = 12 \text{ m/s}
\]
35. **REASONING AND SOLUTION**
   a. The centripetal acceleration of a point on the rim of chamber A is the artificial acceleration due to gravity,
   \[ a_A = \frac{v_A^2}{r_A} = 10.0 \text{ m/s}^2 \]
   A point on the rim of chamber A moves with a speed \( v_A = \frac{2\pi r_A}{T} \) where \( T \) is the period of revolution, 60.0 s. Substituting the second equation into the first and rearranging yields
   \[ r_A = \frac{a_A T^2}{4\pi^2} = 912 \text{ m} \]
   b. Now
   \[ r_B = \frac{r_A}{4} = 228 \text{ m} \]
   c. A point on the rim of chamber B has a centripetal acceleration \( a_B = \frac{v_B^2}{r_B} \). The point moves with a speed \( v_B = \frac{2\pi r_B}{T} \). Substituting the second equation into the first yields
   \[ a_B = \frac{4\pi^2 r_B}{T^2} = \frac{4\pi^2 (228 \text{ m})}{(60.0 \text{ s})^2} = 2.50 \text{ m/s}^2 \]

36. **REASONING** According to Equation 5.3, the magnitude \( F_c \) of the centripetal force that acts on each passenger is \( F_c = mv^2 / r \), where \( m \) and \( v \) are the mass and speed of a passenger and \( r \) is the radius of the turn. From this relation we see that the speed is given by \( v = \sqrt{\frac{F_c r}{m}} \). The centripetal force is the net force required to keep each passenger moving on the circular path and points toward the center of the circle. With the aid of a free-body diagram, we will evaluate the net force and, hence, determine the speed.

**SOLUTION** The free-body diagram shows a passenger at the bottom of the circular dip. There are two forces acting: her downward-acting weight \( mg \) and the upward-acting force \( 2mg \) that the seat exerts on her. The net force is \( +2mg - mg = +mg \), where we have taken “up” as the positive direction. Thus, \( F_c = mg \). The speed of the passenger can be found by using this result in the equation above.

Substituting \( F_c = mg \) into the relation \( v = \sqrt{\frac{F_c r}{m}} \) yields
\[
v = \sqrt{\frac{F_c r}{m}} = \sqrt{\frac{(mg)r}{m}} = \sqrt{gr} = \sqrt{(9.80 \text{ m/s}^2)(20.0 \text{ m})} = 14.0 \text{ m/s}
\]
37. **REASONING AND SOLUTION** Since the tension serves the same purpose as the normal force at point 1 in Figure 5.21, we have, using the equation for the situation at point 1 with $F_{N1}$ replaced by $T$,

$$\frac{mv^2}{r} = T - mg$$

Solving for $T$ gives

$$T = \frac{mv^2}{r} + mg = m \left( \frac{v^2}{r} + g \right) = (2100 \text{ kg}) \left[ \frac{(7.6 \text{ m/s})^2}{15 \text{ m}} + (9.80 \text{ m/s}^2) \right] = 2.9 \times 10^4 \text{ N}$$

38. **REASONING** The normal force (magnitude $F_N$) that the pilot’s seat exerts on him is part of the centripetal force that keeps him on the vertical circular path. However, there is another contribution to the centripetal force, as the drawing at the right shows. This additional contribution is the pilot’s weight (magnitude $W$). To obtain the ratio $F_N/W$, we will apply Equation 5.3, which specifies the centripetal force as $F_c = \frac{mv^2}{r}$.

**SOLUTION** Noting that the direction upward (toward the center of the circular path) is positive in the drawing, we see that the centripetal force is $F_c = F_N - W$. Thus, from Equation 5.3 we have

$$F_c = F_N - W = \frac{mv^2}{r}$$

The weight is given by $W = mg$ (Equation 4.5), so we can divide the expression for the centripetal force by the expression for the weight and obtain that

$$F_c = \frac{F_N - W}{W} = \frac{mv^2}{mg r} \quad \text{or} \quad \frac{F_N}{W} - 1 = \frac{v^2}{gr}$$

Solving for the ratio $F_N/W$, we find that

$$\frac{F_N}{W} = 1 + \frac{v^2}{gr} = 1 + \frac{(230 \text{ m/s})^2}{(9.80 \text{ m/s}^2)(690 \text{ m})} = 8.8$$

39. **REASONING** The centripetal force is the name given to the net force pointing toward the center of the circular path. At point 3 at the top the net force pointing toward the center of the circle consists of the normal force and the weight, both pointing toward the
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center. At point 1 at the bottom the net force consists of the normal force pointing upward toward the center and the weight pointing downward or away from the center. In either case the centripetal force is given by Equation 5.3 as $F_c = \frac{mv^2}{r}$.

**SOLUTION** At point 3 we have

$$F_c = F_N + mg = \frac{mv_3^2}{r}$$

At point 1 we have

$$F_c = F_N - mg = \frac{mv_1^2}{r}$$

Subtracting the second equation from the first gives

$$2mg = \frac{mv_3^2}{r} - \frac{mv_1^2}{r}$$

Rearranging gives

$$v_3^2 = 2gr + v_1^2$$

Thus, we find that

$$v_3 = \sqrt{2(9.80 \text{ m/s}^2)(3.0 \text{ m}) + (15 \text{ m/s})^2} = 17 \text{ m/s}$$

40. **REASONING** As the motorcycle passes over the top of the hill, it experiences a centripetal force, the magnitude of which is given by Equation 5.3 as $F_c = \frac{mv^2}{r}$, where $m$ and $v$ are the mass and speed of the motorcycle, and $r$ is the radius of the circular crest in the road. The speed of the motorcycle is then $v = \sqrt{\frac{F_c r}{m}}$. The centripetal force is the net force acting on the motorcycle and is directed toward the center of the circle. When the motorcycle crests the hill, there are two forces that act along the radial direction, the normal force $F_N$ (upward) that the road exerts on the motorcycle and the weight $mg$ (downward) of the motorcycle and rider. Taking the direction toward the center of the circle (downward) as the positive direction, we have that $F_c = +mg - F_N$. When the motorcycle just loses contact with the road, the normal force becomes zero. With this information, we can find the maximum speed that the cycle can have.

**SOLUTION** Substituting $F_c = +mg - F_N$ into the relation $v = \sqrt{\frac{F_c r}{m}}$ gives

$$v = \sqrt{\frac{(mg - F_N)r}{m}}$$
The maximum speed \( v_{\text{max}} \) occurs when the motorcycle just loses contact with the road. At this instant the normal force becomes zero. Setting \( F_N = 0 \) N, we have

\[
v_{\text{max}} = \sqrt{gr} = \sqrt{(9.80 \text{ m/s}^2)(45.0 \text{ m})} = 21.0 \text{ m/s}
\]

41. **REASONING** When the stone is whirled in a horizontal circle, the centripetal force is provided by the tension \( T_h \) in the string and is given by Equation 5.3 as

\[
\frac{T_h}{r} = \frac{mv^2}{r} \tag{1}
\]

where \( m \) and \( v \) are the mass and speed of the stone, and \( r \) is the radius of the circle. When the stone is whirled in a vertical circle, the maximum tension occurs when the stone is at the lowest point in its path. The free-body diagram shows the forces that act on the stone in this situation: the tension \( T_v \) in the string and the weight \( mg \) of the stone. The centripetal force is the net force that points toward the center of the circle. Setting the centripetal force equal to \( mv^2/r \), as per Equation 5.3, we have

\[
\frac{T_v - mg}{r} = \frac{mv^2}{r} \tag{2}
\]

Here, we have assumed upward to be the positive direction. We are given that the maximum tension in the string in the case of vertical motion is 15.0\% larger than that in the case of horizontal motion. We can use this fact, along with Equations 1 and 2, to find the speed of the stone.

**Solution** Since the maximum tension in the string in the case of vertical motion is 15.0\% larger than that in the horizontal motion, \( T_v = (1.000 + 0.150)T_h \). Substituting the values of \( T_h \) and \( T_v \) from Equations (1) and (2) into this relation gives

\[
T_v = (1.000 + 0.150)T_h
\]

\[
\frac{mv^2}{r} + mg = (1.000 + 0.150)\left(\frac{mv^2}{r}\right)
\]

Solving this equation for the speed \( v \) of the stone yields
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\[ v = \sqrt{\frac{gr}{0.150}} = \sqrt{\frac{(9.80 \text{ m/s}^2)(1.10 \text{ m})}{0.150}} = 8.48 \text{ m/s} \]

42. **REASONING** The drawing at the right shows the two forces that act on a piece of clothing *just before* it loses contact with the wall of the cylinder. At that instant the centripetal force is provided by the normal force \( F_N \) and the radial component of the weight. From the drawing, the radial component of the weight is given by

\[ mg \cos \phi = mg \cos (90^\circ - \theta) = mg \sin \theta \]

Therefore, with inward taken as the positive direction, Equation 5.3 \( (F_c = \frac{mv^2}{r}) \) gives

\[ F_N + mg \sin \theta = \frac{mv^2}{r} \]

At the instant that a piece of clothing loses contact with the surface of the drum, \( F_N = 0 \), and the above expression becomes

\[ mg \sin \theta = \frac{mv^2}{r} \]

According to Equation 5.1, \( v = \frac{2\pi r}{T} \), and with this substitution we obtain

\[ g \sin \theta = \left( \frac{2\pi r}{T} \right)^2 = \frac{4\pi^2 r}{T^2} \]

This expression can be solved for the period \( T \). Since the period is the required time for one revolution, the number of revolutions per second can be found by calculating \( \frac{1}{T} \).

**SOLUTION** Solving for the period, we obtain

\[ T = \sqrt{\frac{4\pi^2 r}{g \sin \theta}} = 2\pi \sqrt{\frac{r}{g \sin \theta}} = 2\pi \sqrt{\frac{0.32 \text{ m}}{(9.80 \text{ m/s}^2) \sin 70.0^\circ}} = 1.17 \text{ s} \]

Therefore, the number of revolutions per second that the cylinder should make is

\[ \frac{1}{T} = \frac{1}{1.17 \text{ s}} = 0.85 \text{ rev/s} \]
43. **REASONING AND SOLUTION** The magnitude of the centripetal force on the ball is given by Equation 5.3: \( F_c = \frac{m v^2}{r} \). Solving for \( v \), we have

\[
v = \sqrt{\frac{F_c r}{m}} = \sqrt{\frac{(0.028 \text{ N})(0.25 \text{ m})}{0.015 \text{ kg}}} = 0.68 \text{ m/s}
\]

44. **REASONING AND SOLUTION** The normal force exerted by the wall on each astronaut is the centripetal force needed to keep him in the circular path, i.e., \( F_c = \frac{m v^2}{r} \). Rearranging and letting \( F_c = \frac{1}{2} mg \) yields

\[
r = 2 \frac{v^2}{g} = 2(35.8 \text{ m/s})^2/(9.80 \text{ m/s}^2) = 262 \text{ m}
\]

45. **REASONING AND SOLUTION** Let \( s \) represent the length of the path of the pebble after it is released. From Conceptual Example 2, we know that the pebble will fly off tangentially. Therefore, the path \( s \) is perpendicular to the radius \( r \) of the circle. Thus, the distances \( r \), \( s \), and \( d \) form a right triangle with hypotenuse \( d \) as shown in the figure at the right. From the figure we see that

\[
\cos \alpha = \frac{r}{d} = \frac{r}{10r} = \frac{1}{10}
\]

or

\[
\alpha = \cos^{-1} \left( \frac{1}{10} \right) = 84^\circ
\]

Furthermore, from the figure, we see that \( \alpha + \theta + 35^\circ = 180^\circ \). Therefore,

\[
\theta = 145^\circ - \alpha = 145^\circ - 84^\circ = 61^\circ
\]
46. **REASONING AND SOLUTION** The force \( P \) supplied by the man will be largest when the partner is at the lowest point in the swing. The diagram at the right shows the forces acting on the partner in this situation. The centripetal force necessary to keep the partner swinging along the arc of a circle is provided by the resultant of the force supplied by the man and the weight of the partner. From the figure

\[
P - mg = \frac{mv^2}{r}
\]

Therefore,

\[
P = \frac{mv^2}{r} + mg
\]

Since the weight of the partner, \( W \), is equal to \( mg \), it follows that \( m = \frac{W}{g} \) and

\[
P = \left( \frac{W}{g} \right) \frac{v^2}{r} + W = \left[ \frac{475 \text{ N}}{(9.80 \text{ m/s}^2)} \right] \left( 4.00 \text{ m/s} \right)^2 + 475 \text{ N} = 594 \text{ N}
\]

47. **SSM REASONING AND SOLUTION** Since the magnitude of the centripetal acceleration is given by Equation 5.2, \( a_c = \frac{v^2}{r} \), we can solve for \( r \) and find that

\[
r = \frac{v^2}{a_c} = \frac{(98.8 \text{ m/s})^2}{3.00(9.80 \text{ m/s}^2)} = 332 \text{ m}
\]

48. **REASONING** The centripetal force is the name given to the net force pointing toward the center of the circular path. At the lowest point the net force consists of the tension in the arm pointing upward toward the center and the weight pointing downward or away from the center. In either case the centripetal force is given by Equation 5.3 as \( F_c = \frac{mv^2}{r} \).

**SOLUTION** (a) The centripetal force is

\[
F_c = \frac{mv^2}{r} = \left( \frac{9.5 \text{ kg}}{0.85 \text{ m}} \right) \left( 2.8 \text{ m/s} \right)^2 = 88 \text{ N}
\]

(b) Using \( T \) to denote the tension in the arm, at the bottom of the circle we have

\[
F_c = T - mg = \frac{mv^2}{r}
\]

\[
T = mg + \frac{mv^2}{r} = \left( 9.5 \text{ kg} \right) \left( 9.80 \text{ m/s}^2 \right) + \frac{(9.5 \text{ kg})(2.8 \text{ m/s})^2}{0.85 \text{ m}} = 181 \text{ N}
\]
49. **SSM REASONING** As the motorcycle passes over the top of the hill, it will experience a centripetal force, the magnitude of which is given by Equation 5.3: \( F_C = \frac{mv^2}{r} \). The centripetal force is provided by the net force on the cycle + driver system. At that instant, the net force on the system is composed of the normal force, which points upward, and the weight, which points downward. Taking the direction toward the center of the circle (downward) as the positive direction, we have \( F_C = mg - F_N \). This expression can be solved for \( F_N \), the normal force.

**SOLUTION**

a. The magnitude of the centripetal force is

\[
F_C = \frac{mv^2}{r} = \frac{(342 \text{ kg})(25.0 \text{ m/s})^2}{126 \text{ m}} = 1.70 \times 10^3 \text{ N}
\]

b. The magnitude of the normal force is

\[
F_N = mg - F_C = (342 \text{ kg})(9.80 \text{ m/s}^2) - 1.70 \times 10^3 \text{ N} = 1.66 \times 10^3 \text{ N}
\]

50. **REASONING AND SOLUTION** The period of the moon's motion (approximately the length of a month) is given by

\[
T = \sqrt{\frac{4\pi^2r^3}{GM_e}} = \sqrt{\frac{4\pi^2 \left(3.85 \times 10^8 \text{ m}\right)^3}{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(5.98 \times 10^{24} \text{ kg})}}
\]

\[
= 2.38 \times 10^6 \text{ s} = 27.5 \text{ days}
\]

51. **REASONING AND SOLUTION** The sample makes one revolution in time \( T \) as given by \( T = \frac{2\pi r}{v} \). The speed is

\[
v^2 = ra_c = (5.00 \times 10^{-2} \text{ m})(6.25 \times 10^3)(9.80 \text{ m/s}^2) \quad \text{so that} \quad v = 55.3 \text{ m/s}
\]

The period is

\[
T = \frac{2\pi (5.00 \times 10^{-2} \text{ m})}{(55.3 \text{ m/s})} = 5.68 \times 10^{-3} \text{ s} = 9.47 \times 10^{-5} \text{ min}
\]

The number of revolutions per minute = \( \frac{1}{T} = 10600 \text{ rev/min} \).

52. **REASONING AND SOLUTION**

a. At the equator a person travels in a circle whose radius equals the radius of the earth, \( r = R_e = 6.38 \times 10^6 \text{ m} \), and whose period of rotation is \( T = 1 \text{ day} = 86400 \text{ s} \). We have
The centripetal acceleration is

$$a_c = \frac{v^2}{r} = \frac{(464 \text{ m/s})^2}{6.38 \times 10^6 \text{ m}} = 3.37 \times 10^{-2} \text{ m/s}^2$$

b. At 30.0° latitude a person travels in a circle of radius,

$$r = R_e \cos 30.0° = 5.53 \times 10^6 \text{ m}$$

Thus,

$$v = 2\pi r/T = 402 \text{ m/s} \quad \text{and} \quad a_c = v^2/r = 2.92 \times 10^{-2} \text{ m/s}^2$$

53. REASONING

a. The free body diagram shows the swing ride and the two forces that act on a chair: the tension $T$ in the cable, and the weight $mg$ of the chair and its occupant. We note that the chair does not accelerate vertically, so the net force $\sum F_y$ in the vertical direction must be zero, $\sum F_y = 0$. The net force consists of the upward vertical component of the tension and the downward weight of the chair. The fact that the net force is zero will allow us to determine the magnitude of the tension.

b. According to Newton’s second law, the net force $\sum F_x$ in the horizontal direction is equal to the mass $m$ of the chair and its occupant times the centripetal acceleration $(a_c = v^2/r)$, so that $\sum F_x = ma_c = mv^2/r$. There is only one force in the horizontal direction, the horizontal component of the tension, so it is the net force. We will use Newton’s second law to find the speed $v$ of the chair.

SOLUTION

a. The vertical component of the tension is $+T \cos 60.0°$, and the weight is $-mg$, where we have chosen “up” as the $+$ direction. Since the chair and its occupant have no vertical acceleration, we have that $\sum F_y = 0$, so
\[ +T \cos 60.0^\circ - mg = 0 \]

Solving for the magnitude \( T \) of the tension gives

\[ T = \frac{mg}{\cos 60.0^\circ} = \frac{(179 \text{ kg})(9.80 \text{ m/s}^2)}{\cos 60.0^\circ} = 3510 \text{ N} \]

b. The horizontal component of the tension is \( +T \sin 60.0^\circ \), where we have chosen the direction to the left in the diagram as the + direction. Since the chair and its occupant have a centripetal acceleration in this direction, we have

\[ \sum F_x = T \sin 60.0^\circ = ma_c = m \left( \frac{v^2}{r} \right) \]

From the drawing we see that the radius \( r \) of the circular path is \( r = (15.0 \text{ m}) \sin 60.0^\circ = 13.0 \text{ m} \). Solving Equation (2) for the speed \( v \) gives

\[ v = \sqrt{\frac{rT \sin 60.0^\circ}{m}} = \sqrt{\frac{(13.0 \text{ m})(3510 \text{ N}) \sin 60.0^\circ}{179 \text{ kg}}} = 14.9 \text{ m/s} \]

54. **REASONING AND SOLUTION**

a. The centripetal force is provided by the normal force exerted on the rider by the wall.

b. Newton's second law applied in the horizontal direction gives

\[ F_N = \frac{mv^2}{r} = \frac{(55.0 \text{ kg})(10.0 \text{ m/s})^2}{(3.30 \text{ m})} = 1670 \text{ N} \]

c. Newton's second law applied in the vertical direction gives \( \mu_s F_N - mg = 0 \) or

\[ \mu_s = \frac{(mg)/F_N = 0.323}{0.323} \]

55. **SSM [WWW] REASONING** If the effects of gravity are not ignored in Example 5, the plane will make an angle \( \theta \) with the vertical as shown in figure A below. The figure B shows the forces that act on the plane, and figure C shows the horizontal and vertical components of these forces.
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From figure C we see that the resultant force in the horizontal direction is the horizontal component of the tension in the guideline and provides the centripetal force. Therefore,

\[ T \sin \theta = \frac{mv^2}{r} \]

From figure A, the radius \( r \) is related to the length \( L \) of the guideline by \( r = L \sin \theta \); therefore,

\[ T \sin \theta = \frac{mv^2}{L \sin \theta} \]  \hspace{1cm} \text{(1)}

The resultant force in the vertical direction is zero: \( T \cos \theta - mg = 0 \), so that

\[ T \cos \theta = mg \]

From equation (2) we have

\[ T = \frac{mg}{\cos \theta} \]  \hspace{1cm} \text{(3)}

Equation (3) contains two unknown, \( T \) and \( \theta \). First we will solve equations (1) and (3) simultaneously to determine the value(s) of the angle \( \theta \). Once \( \theta \) is known, we can calculate the tension using equation (3).

**SOLUTION** Substituting equation (3) into equation (1):

\[ \left( \frac{mg}{\cos \theta} \right) \sin \theta = \frac{mv^2}{L \sin \theta} \]

Thus,

\[ \frac{\sin^2 \theta}{\cos \theta} = \frac{v^2}{gL} \]  \hspace{1cm} \text{(4)}

Using the fact that \( \cos^2 \theta + \sin^2 \theta = 1 \), equation (4) can be written
\[
\frac{1 - \cos^2 \theta}{\cos \theta} = \frac{v^2}{gL}
\]
or
\[
\frac{1}{\cos \theta} - \cos \theta = \frac{v^2}{gL}
\]

This can be put in the form of an equation that is quadratic in \( \cos \theta \). Multiplying both sides by \( \cos \theta \) and rearranging yields:

\[
\cos^2 \theta + \frac{v^2}{gL} \cos \theta - 1 = 0
\]  

(5)

Equation (5) is of the form

\[ax^2 + bx + c = 0\]

with \( x = \cos \theta, a = 1, b = \frac{v^2}{gL}, \) and \( c = -1 \). The solution to equation (6) is found from the quadratic formula:

\[x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\]

When \( v = 19.0 \text{ m/s}, b = 2.17 \). The positive root from the quadratic formula gives \( x = \cos \theta = 0.391 \). Substitution into equation (3) yields

\[T = \frac{mg}{\cos \theta} = \frac{(0.900 \text{ kg})(9.80 \text{ m/s}^2)}{0.391} = 23 \text{ N}\]

When \( v = 38.0 \text{ m/s}, b = 8.67 \). The positive root from the quadratic formula gives \( x = \cos \theta = 0.114 \). Substitution into equation (3) yields

\[T = \frac{mg}{\cos \theta} = \frac{(0.900 \text{ kg})(9.80 \text{ m/s}^2)}{0.114} = 77 \text{ N}\]

56. **CONCEPT QUESTIONS**  

a. The period for the second hand is the time it takes for it to go once around the circle or \( T_{\text{second}} = 60 \text{ s} \).

b. The period for the minute hand is the time it takes for it to go once around the circle or \( T_{\text{minute}} = 1 \text{ h} = 3600 \text{ s} \).

c. The relationship between the centripetal acceleration and the period can be obtained by using Equations 5.2 and 5.1 in the following way:
\[ a_c = \frac{v^2}{r} \quad (5.2) \quad \frac{v}{T} = \frac{2\pi r}{T} \quad (5.1) \]

\[ a_c = \frac{(2\pi r / T)^2}{r} = \frac{4\pi^2 r}{T^2} \]

**SOLUTION** Using the expression for the centripetal acceleration obtained in the answer to concept question c, we have

\[
\frac{a_{c, \text{second}}}{a_{c, \text{minute}}} = \frac{4\pi^2 r / T_{\text{second}}^2}{4\pi^2 r / T_{\text{minute}}^2} = \frac{T_{\text{minute}}^2}{T_{\text{second}}^2} = \frac{(3600 \text{ s})^2}{(60 \text{ s})^2} = 3600
\]

57. **CONCEPT QUESTIONS**

a. Example 2 has the smallest centripetal acceleration. Uniform circular motion on a circle with an infinitely large radius is like motion along a straight line at a constant velocity. In such a case there is no centripetal acceleration.

b. Example 1 has the greatest centripetal acceleration. According to Equation 5.2, the acceleration is \( a_c = \frac{v^2}{r} \). We note that the speed is in the numerator and the radius is in the denominator of this expression. Therefore, the greatest speed and the smallest radius, as is the case for Example 1, produce the greatest centripetal acceleration.

**SOLUTION** With Equation 5.2, we find the following values of the centripetal acceleration.

<table>
<thead>
<tr>
<th>Example</th>
<th>( a_c = \frac{v^2}{r} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Example 1</td>
<td>( \frac{(12 \text{ m/s})^2}{0.50 \text{ m}} = 290 \text{ m/s}^2 )</td>
</tr>
<tr>
<td>Example 2</td>
<td>( \frac{(35 \text{ m/s})^2}{\infty} = 0 \text{ m/s}^2 )</td>
</tr>
<tr>
<td>Example 3</td>
<td>( \frac{(2.3 \text{ m/s})^2}{1.8 \text{ m}} = 2.9 \text{ m/s}^2 )</td>
</tr>
</tbody>
</table>

58. **CONCEPT QUESTIONS**

a. The centripetal acceleration depends only on the speed \( v \) and the radius \( r \) of the curve, according to Equation 5.2 \( (a_c = \frac{v^2}{r}) \). The speeds of the cars are the same, and since they are negotiating the same curve, the radius is the same. Therefore, the cars have the same centripetal acceleration.
b. The centripetal force depends on the mass $m$, as well as the speed and the radius of the curve, according to Equation 5.3 ($F_c = \frac{mv^2}{r}$). Since the speed and the radius are the same for each car, the car with the greater mass, which is car B, experiences the greater centripetal acceleration.

**SOLUTION** Using Equations 5.2 and 5.3, we find the following values for the centripetal acceleration and force:

\[
\begin{align*}
\text{Car A} & \\
\quad a_c &= \frac{v^2}{r} = \frac{(27 \text{ m/s})^2}{120 \text{ m}} = 6.1 \text{ m/s}^2 \\
\quad F_c &= \frac{m_A v^2}{r} = \frac{(1100 \text{ kg})(27 \text{ m/s})^2}{120 \text{ m}} = 6700 \text{ N}
\end{align*}
\]

\[
\begin{align*}
\text{Car B} & \\
\quad a_c &= \frac{v^2}{r} = \frac{(27 \text{ m/s})^2}{120 \text{ m}} = 6.1 \text{ m/s}^2 \\
\quad F_c &= \frac{m_B v^2}{r} = \frac{(1600 \text{ kg})(27 \text{ m/s})^2}{120 \text{ m}} = 9700 \text{ N}
\end{align*}
\]

59. **CONCEPT QUESTIONS**

a. Static, rather than kinetic, friction provides the centripetal force, because the penny is stationary and not sliding relative to the disk.

b. The speed can be determined from the period $T$ of the motion and the radius $r$, according to Equation 5.1 ($v = \frac{2\pi r}{T}$).

**SOLUTION** Using Equation 5.3 for the centripetal force ($F_c = \frac{mv^2}{r}$) and Equation 5.1 for the speed in terms of the period ($v = \frac{2\pi r}{T}$), we have

\[
F_c = \frac{mv^2}{r} = \frac{m(2\pi r/T)^2}{r} = \frac{m4\pi^2 r}{T^2}
\]

According to Equation 4.7, the maximum force of static friction is $f_s^{\text{max}} = \mu_s F_N$, where $F_N$ is the normal force. Since the penny does not accelerate in the vertical direction, the upward normal force must be balanced by the downward-pointing weight, so that $F_N = mg$ and $f_s^{\text{max}} = \mu_s mg$. Using equation (1), we find
60. **CONCEPT QUESTIONS**

a. The satellite with the lower orbit has the greater speed, according to Equation 5.5 \( v = \sqrt{\frac{GM_E}{r}} \), where \( r \) is the radius of the orbit. Thus, satellite A has the greater speed.

b. In Equation 5.5 \( v = \sqrt{\frac{GM_E}{r}} \) the term \( r \) is the orbital radius, as measured from the center of the earth, not the surface of the earth. Therefore, you do not substitute the heights of \( 360 \times 10^3 \) m and \( 720 \times 10^3 \) m for the term \( r \). Instead, these heights must be added to the radius of the earth (\( 6.38 \times 10^6 \) m) in order to get the radii.

**SOLUTION**

First we add the orbital heights to the radius of the earth to obtain the orbital radii. Then we use Equation 5.5 to calculate the speeds.

**Satellite A**

\[
r_A = 6.38 \times 10^6 \text{ m} + 360 \times 10^3 \text{ m} = 6.74 \times 10^6 \text{ m}
\]

\[
v = \sqrt{\frac{GM_E}{r_A}} = \sqrt{\frac{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2 / \text{kg}^2)(5.98 \times 10^{24} \text{ kg})}{6.74 \times 10^6 \text{ m}}} = 7690 \text{ m/s}
\]

**Satellite B**

\[
r_A = 6.38 \times 10^6 \text{ m} + 720 \times 10^3 \text{ m} = 7.10 \times 10^6 \text{ m}
\]

\[
v = \sqrt{\frac{GM_E}{r_A}} = \sqrt{\frac{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2 / \text{kg}^2)(5.98 \times 10^{24} \text{ kg})}{7.10 \times 10^6 \text{ m}}} = 7500 \text{ m/s}
\]

61. **CONCEPT QUESTIONS**

a. Since the speed and mass are constant and the radius is fixed, the centripetal force is the same at each point on the circle.

b. When the ball is at the three o’clock position, the force of gravity, acting downward, is perpendicular to the string and cannot contribute to the centripetal force. (See Figure 5.21, point 2 for a similar situation.) At this point, only the tension of \( T = 16 \) N contributes to the centripetal force. Considering that the centripetal force is the same everywhere, we can conclude that it has a value of 16 N everywhere.

c. At the twelve o’clock position the tension \( T \) and the force of gravity \( mg \) both act downward (the negative direction) toward the center of the circle, with the result that the centripetal force at this point is \( -T - mg \). (See Figure 5.21, point 3.) The magnitude of the
centripetal force here, then, is \( T + mg \). At the six o’clock position the tension points upward toward the center of the circle, while the force of gravity points downward, with the result that the centripetal force at this point is \( T - mg \). (See Figure 5.21, point 1.) The only way for centripetal force to have the same magnitude of 16 N at both of these places is for the tension at the six o’clock position to be greater. The greater tension compensates for the fact that the force of gravity points away from the center of the circle.

**SOLUTION** Assuming that upward is the positive direction, we find at the twelve and six o’clock positions that

- **Twelve o’clock**
  \[ -T - mg = -16 \text{ N} \]
  \[ \text{Centripetal force} \]
  \[ T = 16 \text{ N} - (0.20 \text{ kg})(9.80 \text{ m/s}^2) = 14 \text{ N} \]

- **Six o’clock**
  \[ T - mg = 16 \text{ N} \]
  \[ \text{Centripetal force} \]
  \[ T = 16 \text{ N} + (0.20 \text{ kg})(9.80 \text{ m/s}^2) = 18 \text{ N} \]